

# The Process of Information Propagation in a Traffic Stream with a General Vehicle Headway: a Revisit

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## Abstract

Effective intervehicle communication is fundamental to a decentralized traffic information system based on mobile ad hoc vehicle networks. Here we model the information propagation process through inter-vehicle communication when the vehicle headway follows a general distribution. Equations for the expected value and variance of propagation distance are derived. In addition, we provide simple equations for the expected number of vehicles covered and the probability distribution of propagation distance. This research advances on an earlier study where the vehicle headway is assumed to follow an exponential distribution. This paper generalizes the earlier results and enables a design for robust information propagation by allowing for examination of the impact of different headway distributions. Within the new modeling framework, we also compute connectivity between two vehicles.

**Keywords:** *Inter-vehicle Communication; Stochastic Process; General Vehicle Headway*

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# 1 Introduction

Inter-vehicle communication as an important means to collect and disseminate traffic information has been studied intensively in the most recent years. In this paradigm, vehicles may generate, receive, and transmit information. An important rationale for these studies is the potential benefit from having the mobile automatic vehicular networks (MANET). MANET is essentially an ad hoc self-organizing network, representing a promising venue especially for managing instant and transient traffic information. An abundant information at the microscopic level in a way of distributed management could be potentially made available, establishing a new platform with unmeasurable potentials for traffic improvement and related technology development and deployment. Regarding the paradigm of inter-vehicle communication, many details are available in Yang (2003). Yang (2003) and Yang and Recker (2006) are among the early ones who systematically explore the issues in application of the inter-vehicle communication technology for the prospect of developing a decentralized traffic information system.

Advancing the frontiers of the paradigm of MANET calls for efforts from multiple disciplines. A tremendous amount of research is conducted by electrical and computer engineering researchers on developing protocols (Sun, *et al.* 2000) and selecting radio hardware/channel and routing algorithms (Hartenstein *et al.* 2001; Daizo, 2004; Xu and Barth, 2004; Korkmaz *et al.* 2004; Zang *et al.* 2005) for efficient transmission of information. Altman *et al.* (2005) study cooperative information forwarding between communicating vehicles. Recent availability of a block spectrum from 5.850 to 5.925 GHz for Dedicated Short Range Communications (DSRC) in the United States greatly encourages research in, and development of, this auto-net paradigm based on inter-vehicle communication.

Several recent studies worth our special mention here. Gamal *et al.* (2005) study the throughput delay problems on a mobile ad hoc network (MANET). Hanbali *et al.* (2006) introduce and study the notion of relay throughput, *i.e.* the maximum rate at which a node (e.g. a communicating vehicle) can relay data from the source to the destination, as a function of spatial distribution of transmitting vehicles. The significance of these studies remains in their highlight of work load and possible delay being important parameters in designing the transmitting capacity or computing power of the communicating vehicles. Interestingly, Hanbali *et al.* (2006) find that spatial distribution of vehicles significantly affects throughput and delay. Clearly, explicitly considering vehicle spatial distribution in analyzing propagation effectiveness is an underlying need. As a result, a fundamental issue to these studies is how far information may be propagated along a stream of traffic or on a network where presence of vehicles follows *any* probability distribution.

Study of information propagation along a traffic stream, a fundamental problem to MANET system design, has been attracting a great interest in the research community. Yang (2003) signifies the problem of information propagation distance and conducts numerous simulation studies. The information propagation issue is further studied in Jin and Recker (2006), who develop numerical method to recursively calculate the probability of successful propagation. Wang (2007) further develop analytical models to study the information relay process on an MANET. Although these studies advance the discovery of

information propagation distance, limitations exist. For example, Jin and Recker (2006) only provide numerical method though it allows great flexibility in studying information propagation and inter-vehicle connectivity by considering different traffic flow patterns. Similarly, Wang (2007) is confined to the condition that vehicle presence on a highway follows a homogeneous Poisson process, which is often argued to be in violation of the reality of vehicle spatial distribution on highways (Meng and Khoo, 2007).

