First-In-First-Out Properties of a Commodity-based Kinematic Wave Simulation Model

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Abstract

Computational efficiency and First-In-First-Out (FIFO) are both important issues in applications such as dynamic traffic assignment. In this paper, we first show by example the FIFO properties of a computationally efficient commodity-based kinematic wave (CKW) model of network traffic flow. After developing measurements of FIFO violation among commodities in location and time, based on misplacement of vehicles in total and commodity traffic, we theoretically argue that numerical CKW solutions converge to FIFO ones. Then with numerical examples, we show that FIFO violation in CKW solutions is a function of simulation time and number of commodities, but decreases with decreasing cell lengths, even for infinite number of commodities. Finally we discuss possible implications of this study.

1 Introduction

The LWR type [1, 2] kinematic wave models of network vehicular traffic, in which supply-demand method is used for computing boundary flows [3, 4], have been successfully used in solving dynamic traffic assignment problem in recent years, e.g. [5]. Two such models are the Cell Transmission Model (CTM) [3] and the STRADA model [6]. In the first model, First-In-First-Out (FIFO) discipline is guaranteed on a link by ordering vehicles according to their waiting times; while the STRADA model is based on commodity flows, and FIFO may be violated.

To take full advantages of the supply-demand method, a queue-based kinematic wave (QKW) model and a commodity-based (CKW) model were proposed in [7] and [8, 9] respectively. At the aggregate level, these models are similar to the STRADA model with a general fundamental diagram and various cell lengths. In addition, simpler supply-demand methods for traffic at a merge or a general junction are incorporated. At the disaggregated level, vehicles are ordered as a queue on a link in the QKW model, and the CKW model concerns the evolution of commodity (path) flows. Similar to CTM, the QKW model guarantees FIFO on a link. In contrast, the CKW model yields numerical solutions with FIFO violation as the STRADA model, but solutions in the former converge to FIFO ones, as we will argue in this paper.

Compared to the QKW model, the CKW model does not track individual vehicles explicitly and is thus much more computationally efficient. However, FIFO violation seems to negate the advantage of the CKW model. In [8, 9], it was briefly argued that numerical CKW solutions violate FIFO principle but converge to FIFO ones. That is, the CKW model has both FIFO violation and convergence properties. Since the FIFO properties of the CKW model have been an interesting issue to many colleagues, including a referee of [9], and FIFO in general is an important issue in studies such as dynamic traffic assignment, it is worthwhile to examine the properties of the CKW model in more details both theoretically and computationally. In addition, in this study, we intend to develop a measurement of FIFO violation that can also be used in other studies.

In Section 2, we first demonstrate FIFO violation and FIFO convergence in the CKW model by a simple example, discuss possible FIFO violation in the QKW model at a diverge, and then develop two measurements of FIFO violation. In Section 3, we theoretically show that numerical solution method in the CKW model is a first order Godunov method [10] in nature and thus yields convergent results. In Section 4, we numerically show how FIFO violation are affected by simulation times, cell lengths, and number of commodities. Numerical results support our conclusion on the FIFO properties of the CKW model. Finally, in Section 5, we conclude our study and present some discussions.

2 FIFO violation and its measurements

2.1 Examples of FIFO violation and convergence

As shown in Figure 1(a), all A vehicles are behind B vehicles at time $t = 0$ in a traffic stream. When we update traffic conditions by the CKW model, at $t = \Delta t$, two types of vehicles are mixed in the second cell, but we cannot explicitly observe FIFO violation yet. However, at $t = 2\Delta t$, we can clearly see that 40 B vehicles in cell $(\Delta x, 2\Delta x)$ are left behind by 60 A vehicles in cell $(2\Delta x, 3\Delta x)$. That is, there exists FIFO violation in the CKW solutions. The FIFO violation is caused by the numerical method used in the CKW model, in which proportion of B flow in total flow through $x = 2\Delta x$ is equal to that in cell from Δx to $2\Delta x$. Therefore, FIFO violation is inherited in CKW solutions. However, if we look at Figure 1(b), where we double the number of cells and time intervals, we can see that now only 20 B vehicles in cell $(\Delta x, 2\Delta x)$ are behind 35 A vehicles in cell $(2\Delta x, 3\Delta x)$. This suggests that FIFO violation is less serious if we decrease cell length and time step. That is, the CKW model has FIFO convergence property.

Clearly from Figure 2(a), we can see that solutions from QKW model observe FIFO principle. However, if we look at a diverging case in Figure 2(b), we can see that FIFO principle will be violated if we want to keep flows to two directions consistent with what are obtained from KW models, in which outflow to a branch from a diverge should be proportional to the proportion of vehicles in the upstream cell. However, FIFO violation in the QKW model may not be so serious as those in the CKW model, since they only involve the last cell of a link.

In this paper, we focus on FIFO violation in CKW solutions of traffic on a link, and the results will be the same for a network. In our study, FIFO solutions of the QKW model will be used as a benchmark when exact FIFO solutions are not available.

2.2 Measurements of FIFO violation in location and time

Assume that at time t, the accurate solution of traffic density on a road is given by $\rho_e(x)$, and a numerical solution by $\rho(x)$. Then starting from the upstream of the road, e.g., $x = 0$, we can compute cumulative number of vehicles [11] as

$$N(x) \quad = \quad \int_{y=0}^{x} \rho(y)\mathrm{dy}.$$

Similarly, we can find accurate cumulative number of vehicles as $N_e(x)$. Further assuming vehicle orders are the same in both cases, we can then find the misplacement of each vehicle, e.g. N_0, as $|x_1 - x_2|$ where $N(x_1) = N_e(x_2) = N_0$. As shown in Figure 3, we can see that the total misplacement of a traffic stream is given by the area bounded by two curves of $N(x)$ and $N_e(x)$ in the bottom figure. I.e., we can compute misplacement in location at a time by

$$\epsilon_x \quad = \quad \int_{x=0}^{x=L} |N(x) - N_e(x)|\mathrm{dx}. \tag{1}$$

