CONNECTIVITY OF VEHICULAR AD HOC NETWORKS WITH CONTINUOUS NODE DISTRIBUTION PATTERNS

WEN-LONG JIN[1]

Department of Civil and Environmental Engineering
California Institute for Telecommunications and Information Technology
Institute of Transportation Studies
4038 Anteater Instruction and Research Bldg
University of California
Irvine, CA 92697-3600
Tel: 949-824-1672
Fax: 949-824-8385
Email: wjin@uci.edu

BRUCE (XIUBIN) WANG

Zachry Department of Civil Engineering
Texas A&M University
College Station, TX
Tel: 979-845-9901
Email: bwang@civil.tamu.com or bwang@tamu.edu.

Word Count: 4000+250×4=5000

November 15, 2009

SUBMITTED TO 2010 TRB ANNUAL MEETING

[1] Author for correspondence

ABSTRACT

The connectivity of vehicular ad hoc networks (VANets) can be affected by the special distribution patterns, usually dependent and non-uniform, of vehicles in a transportation network. In this study, we introduce a new framework for computing the connectivity in a VANet for continuous distribution patterns of communication nodes on a line in a transportation network. With this model, we obtain a new closed-form solution to the connectivity when communication nodes follow homogeneous Poisson distributions and a recursive model when communication nodes follow non-homogeneous Poisson distributions. In the same framework, we derive an approximate closed-form solution to the connectivity when distribution patterns of communication nodes are given by spatial renewal processes. With the developed models, we also discussed the impacts on connectivity of road-side stations and different distribution patterns of vehicles. Given continuous traffic conditions, the connectivity model could be helpful for designing routing protocols in VANets and implementing vehicle-infrastructure integration systems. Limitations and future research related to this study are discussed in the conclusion section.

Keywords: Vehicular ad hoc networks; Inter-vehicle communications; Instantaneous connectivity; Continuous traffic conditions; Poisson processes; Renewal processes

List of Figures

1 Introduction

Rapid developments in various frontiers of telecommunications and information technologies could enable the development of next-generation Intelligent Transportation Systems (ITS) that rely on inter-vehicle communications (IVC) to disseminate time-critical and location-based traffic information. In 2004, US Department of Transportation initiated efforts in developing Vehicle Infrastructure Integration (VII) systems [1, 2]. In a VII system or a Vehicular Ad hoc Network (VANet) shown in **Figure** 1, information can be exchanged among IVC-enabled vehicles, traffic management centers, various elements of road infrastructure including traffic signals, message signs, bus stops, and other safety hardware. If we call both IVC-enabled vehicles and road-side stations as nodes, only a portion of the vehicles are nodes in a transportation network. Compared with existing centralized transportation information systems, IVC-based systems are less costly to deploy and use and more resilient to natural disasters. However, we are also facing many challenges for developing such systems [3].

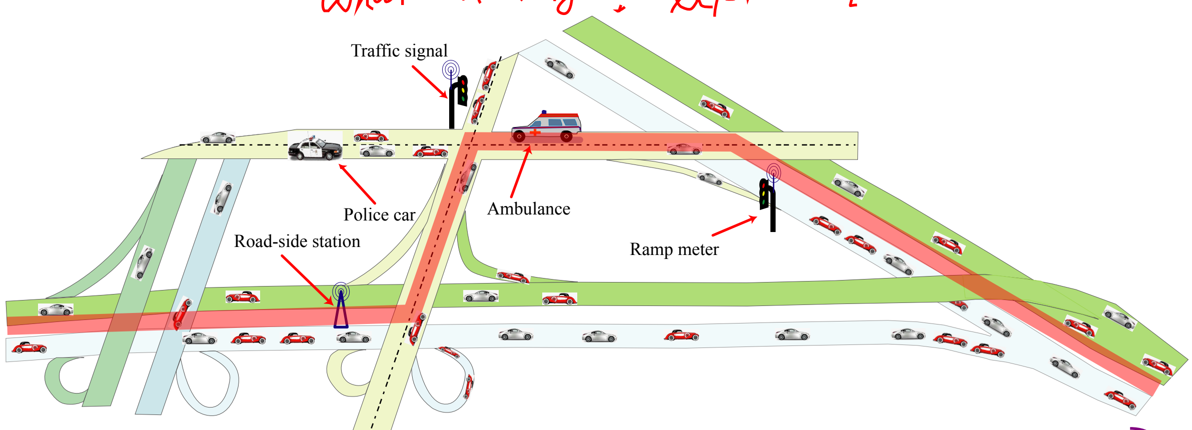


Figure 1: An illustration of vehicular ad hoc networks

In this study, we develop a new modeling framework for the connectivity of VANets, i.e., the probability of establishing a communication path between two nodes. For example, we can use the developed model to estimate the probability of establishing a communication path marked by red in **Figure** 1. An efficient estimation of such connectivity is of practical importance: first, routing protocols would be able to find better information propagation routes by taking routes with higher connectivity; second, connectivity can be used as one metric when determining where and when to deploy stationary or mobile road-side stations. In our study, we are particularly interested in "instantaneous" connectivity at any time instant. Roughly, instantaneous connectivity is determined by the number of communication nodes, their relative locations, and the transmission range of wireless units. In reality, it is possible that both density of vehicles and market penetration rate of communication nodes are known at any location, but the exact locations of equipped vehicles are usually unknown. Therefore, communication nodes follow random distributions with known, location-dependent densities. Note that, in our study of instantaneous connectivity, vehicles of both directions along a communication path are considered to have equivalent contributions. Also we do not differentiate lanes, since for an American standard lane of 12 feet (or 3.7 m) the lateral distances between vehicles on a bidirectional, multilane freeway can be omitted for a DSRC transmission range of up to 1000 m [4]. That is, in our study, we consider a communication path to be a line of communication nodes.