This study focuses on the inter-vehicle communication problem in terms of effective propagation distance with a general vehicle headway. Note that effective propagation distance is critical to estimating throughput and delay on a MANET and to configuration of computing and communication capacity of communicating vehicles, as studied in Hanbali *et al.* (2006). The general idea is similar to Wang (2007), but it differs in that this study generalizes the vehicular spatial presence patterns and allows for any arbitrary vehicle headway distribution in deriving the analytical results. Note that the term headway in this paper means distance between vehicles instead of time as in many time papers. This generalization would offer more insights and allow for a more robust design of a MANET system by allowing for analytical examination of various different headway distributions on successful information propagation. Recently, Ukkusuri and Du (2008) also study robustness of inter-vehicle communication, but largely through simulation and regression analysis without developing closed form analytical expressions as we do here.

In this paper, given a headway distribution, the corresponding results (e.g. propagation distance, variance, probability distribution, number of vehicles covered, and connectivity between two vehicles on the road) are derived. It is nice that the formulas are very simple. In particular, we are able to show that coefficient of variation (CV) of the propagation distance approaches 1.0 infinitely even under a general inter-vehicle headway distribution, which illustrates a general volatility of instantaneous information propagation.

Note that this study only generalizes the earlier results in Wang (2007) in the case when the void distance in front of the first communicating vehicle is zero. A void distance is a proven distance in front of a communicating vehicle without other vehicle presence. Wang (2007) is more general in the sense that it considers the effect of void distance on information propagation. Furthermore, the earlier paper studies the number of relays which cannot be examined in this paper. Therefore, in this sense, the two papers complement each other.

We first present the problem of study as follows.

## 1.1 Problem Statement

A stream of traffic follows a straight road of infinite length. The distance between two consecutive communication capable vehicles (simply referred to as vehicles later) follows a general distribution whose density function is  $f(t)$ ,  $t \geq 0$ . The distance between vehicles is referred to as headway hereafter. Information may be transmitted in a direction between vehicles. The transmission range of vehicles is  $L$ . Each receiving vehicle relays the information over to the next one instantaneously if it is within its transmission range.

If there is no vehicle presence within  $L$ , the information transmission is terminated. The propagation distance equals to the distance of the last receiving vehicle to the origin of information. We primarily study the information propagation distance with this general distribution  $f(t)$  in this paper.

Note that the computing power of each communication vehicle is assumed to be sufficient for instant relay in order to develop insights into the propagation distance, just as in several previous studies (Wang, 2007; Jin and Recker, 2007). Therefore, vehicles in both directions along the line of highway form one stream of traffic for the purpose of information propagation. In other words, we do not concern whether presence of a vehicle in the direction of propagation takes place in the same direction or not. Vehicles take place on multi-lanes, making reasonable the point assumption of vehicles. In addition, we assume that the vehicle headway follows an *i.i.d.* distribution. This distribution could be any arbitrary one, for example, an empirical distribution from field data. The overall propagation distance represents a stochastic process. In this paper, we study the properties of the propagation distance.