If applied to solutions by CKW and QKW models, ϵ_x is then caused by numerical errors in solutions of total traffic density and does not count in FIFO violation among vehicles. Similarly, for traffic of commodity p ($p = 1, \cdots, P$), we can also define its x-misplacement $\epsilon_{x,p}$ as the difference between $N_p(x)$ and $N_{p,e}(x)$, where N_p is cumulative number of commodity p, and $N_{p,e}$ the exact cumulative number of commodity p. In addition, $\epsilon_{x,p}$ denotes numerical error for commodity p but does not count in FIFO violation among vehicles of this commodity. Since all partial flows add up to total flow, misplacement in commodity flows, $\sum_{p=1}^{P} \epsilon_{x,p}$, counts in both numerical error of total traffic and the effect of possible overtaking between commodities. Therefore, the difference of misplacement at the aggregate level and commodity level,

$$\epsilon_{x,FIFO} \quad = \quad \sum_{p=1}^{P} \epsilon_{x,p} - \epsilon_x, \tag{2}$$

can be considered as a measurement of FIFO violation in location among all commodities. The unit of x-misplacement and x-FIFO violation is vehicle miles. Note that both x-misplacement and FIFO violation can be averaged per vehicle or per distance traveled.

Similarly, we can define misplacement in time at a point x by

$$\epsilon_t \quad = \quad \int_{t=0}^{t=T} |F(t) - F_e(t)|\mathrm{dt}, \tag{3}$$

where $F(t)$ and $F_e(t)$ are the computed and accurate cumulative flow at x, and further define FIFO violation in time, $\epsilon_{t,FIFO}$. Note that, if all vehicles have the same speed, then ϵ_x and ϵ_t follow a linear relationship.

Both measurements of FIFO violation have the following properties. First, they are zero iff there is no FIFO violation. For example, in QKW solutions of link traffic, $\sum_{p=1}^{P} \epsilon_{x,p}$ and ϵ_x are equal at any time. Second, the larger $\epsilon_{x,FIFO}$, the more serious FIFO violation. For example, if we take QKW solutions in Figure 2 as exact (i.e., ignore numerical error in total traffic), x-FIFO violations in solutions of Figure 1(a) and (b) are $120\Delta x$ and $50\Delta x$ respectively. That is, the measurement is consistent with our impression on the seriousness in FIFO violations in both solutions. Third, the measurements proposed here are absolute in the sense that exact FIFO solutions, $N_e(x)$ and $N_{p,e}(x)$, are used as the benchmark. When exact solutions are not available, we can use relative measurements by using QKW solutions as the benchmark. Fourth, both measurements are for FIFO violation among commodities and not for FIFO violation among vehicles. This is because, when we compute $\epsilon_{x,p}$, we assume no FIFO violation among vehicles inside a commodity. However, they can be extended for FIFO violation among vehicles if each vehicle is considered as a commodity. Since, in the CKW model for example, the number of commodities is not confined by the number of vehicles and can approach ∞ theoretically, we can determine FIFO violation among vehicles by setting number of commodities sufficiently large.

3 Theory of FIFO convergence in the CKW model

3.1 Analysis of continuous CKW model

Considering a traffic stream of two commodities, we denote total density, density of commodity 1, and the proportion of commodity 1 by ρ, k, and ξ respectively. That is, $k = \xi\rho$. Then, the continuous CKW model can be written in the conservative form [4]

$$\begin{aligned} \rho_t + (\rho V)_x &= 0, \\ k_t + (kV)_x &= 0, \end{aligned} \tag{4}$$

which is a system of two conservation laws, or in the advection form [4]

$$\begin{aligned} \rho_t + (\rho V)_x &= 0, \\ \xi_t + V\xi_x &= 0. \end{aligned} \tag{5}$$

If we denote the state variable by $\vec{u} = (\rho, k)$, Equation 4 has the following quasi-linear form

$$\vec{u}_t + \partial f \vec{u}_x = 0,$$

where the Jacobian $\partial f = \begin{bmatrix} V + \rho V' & 0 \\ kV' & V \end{bmatrix}$. Thus we can obtain the eigenvalues, right and left eigenvectors of Equation 4 as λ_i, $\vec{r}_i$, and $\vec{l}_i$ respectively

$$\begin{aligned} \lambda_1 &= V + \rho V', \quad \vec{r}_1 = \begin{bmatrix} 1 \\ \frac{k}{\rho} \end{bmatrix}, \quad \vec{l}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}; \\ \lambda_2 &= V, \quad \vec{r}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \vec{l}_2 = \begin{bmatrix} -\frac{k}{\rho} & 1 \end{bmatrix}. \end{aligned}$$

Therefore, $\nabla\lambda_1 \cdot \vec{r}_1 = 2V' + \rho V'' \neq 0$, and $\nabla\lambda_2 \cdot \vec{r}_2 = 0$. That is, 1-waves associated with λ_1 are genuinely nonlinear decelerating shock waves or accelerating rarefaction waves, and 2-waves associated with λ_2 are linearly degenerate contact waves.

Following [12], we can solve the Riemann problem of Equation 4 with initial condition

$$\vec{u}(x, t = 0) = \begin{cases} \vec{u}_L, & x < 0; \\ \vec{u}_R, & x > 0. \end{cases}$$

Since $\lambda_1 < \lambda_2$, the Riemann problem is solved by a combination of a 1-wave connecting the upstream state $\vec{u}_L$ to an intermediate state $\vec{u}_M$ and a 2-wave connecting $\vec{u}_M$ to the downstream state $\vec{u}_R$. Further, if denoting the Riemann invariant of i-wave by w_i; i.e., $\nabla w_i \cdot \vec{r}_i = 0$, we obtain $w_1 = k/\rho = \xi$ and $w_2 = V$. That is, $\xi(x,t) = \xi_L$ on 1-waves; and $V = V_R$ on 2-waves. Note that 1-wave covers the affine $x/t = 0$, i.e., $\xi(x = 0, t) = \xi_L$. Also note that 1-wave is exactly the same solution to the LWR model of total traffic. Hence, we have total flux $q(x = 0, t) = \min\{D(\rho_L), S(\rho_R)\}$ and partial flux $(\xi q)(x = 0, t) = \xi_L q(x = 0, t)$.