1.1 Related work

As in other mobile ad hoc networks, multihop connectivity is a fundamental performance measure of VANets. Estimating connectivity a priori is essential for determining specification of appropriate communication devices, routing protocols, database management schemes, and the range of effective applications. In the literature, there have been extensive studies on multihop connectivity of various types of radio networks. Studied radio networks can be one-dimensional [5, 6] or two-dimensional [7, 8, 9]. Research methodologies include theoretical analysis of asymptotic connectivity [10] based on percolation theory [11, 12] as well as Monte Carlo simulations [13]. Performance measures of connectivity include expected propagation distance [5], the probability for having at least one communication path between two nodes [14], the *k*-connectivity [15, 16], or the critical transmission range for asymptotic cases [7, 6, 8].

There have also been some studies of connectivity properties of VANets in recent years. In [14], the probability of establishing a communication path between two nodes are studied for mobile bidirectional traffic with simulations. In [17], an analytical model was proposed for estimating connectivity for vehicles following a Poisson distribution and moving randomly and independently. In [18, 19], the propagation distance was studied also with traffic simulations. In [20, 21, 22], the instantaneous success rate and connectivity of VANets were modeled by considering most-forwarded within-range (MFR) information propagation chains when vehicles' positions are given in a uniform or non-uniform traffic stream. In [23], instantaneous information propagation was modeled as a Markov chain in a uniform traffic stream where vehicles' positions follow a Poisson distribution. In [24], the geometric connectivity of one-dimensional VANets is studied for positions of vehicles following independent distributions but with some disturbance. In [25], a convolution connectivity model was developed for a general distribution of the spacing between two consecutive vehicles. In [26], a connectivity model was developed for the connectivity between two nodes, given arbitrary locations of vehicles and probabilities for vehicles to nodes.

As we know, if vehicles' positions follow a homogeneous Poisson distribution, then vehicles' movements are independent, and the traffic density is location-independent and uniform. However, on a line in a transportation network, e.g., the red line shown in **Figure** 1, on which there can be highways, arterial roads, surface streets, and other kinds of roadways, the density of vehicles and, therefore, the density of nodes, can vary dramatically due to driving behaviors and restrictions of network geometry. For examples, with higher travel demands and more lanes, freeways usually carry much higher density of vehicles than local streets; around a lane-drop or merging area, traffic density is usually significantly higher in the upstream section with the formation of queues; when a shock wave forms [27], traffic density is higher in the downstream part; vehicles tend to form clusters in sparse traffic; and traffic signals can cause gaps between vehicle platoons. Therefore, it is important to consider the impacts of the special distribution patterns and mobility patterns of vehicles in transportation networks when estimating the connectivity between two nodes.

In a road network, traffic patterns can be represented by discrete vehicle positions as in car-following models [28], distribution patterns of the spacings between two consecutive vehicles [29], or continuous densities as in kinematic wave models [27, 30, 31]. With loop detectors, it is also possible to obtain continuous traffic conditions on a road network. In this study, we present a framework for computing multihop connectivity along a communication path in a road network when the distribution patterns of communication nodes are given by spatial Poisson processes or renewal processes [32].

1.2 Contributions

In this study, we will develop a model for the connectivity of VANets along a line of vehicles in a transportation network. We omit the impacts of the mobility of nodes on the connectivity and consider the instantaneous multihop connectivity as in [20, 21, 26]. Different from most existing studies, we do not assume either independent or uniform distributions of nodes. Different from studies in [20, 21, 26], where the positions of vehicles are discrete, we consider continuous distributions of vehicles, where either the distribution function of spacings between two consecutive nodes or the location-dependent density of nodes are given. We will examine the relationship between the *end node probability* and *connectivity* and develop a simplified model for computing connectivity between two nodes. With the model, we will be able to discuss how the distribution patterns of vehicles and traffic dynamics can affect connectivity properties of an IVC system. Although the results are more general since the distribution patterns of communication nodes are arbitrary, the results can also be applied for the case when communication nodes follow Poisson distributions.

The rest of the paper is organized as follows. In Section 2, we give a detailed explanation of the conceptual framework and definitions for our new model and derive an analytical recursive model of connectivity for arbitrary distribution of nodes. In Section 3, we discuss a uniform continuous traffic, where the nodes follow the Poisson distribution and obtain a new closed-form formulation of the connectivity. In Section 4, we study a general continuous traffic, where the spacings between two consecutive nodes follow a general distribution and obtain an approximate closed-form formulation of the connectivity. In Section 5, we study a continuous traffic with a general location-dependent density of nodes and discuss the impacts of variations in densities on the connectivity. In Section 6, we make some conclusions.

2 A recursive model of connectivity based on end-node distributions

We consider vehicles and road-side stations along a line in a transportation network and assume that information propagates in the direction of x-axis. At a time instant, we take a snap-shot of traffic flow on a line and analyze IVC information propagation along the line. The continuous patterns of nodes in spacing distributions or densities, obtained through observations or simulations, can be arbitrarily uniform or non-uniform. Here we assume that all nodes have the same DSRC transmission range r and apply a simple communication model, in which two nodes can communicate with each other when they are within each other's transmission range, and do not consider the effect of signal interferences [33].

2.1 Model derivation

For the information propagation process starting from the sender at $x = x_0$ to any location $x \geq x_0$, we denote the farthest reach of a message through multihop relays by a random variable X. Let $p(x_0, x)$ $(x > x_0)$ be the probability density function of the farthest reach of a message, so that $\mathbf{P}(x_0, x) = Prob(X \leq x)$ is the cumulative distribution of the farthest reach. Therefore, $p(x_0, x)\Delta x$

can be considered as the *end node probability* in $[x, x+\Delta x)$ as in [26]. If the probability that there does not exist another node within the transmission range of the sender is $p(x_0)$, then $\bar{p}(x_0, x)$ has a spike (or discrete component) at x_0 of height p_0. Thus $\bar{p}(x_0, x)$ is a mixed probability density function [29]

$$\bar{p}(x_0, x) \quad = \quad \begin{cases} p(x_0)\delta(x - x_0) + p(x_0, x), & x \geq x_0, \\ 0, & x < x_0, \end{cases}$$

where $\delta(x - x_0)$ is a Dirac function. That is,

$$\mathbf{P}(x_0, x) \quad = \quad \begin{cases} p(x_0) + \int_{y=x_0}^{x} p(x_0, y)dy, & x \geq x_0, \\ 0, & x < x_0. \end{cases} \tag{1}$$

We can see that $\mathbf{P}(x_0, x)$ can be considered as the probability for a message to cover a region of $[x_0, x]$, and $\mathbf{S}(x_0, x) = Prob(X > x) = 1 - \mathbf{P}(x_0, x)$ is the success rate for a message to travel beyond x.

