NETWORK DESIGN EFFECTS OF DYNAMIC TRAFFIC ASSIGNMENT

By Bruce N. Janson

(Reviewed by the Urban Transportation Division)

ABSTRACT: This paper examines the effects of dynamic user-equilibrium (DUE) traffic assignment with scheduled trip arrival times on network design outcomes in comparison to outcomes with steady-state travel demands. The objective is to minimize systemwide travel cost by considering alternative link improvements to an existing network (e.g., select among budget-constrained subsets of link-improvement candidates). DUE is a temporal generalization of static user-equilibrium (SUE) assignment with additional constraints to insure temporally continuous trip paths and first-in first-out (FIFO) trip ordering between all origin-destination pairs. Previous research has not investigated the effects of dynamic travel demands and schedule delay (i.e., shifts by trips to earlier or later arrival times) on network design with multiple trip origins and destinations. DUE is formulated as a bilevel program of two subproblems solved successively by an iterative algorithm that consistently converges to solutions that closely satisfy the necessary optimality conditions of this problem. Examples show the impacts of alternative combinations of network changes affecting capacities and/or free-flow travel times (e.g., ramp metering or road widening) to depend on temporal travel demands and schedule delay distributions.

INTRODUCTION

Many variants of the network design problem can be formulated that differ in their objective function and constraints [see Magnanti and Wong, (1984)]. Network design problems involving static equilibrium travel models have been formulated and solved in previous research (Abdualla and LeBlanc 1979; Boyce and Janson 1979; Boyce 1984). One network design problem is to optimize system-wide performance while assigning traffic to the network according to steady-state or static user-equilibrium (SUE) principles. It is well known for that case that paradoxical or nonintuitive outcomes can occur in which travel supply enhancements, such as capacity expansions to some links, result in greater total travel costs [see LeBlanc (1975a)].

Arnot et al. (1992) show analytically that dynamic traffic flows can give rise to paradoxical assignment outcomes in networks where static flow paradoxes cannot occur. In “timeless” or SUE assignment, a supply increase cannot worsen total travel cost unless at least two used routes between the same O-D pair share at least one link. With the added dimension of time, trips compete for link use over time such that many time-differentiated routes exist along the same physical route between any O-D pair. Indeed, there exist an infinite “number of time-differentiated” routes between any O-D pair in continuous time. In discrete time, there exists a finite number of temporally continuous routes depending on the “fineness” of time intervals or slices. Hence, paradoxical outcomes can occur with dynamic traffic flows even on tree networks with a single source or sink. An example of such a case is given later in this paper.

Steinberg and Stone (1988) show that paradoxical outcomes can also occur when travel demand varies but network supply functions are not altered. Hence, network design decisions can be affected by patterns of travel demand as well as the manner in which trips are assigned to a network. Likewise, network design decisions can be affected by time-of-day changes in travel between origin-destination (O-D) pairs, even if travel demands are held fixed between these zones over the full analysis period. This paper examines the extent to network design and improvement decisions are affected in a few example problems by altering the arrival time schedules of trips in dynamic user-equilibrium (DUE) assignment. Of interest is whether unexpected changes in total travel time occur because of schedule delay changes or adjustments to the supply characteristics of critical network links such as bottlenecks. The outcomes reveal interesting and somewhat nonintuitive results.

Dynamic travel models were being researched over the same time as static equilibrium models and more recently (Merchant and Nemhauser 1978; Carey 1987; Van Aerde and Yagar 1988; Friesz et al. 1989; Wie 1991; Ram et al. 1993). Janson (1991a) presented a path-flow formulation
of DUE with multiple trip origins and destinations in which trips have scheduled departure times and variable arrival times (called DUE1). Janson (1991b) presented a link-flow formulation of DUE1 and a convergent solution algorithm based on decomposing DUE1 into two subproblems. Janson (1992a) presented the temporal opposite of DUE1 in which trips have variable departure times and scheduled arrival times (called DUE2).

In DUE, the full assignment period of several hours is discretized into shorter time intervals of 1–10 min. For each trip is known its departure or arrival time (but not both), and its corresponding trip-end zones. With variable travel times, both the departure and arrival time cannot be fixed for any single trip. Journey-to-home models may have fixed departure times and variable arrival times, while journey-to-work models may have variable departure times and fixed arrival times. DUE1 assumes known departure times for trips from each zone, but only total trip arrivals to each zone over the full analysis period. DUE2 assumes known arrival times of trips to each zone, but only total trip departures from each zone over the full analysis period.

Both DUE1 and DUE2 can be combined into dynamic equilibrium models of many different forms, such as with trip distribution or elastic demand [see Janson (1992b)]. Janson and Southworth (1992) describe the estimation of trip departure times from a peak-period trip matrix using traffic counts and dynamic assignment. The models can also be formulated with departure and arrival times depending on departure and/or arrival time costs [see Janson and Robles (1993)], and path flow formulations of these models allow route choice criteria beyond travel time alone [see Janson (1992c)]. This paper examines the network design effects of alternative schedule delay distributions in DUE2, which represent different levels of peak-period spreading and time-varying travel demands. Schedule delay is defined here as the amount of time by which an arrival is shifted earlier or later from its "desired" arrival time, perhaps in response to congestion reduction policies that induce work schedule changes.

Regarding the solution approach, SUE can be solved efficiently with linear combination methods because of its convex solution space and unimodal objective function. Whereas steady-state flows in SUE allow it to be formulated with all linear constraints, DUE2 has nonlinear mixed-integer constraints with "node time intervals" needed to insure temporally continuous trip paths across multiple time intervals. The link flow form of DUE2 presented here is a bi-level program of two interdependent subproblems solved successively by an iterative solution algorithm. The algorithm described later solves the upper (nonlinear) subproblem by method of linear combinations to find dynamic link volumes, and then the lower (linear) subproblem to find temporally continuous node time intervals. This iterative process continues until sufficient convergence is obtained.