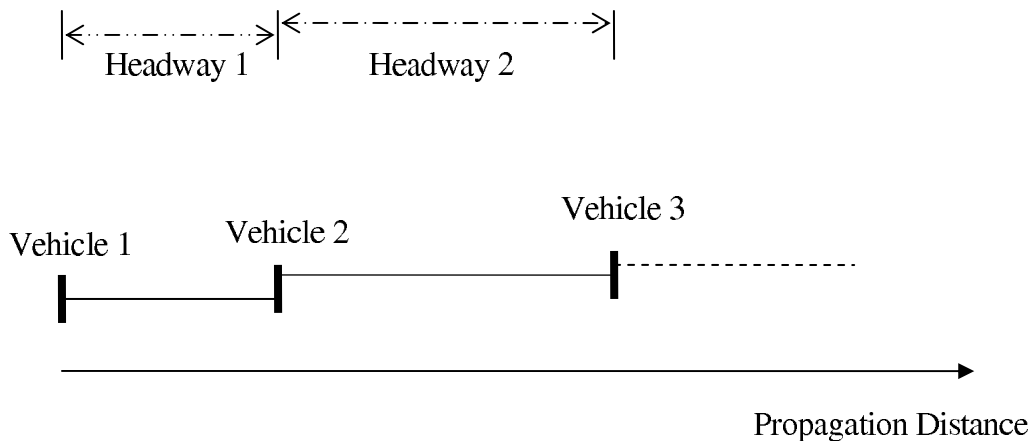


Figure 1: An Illustrative Information Transmission Process

Unlike the previous studies, we do not adopt the most-forwarded-within-range (MFWR) concept in this paper. Instead, we (*hypothetically*) only allow the closest vehicle within range to receive the information and continue the transmission on. Clearly, the overall propagation distance remains the same in this way as from using the MFWR concept. In this paper, if the transmitting vehicle has a distance to its first receiving vehicle within the range  $L$ , information is passed over to the first receiving vehicle. Then the first receiving vehicle that has received the information becomes the transmitting vehicle to continue the transmission forward. This process repeats until no vehicle is present within a transmission range  $L$  in the direction of interest. This is a typical renewal process. Note that each transmitting vehicle has a potential to further propagate the information forward for a same random distance  $X$ .

Figure 1 shows an example propagation process in which information is transmitted from vehicle 1 to 2, and then from vehicle 2 to 3. The process repeats itself until the information

dies out for the lack of vehicle within range  $L$ .

In what follows, Section 2 presents some structural properties of this propagation process. Section 3 develops a recursion for successful probability distribution and connectivity between vehicles. Section 4 provides some numerical results before the paper is concluded.

## 2 Properties of the Propagation Process

Following a popular notion in traffic studies, we assume that vehicles are points on a line.

We first introduce the notation.

### Notation

$X$	the random propagation distance.
$E[\cdot]$	mean function.
$VAR[\cdot]$	variance function.
$x$	expected value of $X$ , i.e., $x = E[X]$ .
$Y$	the random number of vehicles receiving the signal along a direction.
$y$	the expected value of $Y$ , $y = E[Y]$ .
$f(\cdot)$	vehicle headway probability density function over $(0, \infty)$ .
$F(\cdot)$	cdf of $f(\cdot)$ .
$H$	the i.i.d. random vehicle headway.
$\sigma$	variance of vehicle headway. $\sigma = VAR(H)$
$\mu$	the expected headway between two consecutive vehicles, i.e., $\mu = E[H]$ .

We have the following recursion.

$$x = \int_0^L (\tau + x) f(\tau) d\tau. \quad (1)$$

The logic in (1) is simple. The first receiving vehicle taking place at a distance  $\tau$  to the current transmitting vehicle has a presence probability of  $f(\tau)d\tau$ . As the first receiving vehicle repeats the transmission again, it still has the same potential to further transmit forward for an expected distance  $x$ .

Based on (1), we have the following finding.

**Theorem 1** *The expected distance that information is propagated is given by*

$$x = \frac{\int_0^L t f(t) dt}{1 - F(L)}. \quad (2)$$

Where  $F(L) = \int_0^L f(t) dt$ .

An obvious, but important observation is that the propagation distance is irrelevant to the probability distribution of vehicle headway *beyond* the transmission range  $L$ . As will be seen later, the variance of propagation distance is not affected by the distribution beyond  $L$  either.

In the following, we make an attempt to develop bound estimates of the propagation distance in order to obviate the need for estimating the probability density function of the inter-vehicle headway. As seen later, the bound estimates only depend on the cumulative probability of headway within the transmission range  $L$ .

### Corollary 1

$$\frac{\mu}{1 - F(L)} - \frac{\sqrt{\sigma^2 + (L - \mu)^2} - (L - \mu)}{2(1 - F(L))} \leq x \leq \frac{\mu}{1 - F(L)} - L. \quad (3)$$

#### Proof

The result uses Equation (2), and it is mainly about developing bounds for the numerator of the right hand of (2). The first inequality uses a result in Gallego (1992): for any random variable  $Z$  with mean  $\mu$  and finite variance  $\sigma$ , the following holds.

$$\int_z^\infty tf(t)dt \leq \frac{\sqrt{\sigma^2 + (z - \mu)^2} - (z - \mu)}{2}.$$

The second inequality is obvious by considering  $\int_0^L tf(t)dt \leq \int_0^\infty tf(t)dt - \int_L^\infty tf(t)dt \leq \int_0^\infty tf(t)dt - \int_L^\infty Lf(t)dt$ .  $\square$

The advantage of having such an interval estimate of the expected propagation distance relies in the fact that it relieves the need for calibrating the probability density function. Given an arbitrary vehicle headway distribution, the expected propagation distance may be estimated with a known mean and variance of the vehicle headway, both of which can be easily obtained with field data.