Let $\mathbf{C}(x_0, x)$ be the connectivity between two nodes at x_0 and x, i.e., the probability for a message to reach a receiver at x. Then we have that

$$\mathbf{C}(x_0, x) \quad = \quad 1 - \mathbf{P}(x_0, x - r) = \mathbf{S}(x_0, x - r). \tag{2}$$

Therefore, for $x > x_0$

$$p(x_0, x) \quad = \quad -\frac{\mathrm{d}\mathbf{C}(x_0, x + r)}{\mathrm{d}x}. \tag{3}$$

We denote the probability for existing at least one node in $[x, x+\Delta x]$ by $\mathscr{P}_1([x, x+\Delta x])$ and the probability for existing no node in $(x+\Delta x, x+r+\Delta x]$ by $\mathscr{P}_0((x+\Delta x, x+r+\Delta x])$. The farthest reach of a message is in $[x, x+\Delta x]$ if and only if (i) there exists at least one node in $[x, x+\Delta x]$, (ii) the node is connected to the sender, and (iii) there is no node in $(x+\Delta x, x+r+\Delta x]$. Since all these three events are independent, the probability for the farthest reach in $[x, x+\Delta x]$ is

$$p(x_0, x)\Delta x \quad = \quad \mathscr{P}_1([x, x+\Delta x])\mathbf{C}(x_0, x)\mathscr{P}_0((x+\Delta x, x+r+\Delta x]). \tag{4}$$

Letting $\Delta x \to 0$, we thus have

$$p(x_0,x) = \mathbf{C}(x_0,x)\mathscr{P}_0((x,x+r])\lim_{\Delta x\to 0}\frac{\mathscr{P}_1([x,x+\Delta x])}{\Delta x}. \tag{5}$$

If we denote $\lambda(x,r) = \mathscr{P}_0((x,x+r])\lim_{\Delta x\to 0}\frac{\mathscr{P}_1([x,x+\Delta x])}{\Delta x}$, then we can have

$$\lambda(x_0,r) = \mathscr{P}_0((x_0,x_0+r])\delta(x-x_0), \tag{6}$$

since the sender is always equipped and $\mathscr{P}_1([x_0,x_0+\Delta x]) = 1$. From **Equation** 1 we have

$$p(x_0) = \mathscr{P}_0((x_0,x_0+r]). \tag{7}$$

Combining **Equation** 1-**Equation** 2 and **Equation** 4, we have the following models of connectivity and probability density function of farthest reach

$$\mathbf{C}(x_0,x) = \begin{cases} 1-\int_{y=x_0}^{x-r}\mathbf{C}(x_0,y)\lambda(y,r)dy, & x \geq x_0+r, \\ 1, & x < x_0+r. \end{cases} \tag{8}$$

$$\bar{p}(x_0,x) = \begin{cases} (1-\int_{y=x_0}^{x-r}\bar{p}(x_0,y)dy)\lambda(x,r), & x \geq x_0, \\ 0, & x < x_0. \end{cases} \tag{9}$$

In this model, $\lambda(x,r)$ depends on the distribution of nodes along a line and are determined by traffic conditions and the market penetration of equipped vehicles. Further, both equations are recursive, in the sense that both $\mathbf{C}(x_0,x)$ and $p(x_0,x)$ can be computed from the corresponding quantities in $[x_0,x-r]$. In addition, from **Equation** 3 and **Equation** 4, we can obtain a first-order linear delay differential equation for $x-r > x_0$

$$\frac{\mathrm{d}\mathbf{C}(x_0,x)}{\mathrm{d}x} = -\lambda(x,r)\mathbf{C}(x_0,x-r). \tag{10}$$

From **Equation** 8 and **Equation** 9 we can see that the connectivity between the sender at x_0 and a receiver at x only depends on the distribution of nodes in $[x_0,x]$, and $p(x_0,x)$ depends on the distribution of nodes in $[x_0,x+r]$. In addition, $\mathbf{C}(x_0,x)$ is non-increasing with x, but $\bar{p}(x_0,x)$ may not be. The connectivity function is symmetric with respect to the information propagation direction; i.e., $\mathbf{C}(x_0,x) = \mathbf{C}(x,x_0)$.

2.2 Improving connectivity with road-side stations

In VANets, as shown in **Figure** 1, stationary road-side stations can be installed by transportation authorities for the purpose of collecting and disseminating traffic information or by other parties for commercial or other purposes. In emergency situations, police cars, ambulance, and other emergency vehicles could be deployed along a road and serve as road-side stations. Here we assume that the road-side stations have the same communication units as vehicles and are not inter-connected through wired or wireless communications other than inter-vehicle communications. In the following, we study how such road-side stations can improve the connectivity of VANets.

Theorem 2.1 *If there is a road side station at* $x = x_1 \in (x_0, x_2)$*; i.e., if* $\lambda(x_1, r) = \mathscr{P}_0((x_1, x_1 + r])\delta(x - x_1)$*, then*

$$\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1)\mathbf{C}(x_1, x_2). \tag{11}$$

Proof. If $x_2 < x_0 + r$, it is obvious, since $\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1) = \mathbf{C}(x_1, x_2) = 1$. For $x_2 \geq x_0 + r$, we have from **Equation** 8

$$\begin{aligned}\mathbf{C}(x_0, x_2) &= 1 - \int_{y=x_0}^{x_1 - r} \mathbf{C}(x_0, y)\lambda(y, r)dy \\ &\quad - \int_{y=x_1 - r}^{x_1^-} \mathbf{C}(x_0, y)\lambda(y, r)dy \\ &\quad - \int_{y=x_1}^{x_2 - r} \mathbf{C}(x_0, y)\lambda(y, r)dy.\end{aligned}$$