**DYNAMIC USER-EQUILIBRIUM WITH SCHEDULED ARRIVALS (DUE2)**

DUE2 as formulated and solved herein to study the network design effects of dynamic traffic assignment is defined as follows: Given a set of zone-to-zone trip tables with the number of vehicle trips arriving at each destination in successive time intervals of 1–10 min each, and the origin zone but not the departure time of each trip, determine the volume of vehicles on each link in each time interval such that no used path has a higher travel impedance than any other path. At equilibrium, each trip departs from its origin in a time interval and on an equal travel time path that reaches its destination within a scheduled arrival time interval. Note that trips arriving within a given time interval to any node or zone do not arrive at a single point in time. For example, trips arriving from 7:00 to 7:10 a.m. are assigned to the network such that they arrive over the 10 mins between these times. Thus, trips within each time interval are distributed uniformly (both spatially and temporally) across each time interval.

The foregoing condition for dynamic user equilibrium with fixed arrival times is a temporal generalization of Wardrop's (1952) condition for static user-equilibrium (SUE) as will be derived later from the DUE2 formulation shown next. The phrase "arriving at the destination" can be replaced by "departing from the origin" for the case of DUE1 with fixed departure times and variable arrival times as formulated and solved by Janson (1991b). Note that SUE is simply a special case of DUE (more specifically, of the upper subproblem of DUE) with one long assignment period.

DUE2 can be stated equivalently in terms of path flows, but the link flow form shown here does not implicitly assume complete enumeration of all paths between zone pairs. Turn movements at each intersection can be represented by separate links at each node. The exact form of each link's impedance function can be specific to the intersection or link type. The O-D trip matrix can be developed from traffic counts or from survey data and trip distribution models. In DUE2, stated by (1)–(12), link lengths are computed on the basis of monotonically non-decreasing impedance functions of each link's volume in each time interval.

\[
\text{(UP) Minimize } \sum_{i \in K} \sum_{m \in T} \int_{a}^{b} f_{ij}(w) \, dw
\]
subject to

\[ x_{ij}^t = \sum_{s \in Z} \sum_{d} v_{ij}^{s,t} \] \quad \text{for all } i,j \in K; \ t \in T \tag{2} \]

\[ q_{nt}^d = \sum_{i,j \in K} v_{ij}^{s,t} \alpha_{nt}^d - \sum_{i \in n} v_{in}^{s,t} \alpha_{nt}^d \] \quad \text{for all } n \in N; \ s \in Z ; \ d \in T \tag{3} \]

\[ v_{ij}^{s,t} \geq 0 \] \quad \text{for all } s \in Z; \ i,j \in K; \ d \in T; \ t \in T \tag{4} \]

where all \{\alpha_{nt}^d\} are optimal for

(LP) Maximize \[ \sum_{s \in Z} \sum_{i \in N} \sum_{d \in T} b_{it}^d \] \tag{5} \]

subject to

\[ \alpha_{nt}^d = (0, 1) \] \quad \text{for all } s \in Z; \ i \in N; \ d \in T; \ t \in T \tag{6} \]

\[ \sum_{r \in T} \alpha_{nt}^d = 1 \] \quad \text{for all } s \in Z; \ i \in N; \ d \in T \tag{7} \]

\[ b_{it}^d = \max \{e_{it}^d, b_{it}^{d-1} - (1-h)\Delta t\} \] \quad \text{for all } s \in Z; \ i \in N; \ d \in T \tag{8} \]

\[ x_{ij}^t \quad \text{(variable)} \]

\[ v_{ij}^{s,t} = \text{number of vehicle trips arriving at zone } s \text{ in time interval } d \text{ assigned to link } ij \text{ in time interval } t \] \quad \text{(variable)} \]

\[ f_{ij}(x_{ij}^t) = \text{average travel impedance on link } ij \text{ in time interval } t \] \quad \text{(variable)} \]

\[ q_{nt}^d = \text{number of vehicle trips from node } n \text{ to zone } s \text{ arriving in time interval } d \text{ via any path} \]

\[ \text{zero for any node } n \notin Z \text{ (fixed)} \]

\[ e_{it}^d \] \quad \text{(variable)} \]

\[ b_{it}^d \quad \text{(variable)} \]

\[ \alpha_{nt}^d \] \quad \text{(variable)} \]

\[ \Delta t \] \quad \text{(parameter)} \]

\[ h \] \quad \text{(parameter)} \]

\[ J \] \quad \text{(parameter)} \]

\[ K \] \quad \text{(parameter)} \]

\[ N \] \quad \text{(parameter)} \]

\[ T \] \quad \text{(parameter)} \]

\[ x_{ij}^t \] \quad \text{(variable)} \]

\[ v_{ij}^{s,t} \] \quad \text{(variable)} \]

\[ f_{ij}(x_{ij}^t) \] \quad \text{(variable)} \]

\[ q_{nt}^d \] \quad \text{(variable)} \]

\[ e_{it}^d \] \quad \text{(variable)} \]

\[ b_{it}^d \] \quad \text{(variable)} \]

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\[ e_{it}^d \] \quad \text{(variable)} \]

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\[ q_{nt}^d \] \quad \text{(variable)} \]

\[ e_{it}^d \] \quad \text{(variable)} \]

\[ b_{it}^d \] \quad \text{(variable)} \]

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\[ e_{it}^d \] \quad \text{(variable)} \]

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\[ h \] \quad \text{(parameter)} \]

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\[ K \] \quad \text{(parameter)} \]

\[ N \] \quad \text{(parameter)} \]

\[ T \] \quad \text{(parameter)} \]
Albeit counterintuitive, the maximization of node-to-zone travel times in subproblem (LP) is the correct determination of shortest path travel times and node time intervals. The node-to-zone travel times \( b_{ij}^d \) correspond to shortest travel time trees with destinations as roots. If every \( b_{ij}^d \) is not maximized, then temporally discontinuous node-to-zone travel times and node time intervals can result. The shortest path problem is often formulated to find unit link flows that minimize the use of arc lengths. Maximizing node-to-zone travel times subject to arc lengths plus FIFO delay in (9) is the dual of that shortest path problem. Using the mechanical analogy of Minty (1957), Bertsekas (1991) defines this primal-dual relationship as the min-path/max-demand theorem.