The analysis above shows that the discrete CKW model is indeed in the form of Godunov [10] difference and converges to the weak solutions [13, 14]. In addition, from Equation 5, we can see any interface between two commodities travels with vehicles and does not mix in the exact solutions. That is, exact solutions of the continuous CKW model indeed follow FIFO principle and numerical CKW solutions converge to FIFO ones.

3.2 Numerical analysis of the discrete CKW model

The discrete CKW model [4, 8, 9] can be rewritten as

$$\begin{aligned}
\hat{\rho} &= \rho_i^j - \frac{\Delta t}{\Delta x} f_{i+1/2}^{j*}, \\
\tilde{\rho} &= \frac{\Delta t}{\Delta x} f_{i-1/2}^{j*}, \\
\rho_i^{j+1} &= \hat{\rho} + \tilde{\rho}, && (6) \\
\beta &= \frac{\hat{\rho}}{\rho_i^{j+1}}, && (7) \\
\xi_{p,i}^{j+1} &= \beta \xi_{p,i}^j + (1-\beta) \xi_{p,i-1}^j, && (8)
\end{aligned}$$

where subscript p is for commodity number ($p = 1, \cdots, P$), subscript i for cell number, superscript j for time instant number.

Since $f_{i+1/2}^{j*} = \min\{D(\rho_i^j), S(\rho_{i+1}^j)\} \le D(\rho_i^j)$, we have $\hat{\rho} \ge \rho_i^j - \frac{\Delta t}{\Delta x} D(\rho_i^j)$. When $\rho_i^j \le \rho_c$, the critical density, $D(\rho_i^j) = \rho_i^j V(\rho_i^j)$, and we have

$$\hat{\rho} \ge \rho_i^j \left(1 - \frac{\Delta t}{\Delta x} V(\rho_i^j)\right).$$

When $\rho_i^j > \rho_c$, $D(\rho_i^j) = \rho_c V(\rho_c)$, and we have

$$\hat{\rho} \ge \rho_i^j - \frac{\Delta t}{\Delta x} \rho_c V(\rho_c) > \rho_c \left(1 - \frac{\Delta t}{\Delta x} V(\rho_c)\right).$$

In both cases, $\hat{\rho} > 0$ since $\frac{\Delta t}{\Delta x} V(\rho) < 1$ for any ρ, which is the CFL condition [15]. Also since $\tilde{\rho} > 0$ and $\beta \in (0, 1)$, we can see that Equation 8 is an upwind scheme for commodity proportions. From Godunov's theorem [14], the method is first-order accurate in the sense that

$$\begin{aligned}
\sum_{i=1}^n |\rho_i^t - \rho(x_i, t)| &= t\, O(\Delta x), \\
\sum_{i=1}^n |\xi_i^t - \xi(x_i, t)| &= t\, O(\Delta x),
\end{aligned}$$

which also means

$$\sum_{i=1}^n |\rho_i^t \xi_i^t - \rho(x_i, t) \xi(x_i, t)| = t\, O(\Delta x),$$

where $\rho(x_i, t)$ and $\xi(x_i, t)$ are exact solutions. Note that the error increases linearly to the simulation time.

Numerically, we can compute x-misplacement of commodity p as

$$\epsilon_{x,p} = \sum_{i=1}^{n} |N_{p,i}^t - N_p(x_i, t)| = t\, O(\Delta x),$$

where $N_{p,i}^t = \sum_{k=1}^{i} \rho_k^t \xi_k^t \Delta x$ and $N_p(x_i, t) = \sum_{k=1}^{i} \rho(x_k, t)\xi(x_k, t)\Delta x$. We can also compute x-misplacement of total traffic similarly. Further, we can compute FIFO violation among commodities by

$$\epsilon_{x,FIFO} = \sum_{p=1}^{P} \epsilon_{x,p} - \epsilon_x,$$

. From analysis above, we can see that FIFO violation should also be to the order of Δx and solutions converge to FIFO as we decrease Δx, but FIFO violation can be affected by simulation time t and number of commodities P.

Specifically, if we consider free flow scenario, with initial conditions $\rho(x, t = 0) = \rho_0(x) < \rho_c$ and $\xi(x, t = 0) = \xi_0$, then the continuous CKW model is

$$\begin{aligned} \rho_t + v_f \rho_x &= 0, \\ \xi_t + v_f \xi_x &= 0, \end{aligned}$$

whose exact solutions are $\rho(x, t) = \rho_0(x - v_f t)$ and $\xi(x, t) = \xi_0(x - v_f t)$. Note that exact solutions are FIFO, since the relative position between two commodities does not change with time. Correspondingly, the discrete CKW model is

$$\begin{aligned} \rho_i^{j+1} &= (1-\sigma)\rho_i^j + \sigma \rho_{i-1}^j, \\ \beta &= \frac{(1-\sigma)\rho_i^j}{\rho_i^{j+1}}, \\ \xi_{p,i}^{j+1} &= \beta \xi_{p,i}^j + (1-\beta)\xi_{p,i-1}^j \end{aligned}$$

In this case, we can compute x-misplacement and FIFO violation directly, since we have exact, FIFO solutions. Otherwise, for general traffic, in which we do not have analytical solutions, we can use QKW solutions to obtain relative FIFO violation.