Since we can always find a node, i.e., the road-side station, in $(y, y + r]$, $\lambda(y, r) = \mathscr{P}_0((y, y + r]) \lim_{\Delta x \to 0} \frac{\mathscr{P}_1([y, y + \Delta x])}{\Delta x} = 0$ for $y \in [x_1 - r, x_1)$, we have

$$\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1) - \int_{y=x_1}^{x_2 - r} \mathbf{C}(x_0, y)\lambda(y, r)dy.$$

When $x_2 < x_1 + r$, from the equation above we have $\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1)$. Since $\mathbf{C}(x_1, x_2) = 1$ for $x_2 < x_1 + r$, we have $\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1)\mathbf{C}(x_1, x_2)$. For $x_2 \geq x_1 + r$, assuming that $\mathbf{C}(x_0, y) =$

$\mathbf{C}(x_0,x_1)\mathbf{C}(x_1,y)$ for $y \in [x_1, x_2 - r]$, from the equation above we have

$$\begin{aligned}
\mathbf{C}(x_0,x_2) &= \mathbf{C}(x_0,x_1) - \int_{y=x_1}^{x_2-r} \mathbf{C}(x_0,x_1)\mathbf{C}(x_1,y)\lambda(y,r)dy \\
&= \mathbf{C}(x_0,x_1)(1 - \int_{y=x_1}^{x_2-r} \mathbf{C}(x_1,y)\lambda(y,r)dy) \\
&= \mathbf{C}(x_0,x_1)\mathbf{C}(x_1,x_2).
\end{aligned}$$

By mathematical induction, we conclude that **Equation** 11 is always true. ■

For given distributions of nodes, i.e., $\lambda(x,r)$ for $x \in [x_0, x_2]$, the solution of the following optimization problem

$$\max_{x \in [x_0,x_2]} \mathbf{C}(x_0,x)\mathbf{C}(x,x_2)$$

will yield the best location for deploying a road-side station along $[x_0, x_2]$. For more than one road-side stations, we can also formulate the problem similarly. Thus, the solution of the optimization problem will determine the best locations of deploying emergency vehicles when disasters occur.

3 Poisson distributions of nodes

In this section, we consider the case when communication nodes follow spatial Poisson distributions, either homogeneous or inhomogeneous. Here we assume that the sender is at $x_0 = 0$ and denote $\mathbf{C}(x) = \mathbf{C}(0,x)$ and $p(x) = p(0,x)$. If nodes follow the Poisson distribution with a constant node density κ, then the nodes are uniformly and independently distributed on a line, and the spacing between two consecutive nodes follows a negative exponential distribution.

3.1 Homogeneous Poisson processes

When the distributions of communication nodes are given by a homogeneous Poisson process with a location-independent average density κ, we have that $\mathscr{P}_0((x,x+r]) = e^{-\kappa r}$, and $\lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x,x+\Delta x])}{\Delta x} =$

κ. Therefore, $p(0) = e^{-\kappa r}$, $\lambda(x,r) \equiv \lambda(r) = \kappa e^{-\kappa r}$, and the recursive connectivity model **Equation** 8 can be written as

$$\mathbf{C}(x) = \begin{cases} 1, & x < r, \\ \mathbf{C}(r) \equiv 1 - p(0), & x = r, \\ \mathbf{C}(r) - \lambda(r) \int_{y=0^+}^{x-r} \mathbf{C}(y)dy, & x > r. \end{cases} \tag{12}$$

From **Equation** 10 we obtain for $x > r$

$$\frac{\mathrm{d}\mathbf{C}(x)}{\mathrm{d}x} = -p(x-r) = -\lambda(r)\mathbf{C}(x-r). \tag{13}$$

According to [10], we have for $x \geq 2r$

$$\mathbf{C}(r)e^{-(x-2r)\lambda(r)} - \lambda(r) \leq \mathbf{C}(x) \leq \mathbf{C}(r)e^{-(x-r)\lambda(r)}. \tag{14}$$

Here we can study the critical behavior of connectivity by assuming the following critical transmission range

$$r_c = \alpha \log(x\kappa)/\kappa, \tag{15}$$

where $\alpha > 0$. Then $\lambda(r_c) = \kappa(x\kappa)^{-\alpha}$, $\mathbf{C}(r_c) = 1 - (x\kappa)^{-\alpha}$, and

$$\mathbf{C}(r_c)e^{-(x-2r)\kappa(x\kappa)^{-\alpha}} - \kappa(x\kappa)^{-\alpha} \leq \mathbf{C}(x) \leq \mathbf{C}(r_c)e^{-(x-r)\kappa(x\kappa)^{-\alpha}}.$$

For fixed κ and $x \to \infty$, we have

$$\lim_{x\to\infty} \mathbf{C}(r_c)e^{-(x-2r)\kappa(x\kappa)^{-\alpha}} - \kappa(x\kappa)^{-\alpha} =$$

$$\lim_{x\to\infty} \mathbf{C}(r_c)e^{-(x-r)\kappa(x\kappa)^{-\alpha}} = \begin{cases} 1, & \alpha > 1, \\ 0, & \alpha < 1. \end{cases}$$

Therefore,

$$\lim_{x\to\infty} \mathbf{C}(x) = \begin{cases} 1, & \alpha > 1, \\ 0, & \alpha < 1, \end{cases}$$

which is consistent with the results in [6]. For fixed x and $\kappa \to \infty$, we have similar results. In [8], a less strict bound on r_c for two-dimensional wireless networks and $\kappa \to \infty$ was presented, and it is as expected since better connectivity can be achieved in a two-dimensional network.