Eq. (6) defines each node time interval \( \alpha_{ij}^d \) as 0 or 1, which dictates the time interval \( t \) in which arcs incident from node \( i \) are used by trips arriving at zone \( s \) in time interval \( d \). The assumption here is that any link \( ij \) incident from node \( i \) can be used (if at all) only in the time interval \( t \) in which node \( i \) is crossed by trips arriving at zone \( s \) in time interval \( d \). Eq. (7) allows only one time interval \( t \) in which trips arriving at zone \( s \) in time interval \( d \) can cross node \( i \). Note that, although the time interval indices \( t \) and \( d \) are dependent on arrival times, they still advance forward in time such that higher values of \( t \) indicate later times of day. Eqs. (8)–(12) force each time interval \( t \) and node time interval \( \alpha_{ij}^d \) to define trip paths that are temporally continuous with the node-to-zone travel times. Eqs. (8) and (9) also impose first-in-first-out (FIFO) requirements on all O-D trips according to link travel times in successive time intervals as explained next. An equivalent assumption when dealing with aggregate vehicle flows is that vehicles make only one-for-one (or zero-sum) exchanges of traffic positions along any link, which is quite acceptable and even expected in aggregate traffic models.

Eq. (8) is a vehicle-following constraint that prevents later trips from “getting too close” to trips arriving earlier at the same zone in successive time intervals so as to prevent these trips from bunching. The value \( h \) equals the fraction of time interval \( \Delta t \) that trips arriving at zone \( s \) in time interval \( d \) must follow trips arriving at zone \( s \) in interval \( d - 1 \). While not equal to vehicle headway, \( h\Delta t \) can be viewed as the least allowable separation of comparable points (i.e., heads, tails, midpoints) of successive trip streams with common destinations. When solving for \( b_{ij}^d \) on the left-hand side of (8), \( b_{ij}^{d-1} \) on the right-hand side is fixed.

Fig. 1 illustrates the effect of constraint (9) on two trip streams (1 and 2) traversing the same series of nodes and arriving at the same zone in intervals \( d \) and \( d + 1 \), respectively. The node sequence (A,B,C,D,E) denotes a series of links along the trip path. A “trip stream” consists of trips with the same destination and arrival time interval. At node A, trip stream 2 is 0.8\( \Delta t \) behind trip stream 1. The two trip streams traverse node A in different time intervals and thus have different travel times for link (A,B). At node B, the separation between the trip streams has decreased to 0.6\( \Delta t \). The two trip streams traverse node B in different time intervals and have different travel times for link (B,C). Without constraint (8), stream 2 would pass stream 1 and traverse node C earlier. With constraint (8), stream 2 is forced to traverse node C at least \( h\Delta t \) behind stream 1, where \( h = 0.5 \) in Fig. 1.

Continuing in Fig. 1, trip streams 1 and 2 traverse node C in different time intervals and have different travel times for link (C,D). Trip stream 2 lost ground on stream 1 in traversing link (C,D) such that constraint (8) is no longer binding at node D for the two trip streams. Stream 2 gains some ground on stream 1 in traversing link (D,E), and constraint (8) may again become binding at a later node. Other examples might have these two trip streams traversing any of these nodes within the same time interval. If \( h \) equals 0.5, then trailing trips will have node time intervals one later than those of leading trip streams roughly one-half the number of instances that constraint (8) is binding. Allowing that many other paths include the node sequence

**Fig. 1.** Effect of Constraint (8) on Successive Trip Intervals

**Fig. 2.** Effect of Constraint (9) on FIFO Order of All O-D Trips

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(A,B,C,D,E), it can be deduced from Fig. 1 that constraint (8) is less likely to be binding in networks with shorter arc lengths relative to \( \Delta t \). Hence, this representation works best for networks in which most arc lengths have free-flow travel times less than 20% of the interval duration, and in which loadings on the network do not cause arc lengths to exceed \( h \Delta t \). Example runs revealed the solution algorithm explained later to converge more easily if time-varying travel demands do not cause arc travel times to exceed these bounds.

Although constraint (8) prevents successive trip streams from passing each other, the exact specification of \( h \) can be improved in further research. Reasonable values of \( h \) lie between 0.3\( \Delta t \) and 0.7\( \Delta t \), but the exact value of \( h \) depends on traffic densities of arcs incident to a node in each time interval. The value of \( h \) must lie between 0 and 1. If \( h = 0 \), a trailing trip stream can completely overlay (but not overtake) a leading trip stream such that the two streams become coincident, which is not realistic. If \( h = 1 \), then trailing trips can never partly "gain ground on" leading trips, and travel times of later arriving trips can never be less than travel times of earlier arriving trips.

