INSTANTANEOUS CONNECTIVITY OF ONE-DIMENSIONAL INTER-VEHICLE COMMUNICATION NETWORKS FOR GENERAL TRAFFIC CONDITIONS

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ABSTRACT

Rapid developments in various frontiers of telecommunications and information technologies could enable the development of next-generation Intelligent Transportation Systems (ITS) that rely on inter-vehicle communications (IVC) to disseminate time-critical and location-based traffic information. In this study, we present a new model for computing the instantaneous multihop connectivity and end node probability for vehicles on a line in a transportation network, where traffic density may be non-uniform, and vehicles' positions may depend on each other. With given locations of all vehicles, the proposed model can be used to estimate the connectivity when vehicles have different probabilities to be equipped. With the model, we study how the distribution patterns of vehicles can affect information propagation, formulate the choice of the location of a road-side station as an optimization problem, and examine the time-dependent connectivity properties on an inhomogeneous ring road. The new model is simple in formulation and computation and are more general than existing models.

KEYWORDS: Inter-vehicle communication, multihop connectivity, end node probability, traffic dynamics, uniform/non-uniform traffic

List of Tables

List of Figures

1 Introduction

Rapid developments in various frontiers of telecommunications and information technologies could enable the development of next-generation Intelligent Transportation Systems (ITS) that rely on inter-vehicle communications (IVC) to disseminate time-critical and location-based traffic information. Compared with existing centralized transportation information systems, IVC-based systems are less costly to deploy and use and more resilient to natural disasters. In order to harness the power of the latest wireless communication technologies for developing ITS strategies, the Federal Communication Commission (FCC) allocated the spectrum from 5.850GHz to 5.925GHz for dedicated short range communications (DSRC) in 1999 [1]. A corresponding communication protocol, IEEE 802.11p, is being developed to add wireless access in the vehicular environment (WAVE). In 2004, US Department of Transportation initiated efforts in developing Vehicle Infrastructure Integration (VII) systems [2, 3]. In such a VII system, information can be exchanged among IVC enabled vehicles, traffic management centers, various elements of road infrastructure including traffic signals, message signs, bus stops, and other safety hardware. Different from other wireless communication systems, however, IVC systems are subject to restrictions and benefits of special mobility patterns of communication nodes in transportation networks, and it is important to understand the impacts of such special mobility patterns on IVC in order to develop scalable, effective, and efficient IVC technologies.

As in other mobile ad hoc networks, multihop connectivity, the probability of existing a communication path between a sender and a receiver, is a fundamental performance measure of an IVC network, since the connectivity can determine the delivery ratio and delivery delay between a pair of sender and receiver. Estimating connectivity a priori is essential to determine specification of appropriate communication devices, routing protocols, database management schemes, and the range of effective applications. In the literature, there have been extensive studies on multihop connectivity of various types of radio networks. Studied radio networks can be one-dimensional [4, 5] or two-dimensional [6, 7, 8]. Research methodologies include theoretical analysis of asymptotic connectivity [9] based on percolation theory [10, 11] as well as Monte Carlo simulations [12]. Performance measures of connectivity include expected propagation distance [4], the probability for having at least one communication path between two nodes [13], the k-connectivity [14, 15], or the critical transmission range for asymptotic cases [6, 5, 7].

In the literature, there have been some studies of connectivity properties of IVC networks in recent years. In [13], the probability of establishing a communication path between two nodes are studied for simulated, bidirectional traffic. In [16], an analytical model was proposed for estimating connectivity for vehicles following a Poisson distribution and moving randomly and independently. In [17, 18], the propagation distance was studied also with traffic simulations. In [19, 20], success rate and connectivity of IVC networks in instantaneous uniform or non-uniform traffic were modeled by considering most-forwarded within-range (MFR) information propagation. In [21], connectivity of IVC networks in instantaneous traffic was studied with Monte Carlo simulation. In [22], information propagation in instantaneous traffic was modeled as a Markov chain for vehicles following a Poisson distribution. In [23], the geometric connectivity of one-dimensional IVC

networks is studied for positions of vehicles following independent distributions but with some disturbance. In all these models, the probability of a vehicle to be equipped is assumed to the same as the market penetration rate. In addition, except the models developed by the authors in [19, 20, 21], all other models assume independent vehicle positions and uniform traffic density.