Theorem 3.1 *We have that for a natural number $n \geq 1$*

$$\mathbf{C}(x) = \begin{cases} 1, & x \in [0,r), \\ \sum_{m=0}^{n} \mathbf{C}(mr)\frac{(-\lambda(r)(x-nr))^{n-m}}{(n-m)!}, & x \in [nr,(n+1)r]. \end{cases} \tag{16}$$

Proof. It is straightforward to verify that **Equation** 16 satisfies **Equation** 12 or **Equation** 13. The derivation of **Equation** 16 is based on the observation of **Equation** 12 that $\mathbf{C}(x)$ is constant in $[0,r)$, linear in $[r,2r)$, quadratic in $[2r,3r)$, and so on. Note that for $x \in [0,r)$, we can also have $\mathbf{C}(x) = \sum_{m=0}^{n} \mathbf{C}(mr)\frac{(-\lambda(r)(x-nr))^{n-m}}{(n-m)!}$, but we have it separately to emphasize that there is a discontinuity at $x = r$. ■

From **Equation** 16, in particular, we have $C(0) = 1$ and

$$\begin{aligned} \mathbf{C}((n+1)r) &= \begin{cases} 1-p(0), & n=0, \\ \sum_{m=0}^{n} \mathbf{C}(mr)\frac{(-\lambda(r)r)^{n-m}}{(n-m)!}, & n \geq 1. \end{cases} \\ &= \begin{cases} 1-p(0), & n=0, \\ \sum_{m=0}^{n} \mathbf{C}((n-m)r)\frac{(-\lambda(r)r)^{m}}{m!}, & n \geq 1. \end{cases} \end{aligned} \tag{17}$$

It is straightforward to show that **Equation** 16 is equivalent to the closed-form solution in [10]. Also it can be shown that $\mathbf{C}((n+1)r) \leq \mathbf{C}(nr)$ and $\mathbf{C}(x) \in [\mathbf{C}((n+1)r), \mathbf{C}(nr)]$ for $x \in [nr,(n+1)r]$.

From **Equation** 16 and **Equation** 2, we have the cumulative distribution of farthest reach as $x \in [(n-1)r, nr]$

$$\mathbf{P}(x) = 1 - \sum_{m=0}^{n} \mathbf{C}(mr)\frac{(-\lambda(r)(x+r-nr))^{n-m}}{(n-m)!}. \tag{18}$$

Further from **Equation** 4 and **Equation** 13, we have $n = 1,2,\cdots$

$$p(x) = \lambda(r)\mathbf{C}(x) = \begin{cases} \lambda(r), & x \in [0,r), \\ \lambda(r)\sum_{m=0}^{n} \mathbf{C}(mr)\frac{(-\lambda(r)(x-nr))^{n-m}}{(n-m)!}, & x \in [nr,(n+1)r]. \end{cases}$$

Then the average distance for information propagation along an infinitely long traffic stream is

$$\begin{aligned} E(X) &= \int_{x=0}^{\infty} p(x)xdx = -\int_{x=0}^{\infty} xd\mathbf{C}(x+r) \\ &= -x\mathbf{C}(x+r)|_0^{\infty} + \int_{x=0}^{\infty} \mathbf{C}(x+r)dx \\ &= \int_{x=0}^{\infty} \mathbf{C}(x+r)dx = -\frac{\mathbf{C}(x+2r)}{\lambda(r)}|_{x=0}^{\infty} \\ &= \frac{\mathbf{C}(2r)}{\lambda(r)} = \frac{\mathbf{C}(r)}{\lambda(r)} - r, \end{aligned} \tag{19}$$

according to **Equation** 13 and **Equation** 17. Therefore,

$$E(X) = \frac{e^{\kappa r} - 1}{\kappa} - r.$$

Similarly, we can get the variance of the information propagation distance as

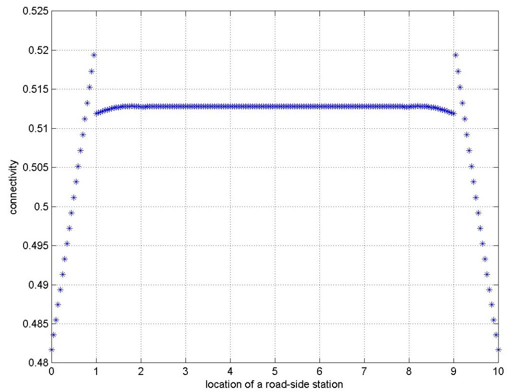
$$V(X) = E(X^2) - [E(X)]^2 = \frac{2}{\lambda(r)^2} C(3r) - [E(x)]^2 = \frac{e^{2\kappa r} - 2\kappa r e^{\kappa r} - 1}{\kappa^2}.$$

Both results are consistent with those in [23].

In **Figure** 2, we show the impact of the location of a road-side station on the connectivity between a sender and a receiver which are 10 km away with $\kappa = 4$ nodes/km and $r = 1$ km. Here the connectivity is computed with **Equation** 16 and **Equation** 11. From the figure we can see that the best locations of the road-side station are just within the transmission range of either the sender or the receiver. Here the discontinuity in the connectivity curve is caused by that in $\mathbf{C}(x)$ in **Equation** 16 at $x = r$.

3.2 Inhomogeneous Poisson processes

In a continuous traffic flow theory, one can track the changes in density $\rho(x,t)$. Further, if we assume that the probability for vehicles to be equipped as $\mu(x,t)$, then the density of nodes is $\kappa(x) = \rho(x)\mu(x)$, where time t is omitted for instantaneous information propagation in a continuous traffic stream. In this case, the distribution patterns of communication nodes follow a



Consistent w/ discrete results

Figure 2: Impact of the location of a road-side station

non-homogeneous Poisson process with location-dependent densities. From the definition of $\kappa(x)$, we can see that

$$\kappa(x) \quad = \quad \lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x, x+\Delta x])}{\Delta x}. \tag{20}$$