4 Numeric studies of FIFO properties of the CKW model

We use the following triangular fundamental diagram [16, 17]:

$$\begin{aligned} V(\rho) &= \begin{cases} v_f, & 0 \le \rho \le \rho_c; \\ \frac{\rho_c}{\rho_j - \rho_c} v_f \frac{\rho_j - \rho}{\rho}, & \rho_c < \rho \le \rho_j, \end{cases} \\ Q(\rho) &= \begin{cases} v_f \rho, & 0 \le \rho \le \rho_c; \\ \frac{\rho_c}{\rho_j - \rho_c} v_f (\rho_j - \rho), & \rho_c < \rho \le \rho_j, \end{cases} \end{aligned}$$

where v_f is the free flow speed, ρ_j the jam density, and ρ_c the critical density where flow-rate, $q = \rho v$, attains its maximum, i.e. the capacity. These values are given as follows [18]: $v_f = 65$ mph, $\rho_j = 240$ vpmpl, and $\rho_c = \frac{1}{6}\rho_j = 40$ vpmpl. As a result, the capacity is $Q_{max} = 2600$ vphpl.

We consider a road link of 200 miles. Initially, we have a platoon in the region from 10 miles to 30 miles. We set the CFL number $v_f \frac{\Delta t}{\Delta x} = 0.91$ [15], and assume no in-flow at the origin. Since no vehicle will cross both upstream and downstream boundaries during the simulation, we do not have to worry about the influence of boundary conditions.

4.1 FIFO violation in free flow of two commodities

Initially, we have a free flow platoon as shown in Figure 4(a), which is split into two ($P = 2$) commodities in the middle. We denote ξ as the proportion of commodity one; i.e., $1 - \xi$ is the proportion of commodity two. The initial condition of ξ is given in Figure 4(b). In this case, we know the exact solutions for both ρ and ξ at T, as shown by solid lines in Figure 4(c)-(d).

4.1.1 x-misplacement and FIFO violation

When $\Delta x = 0.1$ mile, as shown in Figure 4(c), the difference in density means that vehicles are misplaced in the numerical solutions. From Figure 4(d), we can see that two commodities are mixed with each other around the separating interface at 156.5 miles; i.e., FIFO is violated in the numerical solution.

Further, in Figure 5, we show cumulative number of vehicles and their solution errors for $T = 2.1$ hour. From Figure 5(a), we can infer the expected position of a vehicle n_0 as $N_e(x_0) = n_0$; e.g., the position of vehicle 155 is at 156.55 miles, while numerical solutions suggest its position to be 156.65 miles. That is, vehicles are misplaced in numerical solutions, and this is caused by numeric error in computing total traffic density [2]. From Figure 5(b), we can clearly see the mixture of two commodities around 156.5 miles. For example, the position of first commodity two vehicle is at 155.35 miles, in front of which we can find around 21 commodity 1 vehicles, supposed to be behind. In Figure 5(c), we can see that misplacement in both commodities are larger than that in total traffic around the interface. This shows that the difference between misplacement of total traffic and that of commodity traffic, $\epsilon_{x,FIFO}$, is indeed caused by FIFO violation among commodities. In this case, $\epsilon_x = 70.0692$, $\epsilon_{x,1} = 38.9241$, $\epsilon_{x,2} = 44.3779$, and $\epsilon_{x,FIFO} = 13.2328$ vehicle miles. Averagely, since there are 310 vehicles totally, each vehicle misses its position by 0.226 miles after traveling 136.5 miles, and FIFO violation of each vehicle is about 0.0427 miles.

4.1.2 FIFO violation v.s. simulation time

Figure 6 shows ϵ_x,$\epsilon_{x,1}$, $\epsilon_{x,2}$, and $\epsilon_{x,FIFO}$ at ten time instants, evenly distributed from 0.21 to 2.1 hours. In general, we can see that, as expected, x-misplacement increases almost linearly to simulation time. In the figure, the oscillations in ϵ_x, $\epsilon_{x,1}$, and $\epsilon_{x,2}$ are believed to be caused by the averaging effect of the Godunov method, but FIFO violation almost strictly increases linearly in time.

4.1.3 FIFO violation v.s. cell length

Figure 7 shows ϵ_x,$\epsilon_{x,1}$, $\epsilon_{x,2}$, and $\epsilon_{x,FIFO}$ for decreasing cell lengths (and increasing cell numbers) Δx=0.1, $\frac{0.1}{2}$,$\frac{0.1}{4}$,$\frac{0.1}{8}$,$\frac{0.1}{16}$. From the figure, we can see that x-misplacement linearly decreases with cell length. In particular, from the curve of $\epsilon_{x,FIFO}$, FIFO violation among commodities decreases with cell length.

4.2 FIFO violation v.s. number of commodities in free flow

Figure 8 shows FIFO violation for different cell lengths, $\Delta x = 0.1$, 0.05, 0.025 miles, and different number of commodities, $P = 2$, 10, 20, 100, 200. From the figure, we can see that, at $\Delta x = 0.1$ for example, FIFO violation increases with number of commodities, but approaches a constant for very large number of commodities. That is, if we consider infinite number of commodities, FIFO violation remains limited. Further, comparing them for different cell lengths, we find that it is still

[2] In this case, the position of vehicle 155 can also be considered as the interface between two commodities. Thus, there is misplacement in the interface in QKW solutions.

true that x-FIFO violation decreases linearly with cell lengths. From the results, we can see that FIFO violation among vehicles is limited and decreases linearly with cell length in CKW solutions.

4.3 FIFO violation in general traffic

In this subsection, we consider FIFO violation for general traffic conditions, in which vehicles do not always travel at free flow speed. As an example, the initial traffic is shown in Figure 9(a), in which traffic is over-critical in some regions. Here we consider misplacement and FIFO violation in time at $x = 120$ miles in the CKW model.

Since we cannot obtain exact FIFO solutions of flow-rate and cumulative flow of each commodity at $x = 120$, we use the QKW model to provide the solutions with cell length of $\Delta x = \frac{0.1}{32}$, which is small enough to provide almost "exact" solutions. The cumulative flow of total traffic from QKW solutions is shown in Figure 9(b), and that of each commodity not shown.

For given number of commodities and cell length, we can compute misplacement in time, relative to QKW solutions. Further, for increasing numbers of commodities and decreasing cell lengths, we can compute relative FIFO violation in time as in Figure 9(c), which yields the same results as before. That is, FIFO violation is bounded for infinite commodities and decreases to zero with decreasing cell length.