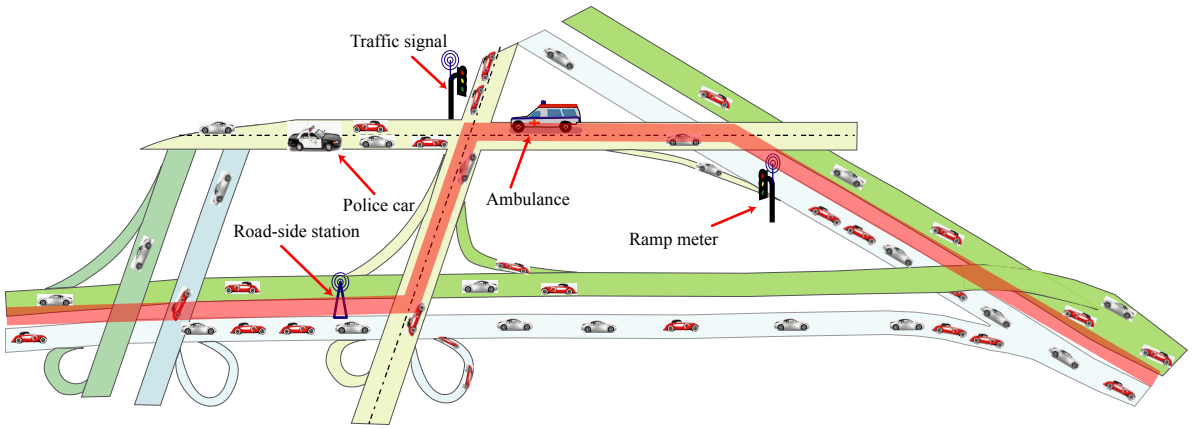

Figure 1: Instantaneous information propagation along a line of vehicles in a transportation network

In a transportation network, the length of a road section between two consecutive junctions is usually much longer than the transmission range of DSRC units, information flows are also confined by transportation network topology. Here we consider instantaneous information propagation on a one-dimensional line in a transportation network, e.g., the red line shown in Figure 1, on which there can be highways, arterial roads, surface streets, and other kinds of roadways. On such a line, the density of vehicles can vary dramatically due to driving behaviors and restrictions of network geometry. For examples, with higher travel demands and more lanes, freeways usually carry much higher density of vehicles than local streets; around a lane-drop or merging area, traffic density is usually significantly higher in the upstream section with the formation of queues; when a shock wave forms [24], traffic density is higher in the downstream part; vehicles tend to form clusters in sparse traffic; and traffic signals can cause gaps between vehicle platoons. Such variations in densities can cause different level of signal interference among nodes [25]. Note that, however, the density of communication nodes in a certain road section is always limited due to the finite lengths of vehicles. For example, the maximum number of vehicles on a lane, i.e., the jam density, is about 150 vehicles/km [26]. Therefore, it is important to consider the impacts of the special distribution patterns and mobility patterns of vehicles in transportation networks when estimating the connectivity between two nodes.

In this study, we will study the IVC connectivity along a line of vehicles in a transportation network in a simplified framework of [19, 20], in which traffic density may not be uniform and vehicles' positions may depend on each other as in the real traffic. Here we assume that different

types of vehicles or vehicles in different regions may have different probabilities to be equipped. Given positions of all vehicles, we will examine the relationship between *end node probability* and *connectivity* and develop a simplified model for computing connectivity between two nodes in general traffic. With the model, we will discuss how the distribution patterns of vehicles and traffic dynamics can affect connectivity properties of an IVC system. Although the results are more general since the distribution patterns of communication nodes are arbitrary, the results can also be applied for the case when communication nodes follow Poisson distributions.

In Section 2, we give a detailed explanation of the conceptual framework and definitions for our new model. In Section 3, we derive an analytical recursive model of connectivity for given vehicle positions and discuss its basic properties. In Section 4, we study the impacts of the distribution of vehicles, road-side stations, and traffic dynamics on connectivity. In Section 5, we make some conclusions.