That is, $\kappa(x)\Delta x$ is the probability for finding at least one equipped vehicle in $[x, x+\Delta x]$. Since the sender is at $x = x_0$ and always equipped, we have have a spike at $x = x_0$ and

$$\bar{\kappa}(x) \quad = \quad \delta(x - x_0) + \kappa(x).$$

For such a continuous traffic stream, we do not have a location-independent spacing distribution function and thus lose the convolution formulation in **Equation** 22. However, since the probability for existing no node in $(x^+, x+r]$ is $\mathscr{P}_0((x^+, x+r]) = e^{-\int_{x^+}^{x+r} \kappa(y)dy}$, we obtain a recursive model in **Equation** 8 and the differential equation in **Equation** 10 with

unconditional node pdf or pdf when $x_0 \in [x-r, x)$

$$\lambda(x, r) = \kappa(x) e^{-\int_{x^+}^{x+r} \kappa(y)dy}. \tag{21}$$

The correctness of the model **Equation** 8 with **Equation** 21 can be proved with the discrete model developed in [26] as follows. We approximate $\kappa(x)$ with a piece-wise constant function as

$$\bar{\kappa}(x) = \begin{cases} \delta(x-x_0), & x = x_0 \\ \kappa_i \equiv \int_{x_0+(i-1)\Delta x}^{x_0+i\Delta x} \kappa(x)dx, & x - x_0 \in ((i-1)\Delta x, i\Delta x] \end{cases}$$

Then in cell i for $x \in (x_0 + (i-1)\Delta x, x_0 + i\Delta x]$, where $\Delta x = (x - x_0)/m$, we place vehicles at $x_{i,j} = x_0 + (i-1)\Delta x + \frac{j-1/2}{n}\Delta x$ for $j = 1, \cdots, n$, and each vehicle has the probability of $\mu_i = \frac{\kappa_i \Delta x}{n}$ to be a node. For this discrete traffic stream, we can apply the results in [26] and obtain the connectivity of $\mathbf{C}_d(x_0, x; m, n)$. Then $\mathbf{C}(x_0, x) = \lim_{m,n\to\infty} \mathbf{C}_d(x_0, x; m, n)$ satisfies **Equation** 8 and **Equation** 21.

We can compute $\mathbf{C}(x_0, x)$ numerically as follows. We divide $[x_0, x]$ into cells with a length of $\Delta x = r/n$ and denote $\mathbf{C}_i \equiv \mathbf{C}(x_0, x_i)$ with $x_i = x_0 + i\Delta x$. We compute $\lambda(x_i, r)$ in **Equation** 21 as

$$\lambda(x_i, r) = \kappa(x_i) e^{-\sum_{j=i+1}^{i+n} \kappa(x_j)\Delta x}.$$

In particular, $\lambda(x_0, r) = e^{-\sum_{j=1}^{n} \kappa(x_j)\Delta x}$. Further we can compute $\mathbf{C}_i$ from **Equation** 8 for $i \geq n$

$$\mathbf{C}_i = \begin{cases} 1, & i < n, \\ 1 - \lambda(x_0, r), & i = n, \\ \mathbf{C}_{i-1} - \mathbf{C}_{i-n}\lambda(x_{i-n}, r)\Delta x, & i > n. \end{cases}$$

In this subsection, we consider a road section with a length of 10 km, on which a sender and a receiver are at the boundaries. The average density of the road section is ρ, and vehicles are uniformly distributed with density ρ_1 and ρ_2 on the first and second halves of the road, respectively. Thus, $\rho_1 + \rho_2 = 2\rho$. When the probability for all vehicles inside the road section to be equipped is the same as $\mu = 0.1$, and the transmission range for all nodes is the same as $r = 1$ km, we have

$$\kappa(x) = \begin{cases} \rho_1\mu, & x \in [0, 5], \\ \rho_2\mu, & x \in (5, 10]. \end{cases}$$

When we divide a transmission range into $n = 100$ cells, the connectivity between the sender and the receiver is shown in **Figure** 3 for different ρ and ρ_1, whose units are nodes/km. From the figure, we can see that, when the total number of nodes inside the road section is the same, the distribution patterns of vehicles can significantly affect the connectivity, and the connectivity reaches its maximum when $\rho_1 = \rho$; i.e., when vehicles are uniformly distributed on the whole road section. In addition, we can see that the connectivity is symmetric in ρ_1.

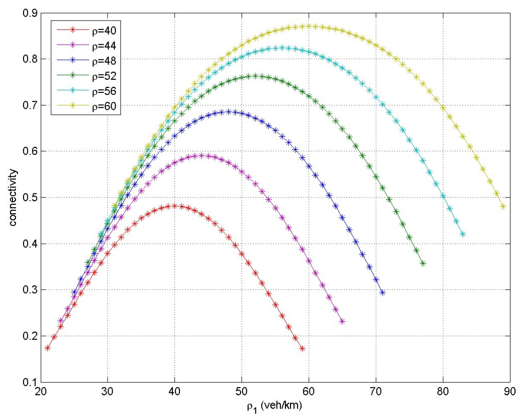


Figure 3: Impacts of the distribution patterns on connectivity

Here, we make the following conjecture: For the same number of nodes between a sender and a receiver, when all nodes have the same transmission range, then the connectivity between the sender and the receiver reaches its maximum when these nodes are uniformly distributed between the sender and the receiver.

4 An approximate connectivity model for renewal processes of communication nodes

When the distribution patterns of communication nodes are governed by renewal processes, we denote the spacing between two communication consecutive nodes by a random variable S. Let $f(s)$ be the probability density function of S, so that $F(s) = Prob(S \leq s)$ is the cumulative distribution

literature review on headway distributions?
→ reviewed by Jaeyong?

of the spacing. As we know, on a multilane road, $s \in [0, \infty)$. In this case, the distribution of nodes is still location-independent, but may lead to non-uniform traffic density.

4.1 A convolution model

For any spacing distribution function $f(s)$, we have for $x \geq r$ [10, 25]

$$\mathbf{C}(x) = \int_{x-r}^{x} \mathbf{C}(y) f(x-y) dy = \int_{0}^{r} \mathbf{C}(x-s) f(s) ds. \tag{22}$$

This yields $\mathbf{C}(r) = \int_0^r f(s)ds$, and

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x) &= \int_0^r \frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x-s) f(s) ds = -\int_0^r f(s) d\mathbf{C}(x-s) \\
&= -f(s)\mathbf{C}(x-s)|_0^r + \int_0^r \mathbf{C}(x-s) f'(s) ds \\
&= -f(r)\mathbf{C}(x-r) + f(0)\mathbf{C}(x) + \int_0^r \mathbf{C}(x-s) f'(s) ds
\end{aligned}$$