5 Conclusion

In this paper, we discussed theoretically and numerically the FIFO properties in a commodity-based kinematic wave model (CKW), with the help of a queue-based model (QKW), in which numerical solutions observe FIFO principle on a link. The attractive properties of the CKW model, and its applicability to analytical/numeric traffic modeling within dynamic network models have been previously discussed and the current paper addresses the extent of problems caused by the FIFO violations in the model. The premise behind the study is that FIFO violations need to be measured quantitatively and viewed in the context of the extent of FIFO violation that naturally occur in traffic, rather than discard the model itself on account of FIFO being not strictly adhered to.

From these studies, we can conclude that FIFO violation in CKW solutions disappears when cell lengths decreases to 0. In this study, we proposed two measurements of FIFO violation among commodities, which can also be effectively used to measure FIFO violation among vehicles. These measurements can be applied in other studies related to FIFO properties.

Since the QKW model yields FIFO solutions, we can compute the travel time of a vehicle by just looking at the cumulative flow of total traffic. That is, the travel time of vehicle n from x_1 to x_2 is $t_2 - t_1$ if cumulative flow reaches n at location x_1 and time t_1 and at location x_2 and time t_2. In the CKW model, we cannot compute the travel time of an individual vehicle in this way due to FIFO violation among commodities (and therefore vehicles). However, we can compute total or average travel times of all vehicles of a commodity from the cumulative flow of total traffic. Since numerical solutions given by CKW and QKW models are exactly the same at the aggregate level, the total or average travel times for all vehicles are the same in both models. Therefore, the QKW model has no advantage over the CKW model if total or average travel times of a platoon are concerned.

Compared to other models such as the Cell Transmission model and the QKW model, the CKW model relaxes the FIFO requirement to some extent but still keeps FIFO violation under control. Therefore, the CKW model could be applied in solving Dynamic Traffic Assignment problem, in which FIFO requirement is found to be too restrictive to cause problems because of nonconvexity of constraint sets [19].

In practice, FIFO violation is allowed in aggregate models such as platoon dispersion models (e.g., [20] and any time vehicles overtake each other in traffic, there is a FIFO violation at the microscopic level as well. Therefore, completely preventing FIFO violations in a traffic model is not necessarily a good property either, as long as the FIFO violations can be shown to be under control with some objective measures such as in this paper. The FIFO violation problems discussed in the Dynamic Assignment literature are pointing to a different problem where a traffic model allows vehicles to be unrealistically held in certain links due to the network optimization constraints. While this paper does not attempt an elaborate discussion of this aspect, it does show that the CKW model allows within-link FIFO violations, but to levels that can be kept under control with proper modeling parameters.

Note that, however, FIFO violation in the CKW model is caused by numerical viscosity, and this is not an inherent feature of the continuous CKW model. Thus, we could develop a theoretical KW model with inherent FIFO violation, for example, simply by adding a second-order viscosity term to the commodity proportion equation in Equation 5. How well we can model FIFO violation in this approach will be a topic of our future research. The final comment is that more research is needed to properly identify and quantify the FIFO violations that occur due to overtaking and platoon dispersion in real traffic and to devise methods to allow only that level of FIFO violations in a model such as the CKW model.

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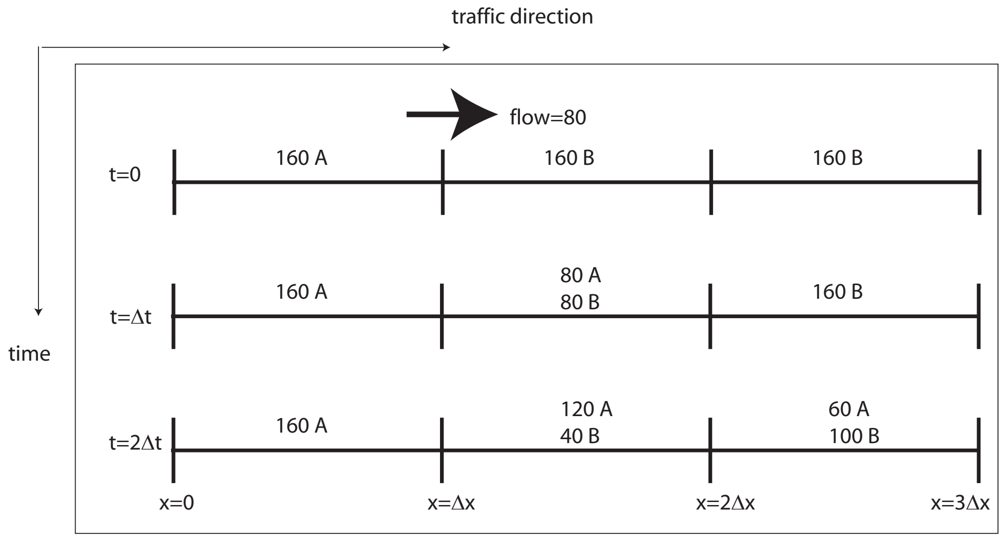
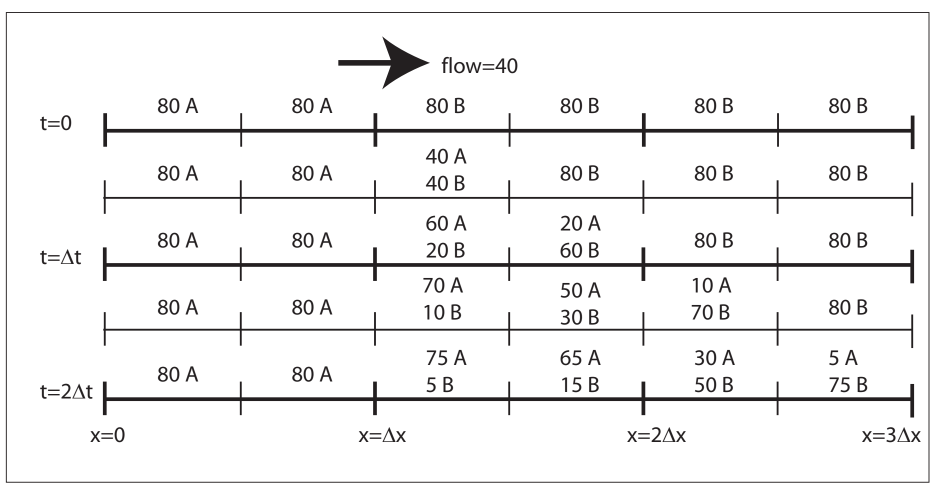
List of Figures