Since constraint (8) also applies to trips arriving in intervals \( d + 1 \) and \( d + 2 \), trips arriving at zone \( s \) in interval \( d + 2 \) must follow trips arriving in interval \( d \) by at least \( 2h \Delta t \) at any node. Note that constraint (8) also applies to all nodes in the network regardless of whether any trips arriving at zone \( s \) in intervals \( d \) and \( d + 1 \) actually cross node \( i \). However, since (8) does not enforce the FIFO order of trips between all O-D pairs, (9) is also required. It follows from a discussion by Carey (1992) of FIFO queuing in dynamic assignment that (13) exactly ensures the FIFO order of all trip arrivals between all O-D pairs. However, as Carey (1992) suggests, the interdependence of O-D travel times in (13) makes its shortest path solution depend on which paths take precedence or are given greater weight.

\[
[e_{ij}^s - \max\{b_{ij}^s, \ \max(b_{ij}^s \alpha_{ij}^s, \ \text{for all } s \in Z, \ \delta \in T) + \Delta f_{ij}^s \} \alpha_{ij}^s \leq f_{ij}^s(x_{ij}) \alpha_{ij}^s; \quad \text{for all } s \in Z; \ \delta \in K; \ \delta \in T; \ t = t + 1; \ \Delta f_{ij}^s = f_{ij}^s(x_{ij}) - f_{ij}^s(x_{ij}) \]  \tag{13}

Because of the solution complication with (13), (9) shown in DUE2 is used instead as a very close substitute. Eq. (9) is more conservative than (13) meaning that it requires slightly more FIFO delay than exactly required by (13) in close situations. Fig. 2 illustrates the effect of constraint (9) on trip stream 2 and 2 between any two zones traversing the same series of nodes arriving in any two intervals.

Without constraint (9), trip stream 2 would reach and depart from node C in time interval 3 with a faster travel speed than trip stream 1 such that it passes stream 1, which departs from node C in intervals with a slower travel speed. With constraint (9), trip stream 2 must depart from node C at least \( \Delta f_{ij}^s \) into time interval 3 and reach node D no earlier than trip stream 1. This FIFO delay may be more than exactly required by (13) to the extent that trip stream 1 departs from node C more than a second before the start of interval 3. If trip stream 1 departs from node C more than \( \Delta f_{ij}^s \) before that start of interval 3, then the constraint is nonbinding. The excess is likely to diminish as the number of O-D pairs increases because some O-D path is likely to reach node C or very close to the start of time interval 3. As shown by Kaufman and Smith (1993), the FIFO constraint (9) is easily added to shortest path label-correcting algorithms (but not label-setting algorithms) so long as labels are properly updated when it occurs.

According to (10)–(12), links are traversed within the time intervals that trip paths cross their tail nodes. For 10-min intervals, interval 1 begins at 0, interval 2 begins at 10 min, interval 3 begins at 20 min, etc. If travel time from node \( i \) to zone \( s \) is within \( d - t + 1) \Delta t \), then \( \alpha_{ij}^s \) equals 1, since these arcs must cross zone \( s \) in time interval \( t \). If any path crosses a node at the exact start of a time interval (to the degree of floating-point precision being used), then the solution algorithm can be coded to have the path use the link in that time interval rather than in the previous interval. In (12), each intrazonal travel time \( b_{ij}^s \) must be set to zero or another fixed value (not necessarily the same for each zone) so as to prevent their maximization in (LP) from having an infinite solution.

Although DUE is nonconvex over the domain of feasible node time intervals for all trip arrivals to all destinations, DUE2 is convex with a unique global optimum for any given set of fixed node time intervals. The optimality conditions of DUE2 stated in the paper’s first section can be derived from (UP) for a given set of node time intervals as given by an optimal solution to (LP) [see Janson and Robles (1993)]. Subproblem (LP) is a shortest path linear program for which there exists an optimal solution for any given solution to (UP). Any set of node time intervals resulting from (LP) defines a directed network for which (UP) is a convex nonlinear program with a global optimum solution. Since node time intervals resulting from (LP) are uniquely determined by a given set of link volumes resulting from (UP), they can be assumed to be known in the derivation of optimality conditions for DUE2. Thus, the optimality conditions of (UP) are for a given set of node time intervals to which all temporally continuous trip paths in the optimal solution must conform.
CONVERGENT DYNAMIC TRAFFIC ASSIGNMENT ALGORITHM

Whereas SUE can be solved quite efficiently by linear combination methods for nonlinear programs with all linear constraints (e.g., Frank-Wolfe and PARTAN), these methods can easily create temporally discontinuous flows if applied directly to DUE2. Instead, the two subproblems of DUE2 are solved successively by a convergent dynamic algorithm. The algorithm first solves (UP) with fixed node time intervals using the Frank-Wolfe (F-W) method of linear combinations (or a similar technique), and then solves (LP) (a linear program) to update all node time intervals for the next F-W solution of (UP). The algorithm terminates when fewer than an acceptable number of node time intervals change from one solution of (LP) to the next. Example results presented here and in previous papers (Janson 1991b, 1992a, 1192b) show this algorithm to consistently converge to solutions that closely satisfy the necessary optimality conditions of these problems.

To clarify, the following steps are performed successively to solve subproblems (UP) and (LP) to near convergence with this algorithm:

1. Input all network data, temporal trip arrival matrices, and initial link flows. Initial link flows are optimal, and can be set to zero, but SUE link flows reduced to the chosen time interval duration may be good starting values. Calculate initial node time intervals by solving (LP) with initial link flows. Set iteration counter \( n = 0 \).
2. Increment iteration counter \( n = n + 1 \).
3. (UP) Minimize (1) subject to (2)–(4), where all \( x_i^t \) are variable and all \( \alpha_{ij}^t \) are fixed to their optimal values from (LP).
4. (LP) Maximize (5) subject to (6)–(12), where all \( \alpha_{ij}^t \) are variable and all \( x_i^t \) are fixed to their optimal values from (UP).
5. Sum NDIFFS = total number of node time interval differences between iterations \( n - 1 \) and \( n \). Compare each \( (\alpha_{ij}^t)^n \) to \( (\alpha_{ij}^t)^{n-1} \). If NDIFFS < \% of all node time intervals \([Z(n - 1)/T]\), then STOP. Otherwise, return to step 2.