Variance of the successful propagation distance is measured as follows.

**Theorem 2** *The variance of information propagation distance is given by*

$$V(X) = \frac{\int_0^L t^2 f(t)dt}{1 - F(L)} + x^2, \text{ and furthermore} \quad (4)$$

$$\lim_{L \rightarrow \infty} \frac{\sqrt{V(X)}}{x} = 1.0. \quad (5)$$

#### Proof

The recursion for variance is given below, conditional on distance  $t$  to the first receiving

vehicle (within or beyond the transmission range) (refer to the Eve's Rule in Ross, 1997).

$$\begin{aligned}
V(X) &= V[E(X|t)] + E(V[X|t]) \\
&= \int_0^L (t+x-x)^2 f(t) dt + \int_L^\infty (0-x)^2 f(t) dt + \int_0^L V(X) f(t) dt \\
&= \int_0^L t^2 f(t) dt + \int_L^\infty x^2 f(t) dt + V(X) F(L) \\
&= \int_0^L t^2 f(t) dt + x^2 - x^2 F(L) + V(X) F(L).
\end{aligned}$$

The second equality uses the fact that if the first vehicle is beyond the range  $L$ , the distance propagated is zero with a variance also being zero, which is therefore ignored in the equation.

From the above, we have the following result.

$$V(X) = \frac{\int_0^L t^2 f(t) dt}{1 - F(L)} + x^2.$$

From here, we can see that practically  $\sqrt{V(X)} \geq x$ . Therefore,  $\frac{\sqrt{V(X)}}{x} \geq 1.0$ . Furthermore,

$$\begin{aligned}
V(X) &\leq \frac{L \int_0^L t f(t) dt}{1 - F(L)} + x^2 \\
&= Lx + x^2 \\
&< \left(x + \frac{L}{2}\right)^2.
\end{aligned}$$

The last inequality above holds when we have a non-trivial value  $x > 0$ . Therefore, we have

$$\frac{\sqrt{V(X)}}{x} < 1 + \frac{L}{2x}.$$

Note that  $\frac{L}{2x} = \frac{L(1-F(L))}{2 \int_0^L t f(t) dt}$ . When  $L \rightarrow \infty$ , the denominator approaches  $2\mu$ , where  $\mu$  is the mean headway. In what follows, we show that  $\lim_{L \rightarrow \infty} L(1 - F(L)) = 0$ .

With simple manipulation, one has

$$\begin{aligned}
\mu &= \int_0^\infty t f(t) dt \\
&\geq \int_0^L t f(t) dt + \int_L^\infty L f(t) dt
\end{aligned} \tag{6}$$

As the mean vehicle headway  $\mu$  is assumed to be finite, and  $\mu = \lim_{L \rightarrow \infty} \int_0^L t f(t) dt$ , we must have  $\lim_{L \rightarrow \infty} \int_L^\infty L f(t) dt = \lim_{L \rightarrow \infty} L(1 - F(L)) = 0$  in light of Equation (6).

Therefore, we can claim that  $\frac{L}{2x} \rightarrow 0$  when  $L \rightarrow \infty$ . In conclusion, we can find that as  $L \rightarrow \infty$ , the CV,  $\frac{\sqrt{V(X)}}{x}$ , approaches 1.0+ from the right hand side.  $\square$

Here in the general case, we have shown that the CV can never be smaller than or equal to 1.0. But it approaches 1.0 infinitely closely from the right side when the transmission range increases. Existence of such a constant limit for the CV illustrates a volatile aspect of inter-vehicle communications in the general case. This also strongly indicates an exponential probability distribution in its limiting case. Note that  $L \rightarrow \infty$  with given  $\mu$  is equivalent to  $\mu \rightarrow 0$  with a given  $L$  by just re-scaling, both meaning  $\frac{L}{\mu} \rightarrow \infty$ .

### Bound of the Variance of Propagation Distance

Let develop a bound for the variance calculated in Theorem 2.

$$\begin{aligned} \int_0^L t^2 f(t) dt &\leq L \int_0^L t f(t) dt \\ &= L\mu - L \int_L^\infty t f(t) dt. \\ &\leq L\mu - L^2(1 - F(L)). \end{aligned}$$

Clearly the following holds as a result of the above analysis.