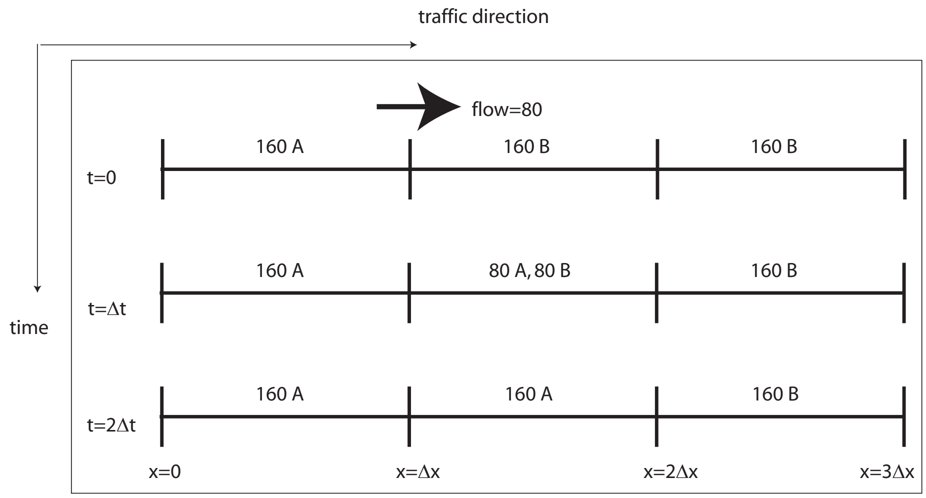


(a) FIFO violation in the CKW model

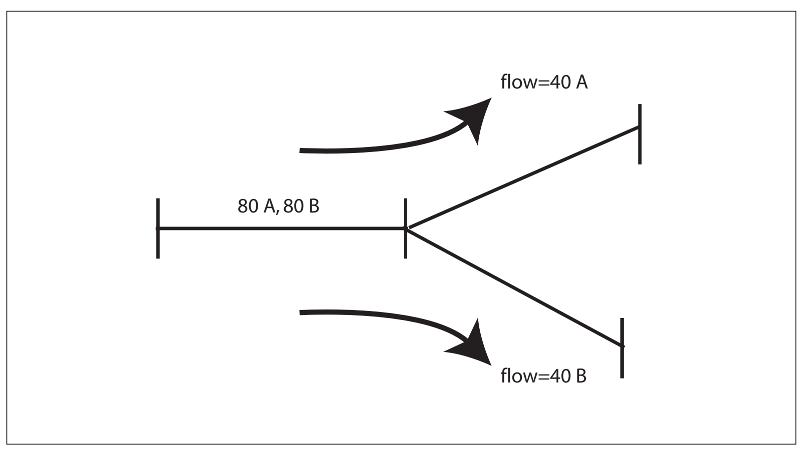


(b) FIFO violation with half cell length

Figure 1: FIFO violation and convergence in the CKW model



(a) FIFO solutions on a link in the QKW model



(a) Possible FIFO violation at a diverge in the QKW model

Figure 2: FIFO properties of the QKW model

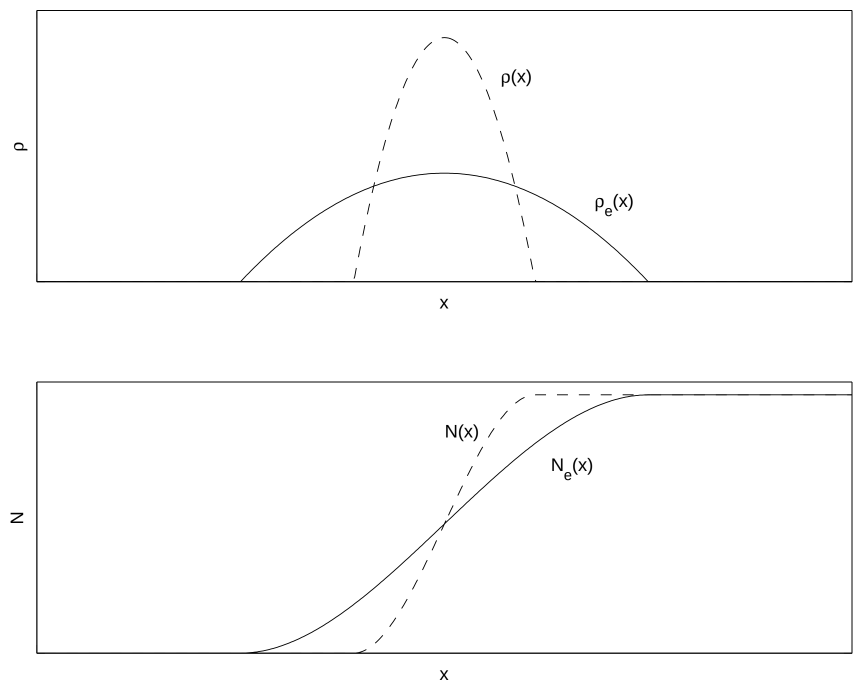


Figure 3: Misplacement in location in a numerical solution compared to the accurate solution