2 Conceptual framework and definitions for instantaneous multihop connectivity

We consider vehicles and road-side stations on a line in a transportation network. A vehicle or road-side station is labeled by an integer number k, and its position on the line is denoted by $x(k)$. Here $x(k+1) \geq x(k)$, and the direction of x can be the same as or different from the direction of information propagation. Without loss of generality, we call both vehicles and road-side stations as vehicles. The probability for vehicle k to be equipped with a wireless communication unit is $\mu(k)$. Thus, the probability for it not to be equipped is $1-\mu(k)$. In particular, $\mu(k)=1$ if k is a road-side station, a sender, or a receiver. We also call all vehicles with $\mu(k)=1$ as communication nodes, and all nodes have the same DSRC transmission range r. If nodes i and j are within each other's transmission range, then information can propagate from i to j or from j to i. That is, the probability for a transmission from i to j, $p(i \to j)$ is

$$p(i \to j) \quad = \quad \begin{cases} 1, & |x(i)-x(j)| \leq r, \\ 0, & |x(i)-x(j)| > r. \end{cases}$$

Note that, in reality, several transmissions at the same time may interfere with each other, and $p(i \to j)$ may not be 1 even when nodes i and j are within the transmission range [27].

In a road network, given initial and boundary conditions, vehicle positions at any time can be observed or predicted by traffic models, such as kinematic wave models [28]. At a time instant, we can then take a snap-shot of vehicles on a line and analyze IVC information propagation along the line. Here, positions of vehicles are determined by traffic models, and the *distribution pattern of vehicles* can be arbitrarily uniform or non-uniform. Although the positions of vehicles are deterministic, a vehicle may be equipped with a wireless communication unit or not, and the probability for a vehicle to be equipped may depend on its type, location, etc. Assuming whether two vehicles are equipped is independent, the distribution of equipped vehicles in a traffic stream can be described by *Bernoulli trials* [29, Chapter VI]. Then for a traffic stream of K vehicles, there can be

2^K different Bernoulli trials. A *realization* of Bernoulli trials corresponding to a traffic stream can be represented by a sequence of 0's and 1's, where 0's stand for non-equipped vehicles and 1's for equipped vehicles, and all possible realizations can be represented by integers from 0 to $2^K - 1$. The probability of the existence of a given Bernoulli trial can be computed from the probabilities for all vehicles to be equipped. For each realization of Bernoulli trials for a traffic stream, we start to transmit a message from the information source until an equipped vehicle, from which information can not be further relayed. We call the sequence of nodes upon which information was relayed as a *communication chain* and the last node of a communication chain as the *end node*. That is, the distance between two consecutive nodes is not larger than transmission range, and an end node is the equipped vehicle within whose transmission range there are no vehicles or equipped vehicles in the direction of information propagation. Note that, for an infinitely long traffic stream, there exists an end node in any realization of Bernoulli trials as long as the probabilities for vehicles to be equipped are smaller than 1.

In an IVC system, two equipped vehicles are connected if and only if there exists a communication chain between them. Obviously, the probability of the existence of a communication chain is determined by vehicle positions $x(k)$, the transmission range r, and probabilities for vehicles to be equipped $\mu(k)$.

2.1 Definitions and properties of probabilities

We denote the *end node probability* for vehicle k to be the end node of a communication chain starting from sender m by $P(m,k)$. For equipped vehicles m and k, we denote the probability for information to propagate from node m to node k by $\mathbf{c}(m,k)$, which is the connectivity between nodes m and n for instantaneous information propagation. For two equipped vehicles m and n on a line, then whether there is a communication chain connecting m and n is independent of the distribution of vehicles or equipped vehicles outside the region $[x(m),x(n)]$. That is, $\mathbf{c}(m,n)$ is independent of vehicles outside $[x(m),x(n)]$. Since there exists a communication chain from m to n if and only if there is a chain from n to m, the connectivity function is symmetric; i.e., $\mathbf{c}(m,n) = \mathbf{c}(n,m)$. On a multi-lane road, it is possible that two vehicles have the same location. If $x(k) = x(i)$, from the definition of the connectivity, we can see that $\mathbf{c}(m,k) = \mathbf{c}(m,i)$. Therefore, we define $\mathbf{c}(m,x(k))$ as the connectivity between the sender m and any node at $x(k)$.