**Corollary 2** *The variance of propagation distance may be bounded as follows.*

$$x^2 \leq V(X) \leq \frac{L\mu}{1 - F(L)} - L^2 + x^2.$$

In addition, the expected number of vehicles receiving the message is often of interest. For example, it may be used to estimate the value of information generated and propagated among vehicles. Denoted by  $y$ , the expected number satisfies the following relationship.

$$y = \int_0^L (1 + y) f(t) dt. \quad (7)$$

Similar to Equation (1), the next vehicle takes place at  $t$  from the current transmitting vehicle at a probability  $f(t) dt$ . Note that when the first vehicle gets the information, the expected number of additional vehicles to cover is still  $y$ .

Solving Equation (7) gives rise to the following result.

**Theorem 3** *The expected number of vehicles covered in a process is given as follows.*

$$y = \frac{F(L)}{1 - F(L)}.$$

Again, clearly  $F(L)$  plays a critical role.

### 3 Probability Distribution of Propagation Distance

#### 3.1 Probability Distribution of Propagation Distance

Denote by  $P(s)$  the probability for  $X > s$ . Note that  $F(\cdot)$  is used as the cdf for vehicle headway distribution. We have the following result.

**Theorem 4**

$$P(s) = \begin{cases} \int_0^L f(t)P(s-t)dt, & L < s \\ F(L) - F(s) + \int_0^s f(t)P(s-t)dt, & 0 < s \leq L \\ F(L), & s = 0. \end{cases}$$

**Proof**

For  $s = 0$ , the result is obvious. For  $0 < s \leq L$ , if the immediate next vehicle is within the range of  $[s, L]$ , whose probability is  $F(L) - F(s)$ , the information would be propagated beyond  $s$ . If, however, the immediate next vehicle is at a location  $t \in [0, s]$  at probability  $f(t)dt$ , the information would have to be further propagated for a distance of at least  $s - t$ , whose probability is  $P(s - t)$ . This explains the second case. For the case that  $s > L$ , the first vehicle at location  $t$  within the range  $L$ , whose presence probability is  $f(t)dt$ , has to continue to propagate for an additional distance of at least  $s - t$  at probability  $P(s - t)$ .  $\square$

With Theorem 4, we are able to recursively calculate the success rate beyond any point  $s$ .

Following the same logic as in Wang (2007), one can configure the two parameters of the Gamma distribution in such a way that yields the same expected value and variance of propagation distance as calculated in Equations (2) and (4). This Gamma distribution could represent a practically good approximation to the true probability distribution described above. When there are more vehicles expectedly within one transmission range, the CV of propagation distance approaches 1.0. And the Gamma approximation becomes exponential. The information propagation has a memoryless property (e.g. a typical renewal process): additional propagation distance from the current vehicle is always the same as from the previous one no matter how far the information has been propagated. With this observation and additional numerical tests along with a CV approaching 1.0, we conjecture that exponential distribution is a limiting distribution in this case. But we have not been able to prove this mathematically.

## 3.2 Multihop Connectivity between Two Equipped Vehicles

Connectivity represents the probability for the existence of a communication path between two given vehicles. We denote by  $C(s)$  the connectivity between two vehicles a distance  $s$  apart from each other. In light of Jin and Recker (2005),  $C(s)$  is calculated with  $P(s)$  as follows.

### Theorem 5

$$C(s) = \begin{cases} P(s - L), & L < s, \\ 1, & 0 \leq s \leq L. \end{cases}$$

*Proof.* When  $0 \leq s \leq L$ , two equipped vehicles are within each other's transmission range and can always communicate with each other. When they are outside of each other's transmission range, if a message from one vehicle travels beyond  $s - L$ , it can always reach the other vehicle. On the other hand, if a message from one vehicle reaches the other vehicle, it must travel beyond  $s - L$ . Therefore we have  $C(s) = P(s - L)$  for  $s > L$ .  $\square$

## 4 Numerical Examples

### 4.1 Gamma Approximation

We assume that the vehicle headway follows a Gamma distribution with parameters  $k = 0.1$  and  $\theta = 2.0$  (See Appendix for the particular expression of Gamma distribution used here). By using Theorem 4, the successful propagation distance can be easily plotted.