If we can still apply the recursive model **Equation** 8 with a location-independent $\lambda(r)$, then we have the following:

$$\mathbf{C}(x) = \begin{cases} 1, & x < r, \\ \mathbf{C}(r) \equiv 1 - p(0) = \int_0^r f(s)ds, & x = r, \\ \mathbf{C}(r) - \lambda(r) \int_{y=0^+}^{x-r} \mathbf{C}(y) dy, & x > r, \end{cases} \tag{23}$$

and $\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x) = -\lambda(r)\mathbf{C}(x-r)$ for $x > r$, which leads to $\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x)|_{r^+} = -\lambda(r)$. For $f(s) = \kappa^2 e^{-\kappa s} s$, we have $f(0) = 0$, and $\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x)|_{r^+} = -f(r)\mathbf{C}(0) + f(r) = 0$. In addition, $\lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x, x+\Delta x])}{\Delta x} = 0$. Therefore, $f(r) = 0$, and we cannot obtain a recursive formulation as in **Equation** 23.

4.2 An approximate closed-form solution

We derive an approximate solution of **Equation** 22 in the form of **Equation** 23 and choose $\lambda(r)$ such that the average information propagation distance is the same. From **Equation** 19, we have

$$E(\lambda) = \frac{\int_0^r f(s)ds}{\lambda(r)} - r.$$

The average information propagation distance for **Equation** 22 is [25]

$$E(x) = \frac{\int_0^r s f(s) ds}{1 - \int_0^r f(s) ds}.$$

Therefore, we choose

$$\lambda(r) = \frac{\int_0^r f(s) ds (1 - \int_0^r f(s) ds)}{r - r \int_0^r f(s) ds + \int_0^r s f(s) ds}. \tag{24}$$

Then the closed-form solution of **Equation** 23 is the same as in **Equation** 16 with $\lambda(r)$ given in **Equation** 24.

When $f(s) = \kappa^2 e^{-\kappa s} s$, $r = 1$ km, and $\kappa = 5.8$ nodes/km, the results of the connectivity at x obtained by the accurate model in **Equation** 22 and the approximate model in **Equation** 16 with **Equation** 24 are shown in **Figure** 4, from which we can see the approximate model is quite accurate.

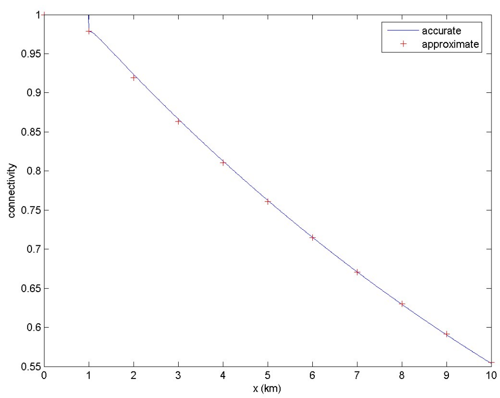


Figure 4: Accurate and approximate connectivity from **Equations** 22 and 17 respectively

5 Conclusion

In this paper we presented a recursive model for the connectivity in a VANet with continuous node distributions governed by Poisson or renewal processes and considered the improvement of connectivity by road-side stations. Given homogeneous Poisson distributions of nodes, we discussed the asymptotic properties of the connectivity and obtained a closed-form formulation of the connectivity. Given non-homogeneous Poisson distributions of nodes, we applied the recursive model to study the impact of variations in node densities. Given renewal processes of nodes, we showed that there does not exist an exact recursive model but presented an approximate solution. With the developed models, we also discussed the impacts on connectivity of road-side stations and different distribution patterns of vehicles.

In the developed model, we assume that a traveler or system operator can have full information on $\rho(x)$, the traffic density, and $\mu(x)$, the probability that vehicles are equipped with wireless communication units, at x along a communication path. Such an assumption can be too strict in reality. In the future, we will investigate how the accuracy in estimating traffic densities can impact the estimation of connectivity.

In a traffic stream, the evolution of traffic density on a homogeneous road-link can be described by the following LWR theory [27, 30]

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial Q(\rho(x,t))}{\partial x} = 0,$$

where $Q(\rho(x,t))$ is a unimodal flux-density relation, or a fundamental diagram. Thus we can also study the dynamics of the instantaneous connectivity when we know the probability for vehicles to be equipped, $\mu(x,t)$, such that the density of nodes $\kappa(x,t) = \rho(x,t)\mu(x,t)$. In addition, mobility of vehicles can improve the connectivity of VANets [34], and we will be interested in studying the impact of traffic dynamics on the connectivity during a time interval. The model could be used to develop more efficient communication routing protocols based on connectivity estimations [35] and design vehicle-infrastructure integration systems [2].

The developed modeling framework also bears certain limitations. First, in this model, all nodes are assumed to have the same transmission range; it will be interesting to check how heterogeneous transmission ranges would impact connectivity. Second, we omit the impacts of lateral distances between vehicles on bidirectional, multilane roadways; this assumption may not be acceptable when transmission ranges are much smaller than 1000 m. Third, a very simple underlying communication model is assumed for this study; i.e., two nodes are connected when they are within the transmission range. But at the physical level the connectivity between two nodes is a random process determined by transmission power, and at the MAC level communications between two nodes may not be established due to signal contention and other interferences. In the future, we will also be interested in studying how transportation network topology and vehicle mobility would impact connectivity.

Acknowledgments

The author would like to thank Will Recker, Amelia Regan and Rex Chen at the University of California, Irvine for their helpful comments and discussions. Several anomalous reviewers' comments are also appreciated. This research was supported in part by a grant from the University of California Transportation Center (UCTC). The views and results herein are the authors' alone.

References

[1] USDOT. USDOT'S ITS Program - Major Initiatives — 2004, 2004. URL `http://www.its.dot.gov/press/Initiatives4.htm`. Accessed January 30, 2007.

[2] X. Dong, K. Li, J. Misener, P. Varayia, and W. Zhang. Expediting Vehicle Infrastructure Integration (EVII). Technical report, 2006. UCB-ITS-PRR-2006-20.

[3] J. J. Blum, A. Eskandarian, and L. J. Hoffman. Challenges of intervehicle ad hoc networks. *IEEE Transactions on ITS*, 5(4), December 2004.