Vehicle k is the end node, if and only iff (i) vehicle k is equipped, (ii) there is a communication chain connecting vehicle k and sender m, and (iii) vehicles in $(x(k),x(k)+r]$ are not equipped. Since the three events are independent, we obtain

$$P(m,k) \quad = \quad \mathbf{c}(m,k)\mu(k) \prod_{x(j)\in(x(k),x(k)+r]} (1-\mu(j)), \quad k = m,\cdots,n. \tag{1}$$

Since $\mathbf{c}(m,k)$ only depends on the distribution of vehicles in $[x(m),x(k)]$, from this equation, we can see that $P(m,k)$ is independent of the distribution of vehicles or nodes outside $[x(m),x(k)+r]$. Note that, if $x(k) \neq x(i)$, k and i cannot be the end node at the same time. However, two vehicles at the same location can be the end node at the same time. That is, if $x(k) = x(i)$ for $k \neq i$, $P(m,k)$ and

$P(m,i)$ are not mutually exclusive. In this case, we can merge all vehicles at $x(k)$ into one vehicle but with a combined location-dependent probability $\mu(x(k))$

$$\mu(x(k)) \quad = \quad 1 - \prod_{x(i)=x(k)} (1-\mu(i)).$$

Then, the probability for a vehicle at $x(k)$ to be the end node is

$$P(m,x(k)) \quad = \quad \mathbf{c}(m,x(k))\mu(x(k)) \prod_{x(j)\in(x(k),x(k)+r]} (1-\mu(j)).$$

In particular, if $x(i) = x(m)$, it does not contribute to $\bar{\mu}(m)$ since $\mu(m) = 1$. That is, all vehicles too close to the sender or the receiver can be removed without harming the information propagation process.

Since whether vehicle j with $x(j) \geq x(k)$ is equipped or not does not affect the end node probability of vehicle i with $x(i) \in [x(m), x(k) - r)$, we can set $\mu(k) = 1$ and remove all other vehicles j with $x(j) \in [x(k), x(n)]$, $\mathbf{c}(m,k) = P(m,k)$. Then, nodes m and k are connected if and only vehicle k is the end node. Since information will stop at one and only one location for any realization of Bernoulli trials of all vehicles, we have

$$\sum_{x(i)\in[x(m),x(k)-r)} P(m,x(i)) + \mathbf{c}(m,x(k)) \quad = \quad 1.$$

Note that vehicles in $[x(k) - r, x(k))$ cannot be the end node since vehicle k is surely equipped. Therefore,

$$\mathbf{c}(m,x(k)) = 1 - \sum_{x(i)\in[x(m),x(k)-r)} P(m,x(i)). \tag{2}$$

For the purpose of simplicity, we hereafter assume that $x(k) \neq x(i)$ for $k \neq i$: if $x(k) = x(i)$, we combine them into one vehicle with the combined probability to be equipped. Therefore, $x(k+1) > x(k)$.

2.2 Upstream and downstream reaches

For vehicle k between the sender m and the receiver n, we define its *downstream reach* $d(k)$ as the farthest downstream vehicle within its transmission range:

$$d(k) = \max\{i | x(i) - x(k) \leq r, x(i) \in [x(m), x(n)]\}.$$

Similarly, we define the *upstream reach* of vehicle k as the farthest upstream vehicle within its transmission range:

$$u(k) = \min\{i | x(k) - x(i) \leq r, x(i) \in [x(m), x(n)]\}.$$

Then we have the following properties of $d(k)$ and $u(k)$:

1. Both $d(k)$ and $u(k)$ are functions of k. Both $d(k)$ and $u(k)$ belong to $\{m, \cdots, n\}$, which is an invariant set for both functions.

2. For all vehicles within the downstream transmission range of the sender m, their upstream reach is m; for all vehicles within the upstream transmission range of the receiver n, their downstream reach is n. That is, $u(k) = m$ for $k = m, \cdots, d(m)$, and $d(k) = n$ for $k = u(n), \cdots, n$.