By using Equation (2) and (4), we can have the mean and variance of successful propagation distance being 0.188 and 0.056 respectively. Let configure a Gamma function so that it yields the same mean and variance of successful propagation distance as calculated. One can find that  $\theta = 0.2955$  and  $k = 0.636$ . With these two parameters, an approximation with Gamma distribution can be plotted as opposed to the true success probability distribution as in Figure 2.

### 4.2 Multihop Connectivity of Two Equipped Vehicles in Uniform Traffic Streams

In order to illustrate the validity of our proposed formulas by comparing with the existing numerical results in literature, we consider a uniform traffic flow, in which the distribution of headways follows an exponential distribution:

$$f(t) = \lambda e^{-\lambda t},$$

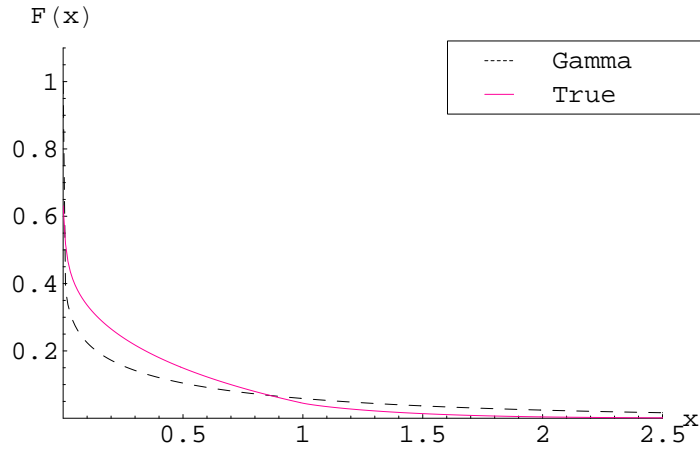


Figure 2: Gamma Approximation to Successful Propagation

where  $\lambda$  is the expected number of equipped vehicles on a piece of road of unit length. Clearly,  $\lambda = \frac{1}{\mu}$ . We study the impact of  $\lambda$  and transmission range  $L$  on connectivity between vehicles with varying distance  $m$  apart.

In Figure 3, the connectivity between an equipped vehicle and the information source is shown for a uniform traffic stream with density of  $\lambda = 5.8$  veh/km, for four transmission ranges  $R=1, 0.5, 0.2, 0.1$  km. In the figure, lines with marks are from Figure 1 of Hartenstein *et al.* (2001)<sup>1</sup>, and the corresponding solid lines are from the analytical model developed in the preceding section. As expected, the connectivity increases with transmission range. In addition, the analytical model yields results highly consistent with those in Hartenstein *et al.* (2001). Considering that Hartenstein *et al.* (2001) accounts for traffic dynamics explicitly, it is seen that vehicle mobility has a very limited impact on multihop connectivity in this case. This observation is also supported by results shown in Figure 4, which compares Figure 2 of Hartenstein *et al.* (2001)<sup>2</sup> with the analytical results from this paper at a traffic density of 1.9 veh/km, a sparser traffic stream.

## 5 Conclusion and Further Discussions

We examine instantaneous information propagation through inter-vehicle communication along a stream of traffic when the vehicle headway follows a general distribution. This generalization of headway distribution is necessary as people often argue that inter-vehicle

<sup>1</sup>Figure 1 of Hartenstein *et al.* (2001) considered a traffic stream on  $2 \times 2$  lanes with average distances between two vehicles at 69m and a market penetration rate of 10%. That is, the total density is approximately  $58 \approx 1000/69 \cdot 4$ . This traffic stream can be approximated by a uniform traffic with  $\lambda = 5.8$  veh/km.

<sup>2</sup>Figure 2 of Hartenstein *et al.* (2001) considered a traffic stream on  $2 \times 2$  lanes with average distances between two vehicles at 208m and a market penetration rate of 10%. That is, the total density is approximately  $1.9 \approx 1000/208 \cdot 4$ . This traffic stream can be approximated by a uniform traffic with  $\lambda = 1.9$  veh/km.