With fixed-node time intervals, subproblem (UP) is solved without fixing which links are used but only fixing the time intervals in which links are used by trips depending on their destinations and arrival times. Subproblem (LP) is solved with a label-correcting shortest-path algorithm adapted for temporally dependent arc lengths, which will correctly find temporally continuous shortest paths given dynamic arc lengths with the restriction that all vehicle streams maintain FIFO ordering and do not pass each other along any link as explained earlier.

The algorithm converges toward an equilibrium solution for the following reasons. First, if node time intervals corresponding to the true equilibrium are known, then solving (UP) will reproduce the equilibrium link volumes from which these node time intervals can be calculated. That convergence proof follows from the fact that any set of node time intervals resulting from (LP) defines a directed network for which (UP) is a convex nonlinear program for which a global optimum exists. Second, given node time intervals that do not correspond to a true dynamic equilibrium, then solving (UP) with the F-W algorithm will produce link volumes that shift the node time intervals toward their correct values. For example, if a node time interval is too early, then solving (UP) will assign more traffic to paths leading to that node such that the node time interval is shifted later when recalculated in (LP). Oppositely, if a node time interval is too late, then solving (UP) will assign less traffic to paths leading to that node such that the node time interval is shifted earlier when recalculated in (LP). Hence, the algorithm converges toward a set of node time intervals that when used to assign trips to the network in solving (UP) result in temporal link volumes that give rise to the same node time intervals when recalculated in (LP).

Although many bilevel programming problems are not globally convex (such as DUE2, except with steady-state flows), solving some bilevel programs to near optimality is less difficult when the (LP) solution pushes the (UP) solution toward a global optimum rather than away from it. In DUE2, link travel times (first-order gradients of UP) shift the node time intervals found by solving (LP) in the proper directions. Perfect convergence of the algorithm to optimal solutions with temporally continuous trip paths is not assured in all cases, but close convergence has been obtained in all test applications.

Perfect convergence of the algorithm to steady-state flows with uniform trip arrivals over all time intervals always occurs rapidly regardless of initial link flows. In other cases, convergence difficulty increases as travel demands vary more greatly over time and/or high volumes cause many loaded link lengths to approach or exceed one-half the time interval duration. An approximate rule of thumb mentioned earlier is to keep most free-flow link lengths less than 20% of the time interval duration. An acceptable degree of convergence was obtained in each of the examples given next. Hence, although convergence is not definitely assured, the algorithm appears to be an efficient approach to solving DUE2.

How well the assignment for each set of arrival rates satisfies dynamic UE conditions must
be assessed. A standard UE measure is the duality gap (DG), which for a dynamic assignment is the difference between assigned trip impedances and shortest path impedances based on assigned link loadings for trips with the same O-D pairs and arrival or departure times. The time dimension of the duality gap defined by (14) can be disregarded for a static assignment with only one time period.

\[
\text{DG} = -100 + (100/\text{LB}) \sum_{a \in K} \sum_{i \in T} x_i f_a(x_i)
\]

(14)

where DG = duality gap of a dynamic assignment; \( q^d_{rs} \) = number of trips from zone r to zone s arriving in time interval d via any path; \( b^d_{rs} \) = shortest path impedance (here equal to travel time) from zone r to zone s for trips arriving in time interval d through network of assigned link loadings; and LB = total trip impedance if all trips have minimum impedances through network of assigned link loadings = \( \sum_{i \in T} \sum_{r \in Z} \sum_{s \in Z} q^d_{rs} b^d_{rs} \).

DG is the difference between the objective functions of the primal and dual formulations of the linear program solved in each iteration of the F-W algorithm used to solve subproblem (UP) [see Hearne (1982)]. The summation term in (14) is the total trip impedance of the assigned flows, and LB is a strict lower bound on the (UP) objective function after each F-W iteration when solving subproblem (UP) with fixed node time intervals. DG decreases toward zero (not strictly monotonically) as the F-W algorithm converges toward the solution of (UP). DG equals zero for a true equilibrium solution in which the impedance of every used path between each pair of zones equals the shortest path impedance. LB is not a strict lower bound on the optimal (UP) objective function value if based on an infeasible solution with temporally discontinuous paths. Hence, the number of node time interval changes between (LP) solutions, which decreases towards zero as the solution converges, is used as the stopping criterion for the solution algorithm. The algorithm converged rapidly to a duality gap of less than 5% for each of the DUE2 solutions presented next.

EFFECTS OF LINK CAPACITY CHANGES AND ARRIVAL DISTRIBUTIONS

This section illustrates the network design effects of link capacity changes on DUE2 solutions with different schedule delay distributions for morning peak-period work trips. In these examples, alternative trip arrival distributions applied to the entire peak-period trip matrix are used to represent peak-period spreading due to flextime work schedules, staggered work hours, or telecommuting for partial days so as to commute at less congested hours. Of interest is whether unexpected changes in total travel time occur because of schedule delay changes or adjustments to the supply characteristics of critical network links such as bottlenecks. The outcomes reveal some interesting and somewhat nonintuitive results.