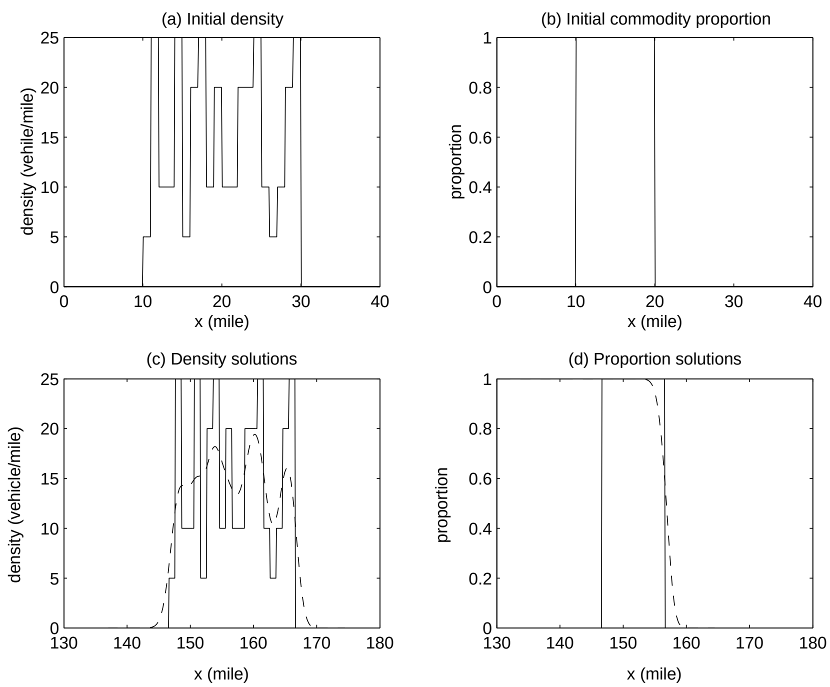


Figure 4: Initial conditions (a,b) and solutions at $T = 2.1$ hours: In (c,d): solid lines for exact solutions, dashed lines for solutions when $\Delta x = 0.1$ mile

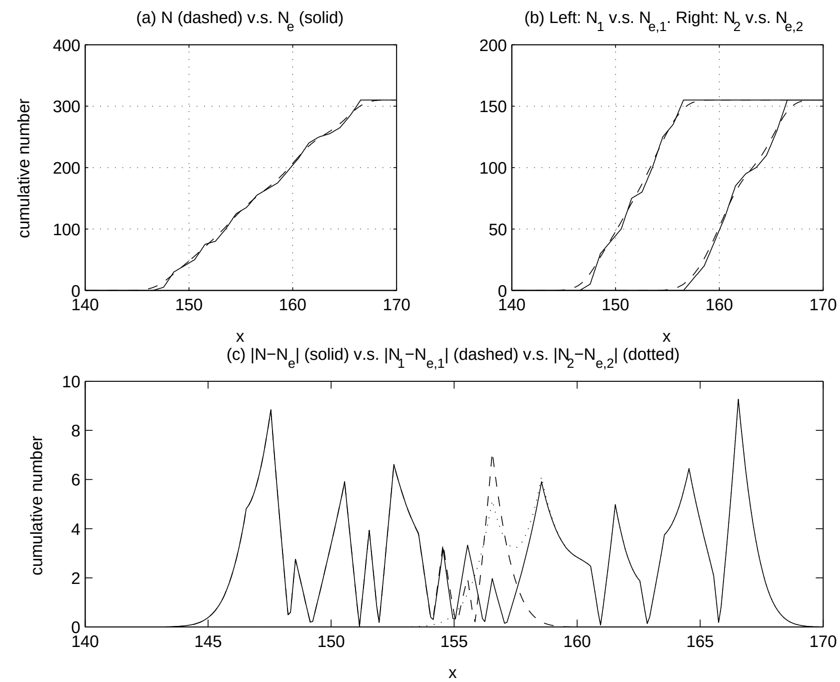


Figure 5: Cumulative number at $T = 2.1$ hours for two commodities when $\Delta x = 0.1$

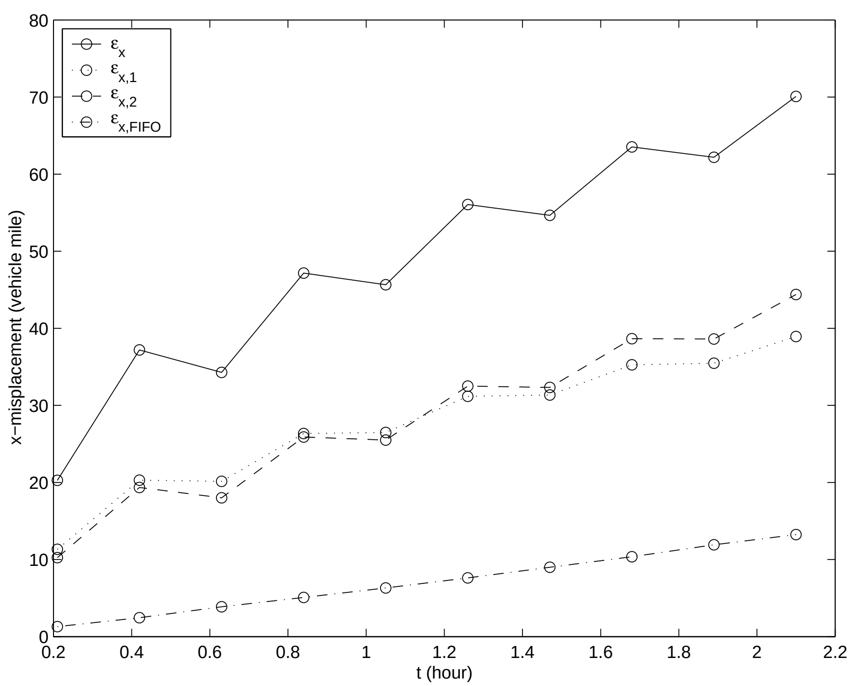


Figure 6: x-misplacement at different time instants for two commodities when $\Delta x = 0.1$