[4] FCC. FCC allocates spectrum in 5.9 ghz range for Intelligent Transportation Systems

uses, 1999. URL http://www.fcc.gov/Bureaus/Engineering_Technology/News_Releases/1999/nret9006.html. Accessed September 26, 2007.

[5] Y.-C. Cheng and T. G. Robertazzi. Critical connectivity phenomena in multihop radio models. *IEEE Transactions on Communications*, 37(7):770–777, July 1989.

[6] Philippe Piret. On the connectivity of radio networks. *IEEE Transactions on Information Theory*, 37(5):1490–1492, September 1991.

[7] T. J. Philips, S. S. Panwar, and A. N. Tantawi. Connectivity properties of a packet radio network model. *IEEE Transactions on Information Theory*, 35(5):1044–1047, September 1989.

[8] P. Gupta and P. R. Kumar. Critical power for asymptotic connectivity in wireless networks. In W. M. McEneaney, G. Yin, and Q. Zhang, editors, *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming*. Birkhäuser, Boston, 1998.

[9] M Desai and D Manjunath. On the connectivity in finite ad hoc networks. *IEEE COMMUNICATIONS LETTERS*, 6(10):437–439, October 2002.

[10] Olivier Dousse, Patrick Thiran, and Martin Hasler. Connectivity in ad-hoc and hybrid networks. In *Proc. IEEE Infocom*, New York, June 2002.

[11] E. N. Gilbert. Random plane networks. *SIAM Journal on Applied Mathematics*, 9(4):533–543, December 1961.

[12] R. Meester and R. Roy. *Continuum Percolation*. Cambridge University Press, 1996.

[13] A Tang, C Florens, and S H Low. An empirical study on the connectivity of ad hoc networks. In *IEEE Proceedings Aerospace Conference*, volume 3, pages 1333–1338, March 2003.

[14] Hannes Hartenstein, Bernd Bochow, André Ebner, Mathhias Lott, Markus Radimirsch, and Dieter Vollmer. Position-aware ad hoc wireless networks for inter-vehicle communications: the fleetnet project. In *Proceedings of the 2nd ACM international symposium on Mobile ad hoc networking & computing*, pages 259 – 262, Long Beach, CA, USA, 2001.

[15] Mathew D. Penrose. On k-connectivity for a geometric random graph. *Random Structures and Algorithms*, 15(2):145 – 164, 1999.

[16] Christian Bettstetter. On the minimum node degree and connectivity of a wireless multihop network. In *MobiHoc*, pages 80–91, 2002.

[17] Hao Wu, R. Fujimoto, and G. Riley. Analytical models for information propagation in vehicle-to-vehicle networks. *Vehicular Technology Conference*, 6:4548 – 4552, 2004.

[18] H. Wu, J. Lee, M. Hunter, R. Fujimoto, R. Guensler, and J. Ko. Simulated Vehicle-to-Vehicle Message Propagation Efficiency on Atlanta's I-75 Corridor. *Transportation Research Record (TRR)*, 2005.

[19] Xu Yang and W. W. Recker. Simulation studies of information propagation in a self-organizing distributed traffic information system. *Transportation Research Part C: Emerging Technologies*, 13(5-6):370–290, 2005.

[20] Wen-Long Jin and Wilfred W. Recker. Instantaneous information propagation in a traffic stream through inter-vehicle communication. *Transportation Research Part B: Methodological*, 40(3):230–250, March 2006.

[21] Wen-Long Jin and Wilfred W. Recker. An analytical model of multihop connectivity of inter-vehicle communication systems. Submitted to *IEEE Transactions on Wireless Communications*, 2005.

[22] Wen-Long Jin and Wilfred W. Recker. A Monte Carlo Simulation Model of Inter-Vehicle Communication. *Transportation Research Record: Journal of the Transportation Research Board*, 2000:8–15, 2007.

[23] Xiubin Wang. Modeling the process of information relay through inter-vehicle communication. *Transportation Research Part B: Methodological*, 41(6):684–700, July 2007.

[24] S. Ukkusuri and L. Du. Geometric connectivity of vehicular ad hoc networks: Analytical characterization. *Transportation Research Part C*, 2008. Article in press.

[25] Xiubin Wang, Teresa M. Adams, Wen-Long Jin, and Qiang Meng. The Process of Information Propagation in a Traffic Stream with a General Vehicle Headway: a Revisit. 2008. Submitted to Transportation Research Board Annual Meeting.

[26] Wen-Long Jin and Wilfred W. Recker. Instantaneous connectivity of one-dimensional inter-vehicle communication networks for general traffic conditions. *Proceedings of Transportation Research Board Annual Meeting*, 2009.

[27] M. J. Lighthill and G. B. Whitham. On kinematic waves: II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London A*, 229(1178):317–345, 1955.

[28] D. C. Gazis, R. Herman, and R. W. Rothery. Nonlinear follow-the-leader models of traffic flow. *Operations Research*, 9(4):545–567, 1961.

[29] Frank A. Haight. *Mathematical Theories of Traffic Flow*. Academic Press New York, 1963.

[30] P. I. Richards. Shock waves on the highway. *Operations Research*, 4:42–51, 1956.

[31] Wen-Long Jin. *Kinematic Wave Models of Network Vehicular Traffic*. PhD thesis, University of California, Davis, September 2003. http://arxiv.org/abs/math.DS/0309060.

[32] G. Grimmett and D. Stirzaker. *Probability and random processes*. Oxford University Press, USA, 2001.

[33] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404, March 2000.

[34] M. Grossglauser and D.N.C. Tse. Mobility increases the capacity of ad hoc wireless networks. *IEEE/ACM Transactions on Networking (TON)*, 10(4):477–486, 2002.

[35] Q. Yang, A. Lim, S. Li, J. Fang, and P. Agrawal. ACAR: Adaptive Connectivity Aware Routing Protocol for Vehicular Ad Hoc Networks. In *Computer Communications and Networks, 2008. ICCCN'08. Proceedings of 17th International Conference on*, pages 1–6, 2008.