Network performance results are compared next for a range of trip arrival rate distributions ranging from steady-state travel demands (i.e., uniform trip arrivals) to a steeply peaked distribution in which trip arrival rate varies by 100% over a 2.5-hr analysis period containing fifteen 10-min intervals. Time intervals 6–11 represent the peak hour, with additional intervals at each end of the period to establish a base level of travel demand that uses the

![Graph](image-url)

**FIG. 3.** Trip Arrival Rate Distributions Used in Examples

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TABLE 1. Trip Arrival Rate Distributions Used in the Examples

<table>
<thead>
<tr>
<th>Time interval (a.m.)</th>
<th>Distribution #1</th>
<th>Distribution #2</th>
<th>Distribution #3</th>
<th>Distribution #4</th>
<th>Distribution #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 6:40–6:50</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>2 6:50–7:00</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>3 7:00–7:10</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>4 7:10–7:20</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>5 7:20–7:30</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>6 7:30–7:40</td>
<td>8.25%</td>
<td>8.33%</td>
<td>8.50%</td>
<td>8.25%</td>
<td>8.33%</td>
</tr>
<tr>
<td>7 7:40–7:50</td>
<td>10.50%</td>
<td>10.00%</td>
<td>9.75%</td>
<td>9.25%</td>
<td>8.33%</td>
</tr>
<tr>
<td>8 7:50–8:00</td>
<td>12.50%</td>
<td>11.67%</td>
<td>10.75%</td>
<td>10.00%</td>
<td>8.33%</td>
</tr>
<tr>
<td>9 8:00–8:10</td>
<td>12.50%</td>
<td>11.67%</td>
<td>10.75%</td>
<td>10.00%</td>
<td>8.33%</td>
</tr>
<tr>
<td>10 8:10–8:20</td>
<td>10.50%</td>
<td>10.00%</td>
<td>9.75%</td>
<td>9.25%</td>
<td>8.33%</td>
</tr>
<tr>
<td>11 8:20–8:30</td>
<td>8.25%</td>
<td>8.33%</td>
<td>8.50%</td>
<td>8.25%</td>
<td>8.33%</td>
</tr>
<tr>
<td>12 8:30–8:40</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>13 8:40–8:50</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>14 8:50–9:00</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>15 9:00–9:10</td>
<td>6.25%</td>
<td>6.67%</td>
<td>7.00%</td>
<td>7.50%</td>
<td>8.33%</td>
</tr>
<tr>
<td>For intervals 3–14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution sum</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Mean delay (min)</td>
<td>0.00</td>
<td>0.93</td>
<td>1.75</td>
<td>2.75</td>
<td>4.60</td>
</tr>
<tr>
<td>Percent trips delayed*</td>
<td>0.0%</td>
<td>9.3%</td>
<td>17.5%</td>
<td>27.5%</td>
<td>46.0%</td>
</tr>
</tbody>
</table>

Note: Mean delay equals the average minutes that trips arrive before or after their “desired” arrival times given by distribution #1.
*Percent trips delayed equals percent of all trip arrivals shifted earlier or later by one 10-min interval from distribution #1.

network concurrently with peak-hour trips. Fig. 3 and Table 1 show the five trip-arrival distributions used in the examples.

Each distribution of trip arrivals sums to 100% over the 2-hr period between intervals 3 and 14. Only trips arriving over this 2-hr period are included in comparisons of average travel times in the later examples so that equal numbers of trips are represented in each average. Each arrival rate distribution for these intervals is symmetrical about the same mean arrival time of 8 a.m. (interval 8.5), meaning that equal numbers of commuters switch to earlier and later arrival times as the distribution shifts from most peaked (#1) to uniform arrival rates (#5). Other examples can be skewed towards earlier or later arrivals, implying that commuters have unequal preferences for early or late schedule delays.

Mean schedule delay for each distribution equals the average number of minutes that trips arrive before or after their “desired” arrival times, taken here to be distribution #1 for example purposes. For these distributions, mean schedule delay can also be calculated as the percent of all trip arrivals shifted earlier or later by one 10-min interval from arrival times #1, multiplied by 10 min, as shown at the bottom of Table 1. Thus, “percent trips delayed” indicates how many trips must be schedule delayed to earlier or later arrival times by exactly 10 min (or one time interval) for a given amount of peak-period spreading. Other combinations of individual trip adjustments to distribution #1 (e.g., delaying some arrivals by 20–30 min and others by none) can also be made to obtain the other distributions.

Two Bottlenecks in Series along Freeway

Arnott et al. (1993) show analytically that dynamic traffic flows can give rise to paradoxical assignment outcomes in networks where static flow paradoxes cannot occur. Fig. 4 shows a network similar to the one in their paper with two bottlenecks, except that this example network has one origin and two destinations, instead of the other way around. In SUE assignment, a supply increase cannot worsen total travel cost unless at least two used routes between the same O-D pair share at least one link. Here, there is only one route from node 1 to node 7, and one route from node 1 to node 8. Thus, a capacity expansion to bottlenecks A, B, or both, can only decrease total or average travel time with SUE assignment.

Fig. 5 shows mean trip times of static SUE and DUE2 assignments “before” and “after” the capacity of bottleneck B was expanded from 1,500 to 2,500 vehicles per hour (vph), an increase of 67%. This capacity expansion decreased the SUE mean trip time from 42.8 to 42.0 min. The SUE assignments were found by applying trip arrival rates #5 to a total of 2,400 trip arrivals to node 7 and 2,400 trip arrivals to node 8 over a 2-hr period. The SUE results were also confirmed by solving SUE with a static assignment code for both the before and after cases. As shown in Fig. 4, a travel time function of the typical BPR (Bureau of Public Roads) form was used, except that the volume/capacity ratio was multiplied by 0.85, which is often used for level-of-service (LOS) E capacities as assumed here. Any monotonically nondecreasing travel time function of reasonable form can be used, and can be specific to link types.