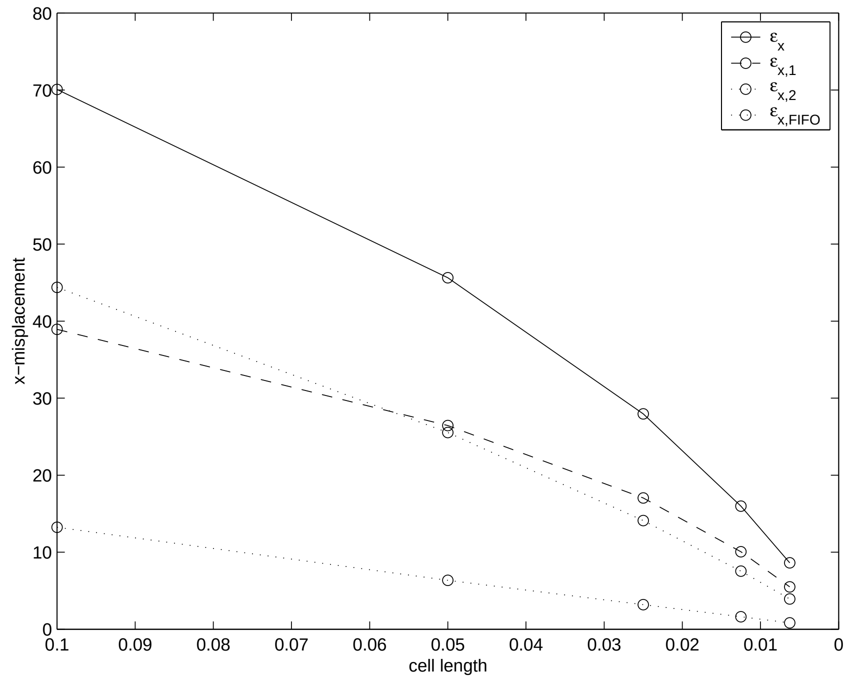


Figure 7: x-misplacement for different cell lengths for two commodities

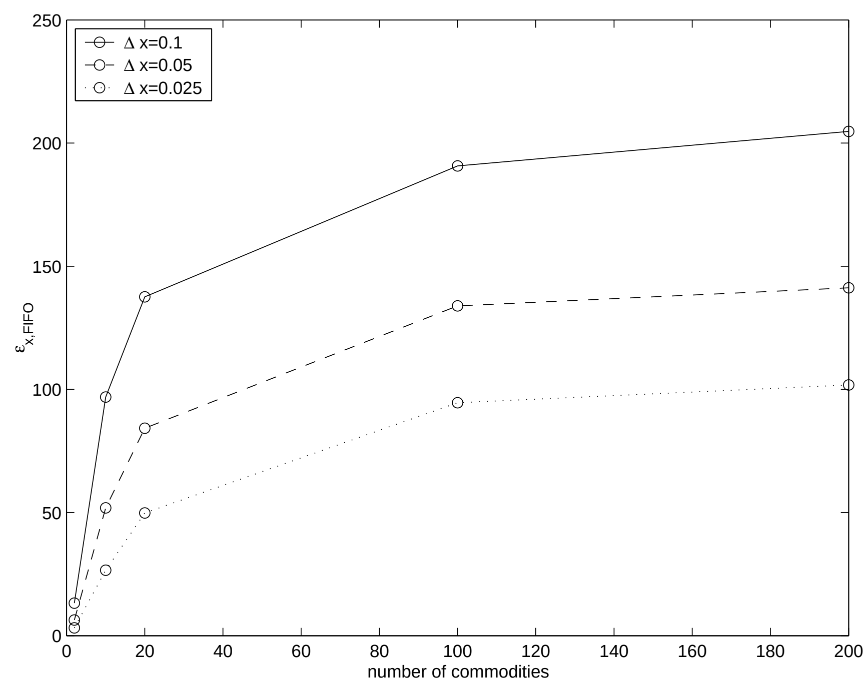


Figure 8: FIFO violation in location for different number of commodities and cell lengths

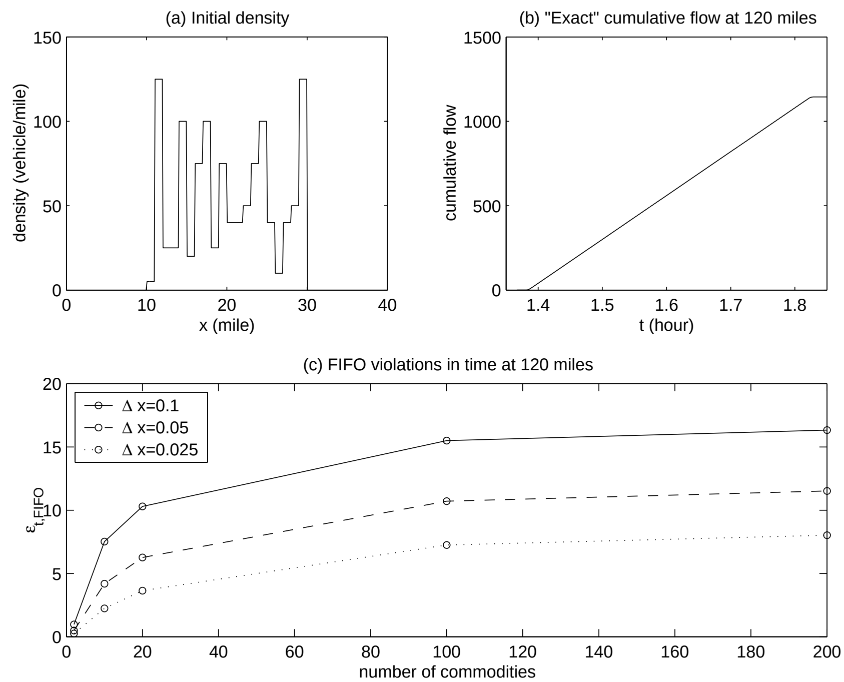


Figure 9: FIFO violation in time for general traffic conditions