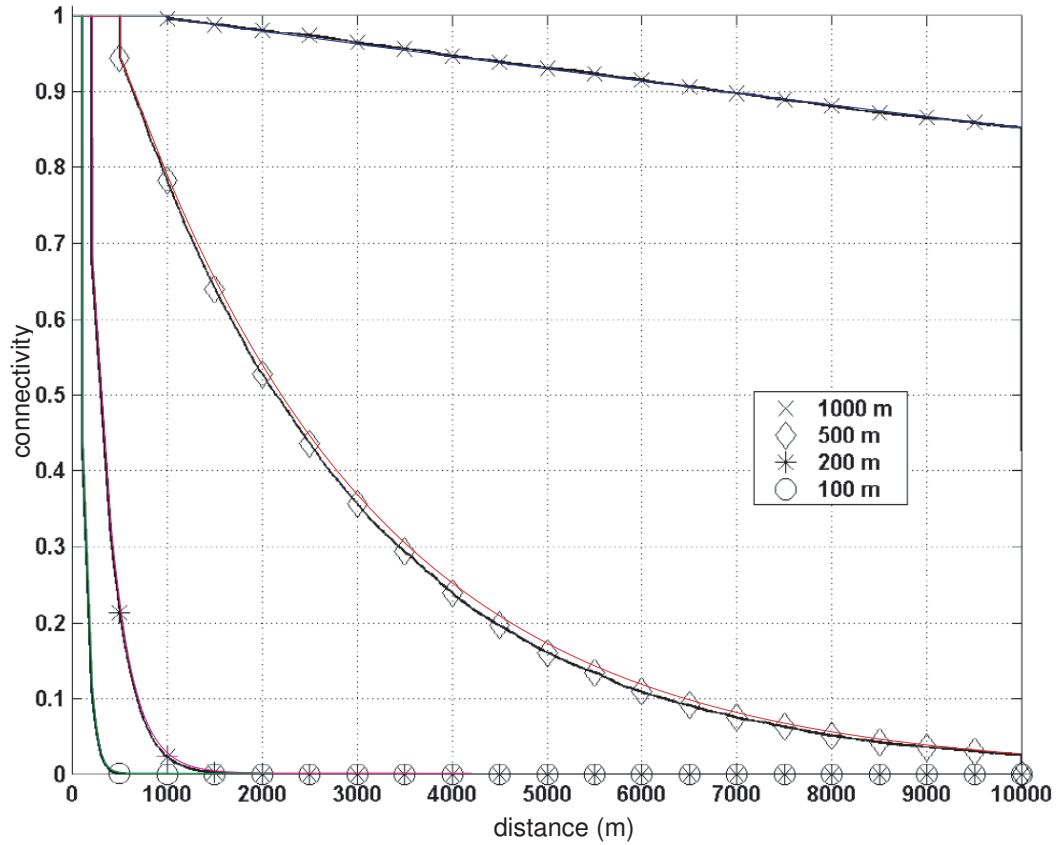


Figure 3: Multihop connectivity in a uniform traffic with  $\lambda = 5.8$  veh/km

headway does not always follow an exponential distribution. Furthermore, the general results enable a robust design by examining impacts on propagation from different headway distributions, including empirical distributions from field data. This study is a follow-up to an earlier work by Wang (2007) which assumes an exponential distribution for the inter-vehicle headway.

Interestingly, the coefficient of variation (CV) of propagation distance in this general case is proved to approach 1.0 infinitely from the right hand side. This indicates that increasing the expected propagation distance does not decrease the variance of distance relative to its mean, which means the reliability of reaching its mean distance does not improve no matter how far the mean distance is. Importantly, Gamma (exponential) approximation to the propagation could still be practical for adoption. A CV approaching 1.0 in this case strongly indicates an asymptotic exponential distribution.