Fig. 5 shows mean trip times of DUE2 assignments using arrival rates #1 (most peaked) for
4800 trips $\rightarrow$ 1 $\rightarrow$ 2 $\rightarrow$ 3 A $\rightarrow$ 4 $\rightarrow$ 5 $\rightarrow$ 6 B $\rightarrow$ 8 $\rightarrow$ 2400 trips
over 2 hours

<table>
<thead>
<tr>
<th>Link Data</th>
<th>Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>FN</td>
<td>TN</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Expansion increases capacity of link (5,6) from 1500 to 2500 vph.

Link Travel Time = FFTT [ 1.0 + 0.85 (Volume/CAP)$^4$ ]

FIG. 4. Two Bottlenecks in Series along Freeway

FIG. 5. Mean Trip Times for Two Bottlenecks in Series

the same two cases of bottleneck B before and after expansion. For these dynamic assignments, the mean trip time increased from 46.3 min before expansion to 47.1 min after expansion. Comparing mean trip times across arrival time intervals shows that expanding bottleneck B shifted congestion upstream to bottleneck A. Expanding bottleneck B creates greater delays for later trips at bottleneck A, and higher mean trip times persist longer (while queues dissipate) before eventually falling below the unexpanded case. Trips of 40–60 min duration affect congestion levels on the network four to six intervals ahead of their arrival times. As a result, no trips over the 2.5-hr period gained any significant decrease in travel time due to capacity expansion.

Similar results occur for trip arrival rates #2–#4, but not so as dramatically as for #1. Arnott et al. (1993) define ranges of supply and demand over which total travel time will increase (given their assumptions) when an upstream bottleneck is expanded in network with two origins and one destination. These results show that it can also occur when a downstream bottleneck is expanded in network with one origin and two destinations.

Effects of Shortcut Link on Two-Path Network

Since the UE objective (static or dynamic) is not to minimize total travel cost [which is the system optimal (SO) objective], it is not wholly unexpected that constraint relaxation in the form capacity expansion does not necessarily reduce total travel cost. The outcome of worse overall network performance contradicts our intuition that more supply cannot make the average condition of all travelers worse no matter how that supply is used. Braess-type paradoxes can
occur in many varied situations, although the network configuration and link performance functions must have certain relationships for such paradoxical outcomes to occur [see Steinberg and Zangwill (1983)].

Steinberg and Stone (1988) show analytically that changes in travel demand can also cause paradoxical outcomes. Because of nonlinear congestion effects, more steeply peaked arrival rates (for a given total travel demand) have the effect of greater travel demand and cause greater mean trip times. Gains in travel time achieved by lower-than-uniform demand in early and late arrival intervals are more or less offset by higher-than-uniform demand in middle arrival intervals plus the queuing delays that these higher demands create.

Fig. 6 shows a two-path network between a single O-D pair where adding a “shortcut” link from node 2 to node 3 can cause the classic Braess paradox to occur. LeBlanc (1975) and Steinberg and Stone (1988) both use similar networks for their examples. As in the previous example, the volume/capacity ratio was multiplied by 0.85 in each link’s travel time function. For the link supply attributes shown in Fig. 6, Braess’ paradox [i.e., a higher mean trip time when link (2,3) is added] occurs for any SUE assignment below 4,000 vph (8,000 vehicles over 2 hr), but not in any SUE assignment above 4,000 vph. The paradox does not occur for any SUE assignment below 2,500 vph. The largest paradoxical difference in mean trip times occurs at 2,800 vph (not shown on graph)—the demand below which nearly all trips use the path containing link (2,3) if available.

Of further interest is whether any DUE2 assignments will cause the paradox to occur. Fig. 7 shows that for trip arrival rates #3, the paradox occurs for any DUE2 assignment below 7,350 vehicles over 2 hr, but not in any DUE2 assignment above 7,350 vehicles over 2 hr. The DUE2

![Diagram of network and key data]

Network improvement adds link (2,3) with the above parameters.

\[
\text{Link Travel Time} = \text{FFTT} \left[ 1.0 + 0.85 \left( \frac{\text{Volume}}{\text{CAP}} \right)^4 \right]
\]

**FIG. 6. Two-Path Network for Adding Shortcut Link**

![Graph showing trip times for different networks]

**FIG. 7. Mean Trip Times for Two-Path Network**
mean trip times always exceed the corresponding SUE mean trip times at each level of travel demand, except that the static and dynamic cases with link (2, 3) added have equal mean trip times at 6,400 vehicles over 2 hr. Note that in the range of 7,550 to 8,000 vehicles over two hours, adding link (2, 3) to the network increases mean trip time with SUE assignment, but decreases mean trip time with DUE2 assignment.

Effects of Multiple Link Improvements in Larger Network

The network design analysis presented by LeBlanc (1975) for SUE assignment is now revisited with DUE2 assignment. The Sioux Falls network (with 24 nodes, 76 one-way links, 24 zones, and 552 O-D pairs with positive demand) is well known and used frequently for example purposes. When solving DUE2 for the Sioux Falls network, the entire algorithm was halted when the number of node time interval changes between (LP) solutions was less than 3% as a percentage of total node time intervals equal to \(Z(N-1)T\). In addition, each (UP) subproblem was halted when the greatest single link volume change was less than 3% between F-W iterations. Rose et al. (1988) found final SUE link volumes for Sioux Falls to have less than a 0.5% coefficient of variation between solutions when they applied the 1% link volume change stopping criterion to the F-W algorithm with different starting solutions.

Table 2 lists the unimproved and improved parameters for five links in the Sioux Falls network, where improvements always apply to both directions of a link. Each example was run using the aforementioned five arrival-rate distributions. Eight link improvement subsets can fit within the budget constraint of $3,000,000 used by LeBlanc (1975). The eight link improvement subsets and their combined capital costs are also listed in Table 2. Fig. 8 shows the Sioux Falls results for arrival rates #1–#5 applied to twice the trip matrix given by LeBlanc (1975), assuming that trip matrix to represent 1 hr. As did LeBlanc (1975), the standard BPR travel time function was used with the volume/capacity ratio multiplied by 0.15, as was meant for LOS C capacities instead of 0.85 used earlier.

Points connected in Fig. 8 represent the five arrival rate distributions, where each point along a line is the mean trip time for a different subset of link improvements. (Line segments between points do not imply that mix of link improvement subsets are possible.) Fig. 8 shows that

<table>
<thead>
<tr>
<th>Link number</th>
<th>Directed arc pair</th>
<th>Unimproved FF-TT (hr)</th>
<th>Cap (vph)</th>
<th>Improved FF-TT (hr)</th>
<th>Cap (vph)</th>
<th>Improve cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(6, 8) and (8, 6)</td>
<td>0.0217</td>
<td>5.0</td>
<td>0.0135</td>
<td>6.0</td>
<td>650,000</td>
</tr>
<tr>
<td>2</td>
<td>(7, 8) and (8, 7)</td>
<td>0.0250</td>
<td>7.5</td>
<td>0.0155</td>
<td>9.0</td>
<td>1,000,000</td>
</tr>
<tr>
<td>3</td>
<td>(9, 10) and (10, 9)</td>
<td>0.0275</td>
<td>13.5</td>
<td>0.0162</td>
<td>16.0</td>
<td>625,000</td>
</tr>
<tr>
<td>4</td>
<td>(10, 16) and (16, 10)</td>
<td>0.0450</td>
<td>5.0</td>
<td>0.0280</td>
<td>6.0</td>
<td>1,200,000</td>
</tr>
<tr>
<td>5</td>
<td>(13, 24) and (24, 13)</td>
<td>0.0372</td>
<td>5.0</td>
<td>0.0231</td>
<td>6.0</td>
<td>850,000</td>
</tr>
</tbody>
</table>

Note: Link improvement subsets with total costs below $3,000,000 (total costs shown in millions of dollars) are as follows: (1, 2, 3) $2.275; (1, 2, 4) $2.850; (1, 2, 5) $2.500; (1, 3, 4) $2.475; (1, 3, 5) $2.125; (1, 4, 5) $2.700; (2, 3, 4) $2.625; (2, 3, 5) $2.475; and (3, 4, 5) $2.675.

FIG. 8. Mean Trip Times for Sioux Falls Network

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instances of Braess's paradox do not occur for any of these budget-feasible sets of link improvements with either static or dynamic assignment. Fig. 8 also shows the best budget-feasible subset of link improvements (1,4,5) with static travel demands to remain best for each dynamic arrival rate distribution. However, the results show an interesting change in the benefit/cost ratios of link improvement subsets. With static arrival rates #5, subset (1,3,5) has the highest ratio of mean trip time reduction to capital cost as compared to the base case. With dynamic arrival rates #1, subset (1,4,5) has the highest ratio of mean trip time reduction to capital cost as compared to the mean trip time of the base case with arrival rates #1. In capital rationing, projects are ranked and programmed within a budget so as to maximize their total net benefit to capital cost ratio. Hence, in terms of capital rationing, subset (1,3,5) is best with SUE assignment, but subset (1,4,5) is best with DUE2 assignment and arrival rates #1.

Regarding computational burden, the algorithm required less than 2.5 min to converge acceptably for the most demanding of these Sioux Falls cases on a 33 MHz 486/DX computer. For 5 hr of 5-min intervals on a Denver network (220 nodes, 450 links, 26 zones, 650 O-D pairs), the algorithm required 12 min on the same computer.

CONCLUSIONS AND FUTURE RESEARCH

Three cases were presented in which the effects on network design with DUE2 assignment lead to different conclusions than with SUE assignment. The first example showed a case in which expanding the capacity of a downstream bottleneck did not reduce total travel time with DUE2, whereas this outcome is possible for SUE assignment to the same network. A second example showed Braess’s paradox (i.e., link addition causing an increase in mean trip time) to occur with either SUE or DUE2 assignment depending on total travel demand over the analysis period. In the third example, the preferred subset of budget-feasible link improvements according to their combined benefit/cost ratio (i.e., the project selection criterion of capital rationing) was different with SUE than with DUE2 and arrival rates #1. These differences in SUE and DUE2 outcomes were due to temporal travel demand changes only, since total vehicle trips over two hours were held constant in each case.

The examples in this paper indicate that decisions affecting supply changes such as road widening and ramp metering cannot be considered independent of policies affecting peak-period spreading of commuting trips. Such programs often provide greater incentives for employers to offer staggered work hours, flextime schedules, or partial-day telecommuting options to employees. Systemwide gain per unit of strategy cost must be evaluated in order to design and implement a cost-effective program of both travel demand management strategies and network supply enhancements. Of further interest are potential gains of providing departure time and route choice information to motorists to aid their trip scheduling decisions. Janson (1991c) showed the use of DUE models to assess travel guidance impacts on peak-period travel times and fuel consumption, particularly during temporary lane closures for accidents or road work. To that end, these dynamic models can be run on high-speed computers for large networks in order to implement real-time traffic modeling and route guidance systems in traffic operation centers. This approach can also incorporate reductions in link capacities due to accidents and spillback queuing in time intervals when these events occur. Such model advances and applications are endeavors of future research.

APPENDIX. REFERENCES