Although vehicular headway distributions might not be independent as vehicles often travel in platoons, the numerical tests indicate that the assumption of independent headway distribution in this paper leads to results of high accuracy. However, limitations exist in this study due to the assumption on instantaneous transmission. For example, individual transmitting vehicle's computing power might be a limiting factor when the

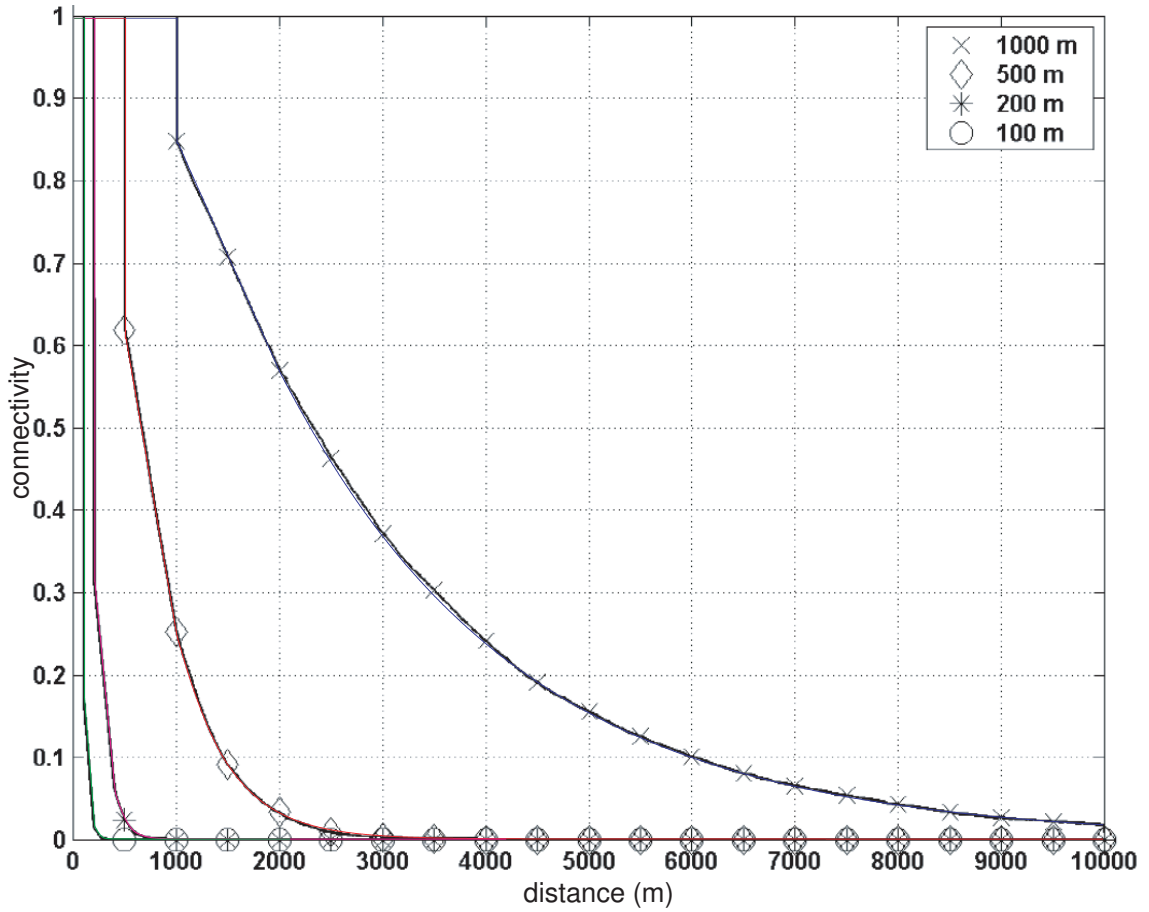


Figure 4: Multihop connectivity in a uniform traffic with  $\lambda = 1.9$  veh/km

propagation distance becomes large as this could dramatically increase the amount of information processed. This limitation can be examined through study of the curbing effect of distance itself.

## 6 Acknowledgements

## 7 Appendix

### 7.1 Gamma Distribution

Gamma distribution has the following probability density function.

$$f(x) = \frac{1}{\theta^k \Gamma(k)} x^{k-1} e^{-x/\theta}$$

Where  $x > 0, k > 0, \theta > 0$ , and  $\Gamma(k)$  is the Gamma function defined as follows.

$$\Gamma(k) = \int_0^{\infty} t^{k-1} e^{-t} dt.$$

Note that the Gamma distribution above has a mean  $k\theta$  and a variance  $k\theta^2$ .

When  $k \rightarrow 1$ , the Gamma distribution approaches an exponential one of the following form with the mean and variance both being  $\theta$ .

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

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