3. $u(k) \leq k$ and $d(k) \geq k$. If $u(k) = k$ for $k > m$ or $d(k) = k$ for $k < n$, then there exists a gap larger than r on the line of vehicles, and $\mathbf{c}(m,n) = 0$. Hereafter we omit these trivial scenarios and assume $u(k) < k$ for $k > m$ and $d(k) > k$ for $k < n$.

4. Both $d(k)$ and $u(k)$ are non-decreasing in k. That is, $d(k+1) \geq d(k)$, and $u(k+1) \geq u(k)$.

5. Vehicle k is always within the transmission range of $u(k)$ and $d(k)$. That is, $d(u(k)) \geq k$ and $u(d(k)) \leq k$.

6. Vehicle k is not within the transmission range of $u(k) - 1$ or $d(k) + 1$, otherwise $u(k) \leq u(k) - 1$ or $d(k) \geq d(k) + 1$. Therefore, $d(u(k) - 1) < k$, and $u(d(k) + 1) > k$.

3 A recursive model of multihop connectivity

3.1 Model derivation

For vehicles with distinct locations, (1) and (2) can be rewritten as

$$P(m,k) = \mathbf{c}(m,k)\mu(k) \prod_{j=k+1}^{d(k)} (1 - \mu(j)), \tag{3}$$

$$\mathbf{c}(m,k) = 1 - \sum_{i=m}^{u(k)-1} P(m,i). \tag{4}$$

Further, we can have the following theorem.

Theorem 3.1 *For vehicles on a line with $x(k+1) > x(k)$, we can compute end node probabilities and connectivity recursively as follows ($k = m, \cdots, n$)*

$$P(m,k) = \left(1 - \sum_{i=m}^{u(k)-1} P(m,i)\right) \mu(k) \prod_{j=k+1}^{d(k)} (1 - \mu(j)), \tag{5}$$

$$\mathbf{c}(m,k) = 1 - \sum_{i=m}^{u(k)-1} \mathbf{c}(m,i)\mu(i) \prod_{j=i+1}^{d(j)} (1 - \mu(j)), \tag{6}$$

where $\mu(m) = 1$ and $\mu(n) = 1$ for the sender m and the receiver n. Further, if we let

$$\lambda(k) = \mu(k) \prod_{j=k+1}^{d(k)} (1-\mu(j)),$$

then (6) and (5) can be rewritten as

$$P(m,k) = \left(1 - \sum_{i=m}^{u(k)-1} P(m,i)\right) \lambda(k), \tag{7}$$

$$\mathbf{c}(m,k) = 1 - \sum_{i=m}^{u(k)-1} \mathbf{c}(m,i)\lambda(i). \tag{8}$$

From (6), we have $c(m,k) = 1$ for $k = m, \cdots, d(m)$, since $u(k) - 1 = m - 1 < m$ and the second term on the right hand-side is 0. Similarly, from (5), we have $P(m,m) = \prod_{j=m+1}^{d(m)}(1-\mu(j))$, which is the probability that none of $m+1, \cdots, d(m)$ is equipped. For other k, we can compute both $\mathbf{c}(m,k)$ and $P(m,k)$ recursively by following (6) and (5).

Since $u(k+1) \geq u(k)$, we have $\mathbf{c}(m,k+1) \leq \mathbf{c}(m,k)$ from (6). That is, on the same line of vehicles, the farther the receiver is from the sender, with the smaller probability they will be connected. Note that, however, $P(m,k)$ may not be decreasing with k. For example, when $\mu(n) = 1$, $P(m,n) = \mathbf{c}(m,n)$, but $P(m,i) = 0$ for $i = u(n), \cdots, n-1$.

From (8), we can see that $\mathbf{c}(m,k)$ is only related to the distribution of vehicles $i = m, \cdots, d(u(k)-1)$. Since $d(u(k)-1) < k$, $\mathbf{c}(m,k)$ is independent of vehicles' positions and probabilities to be equipped beyond and including vehicle k. From (7), $P(m,k)$ is independent of the status of vehicles beyond vehicle $d(k)$. These properties are consistent with those obtained with communication chains.

3.2 Systems of linear equations

Both (3) and (4) are the following systems of linear equations

$$\begin{aligned} \vec{P} &= A\vec{\mathbf{c}}, \\ \vec{\mathbf{c}} &= \vec{1} - B\vec{P}, \end{aligned}$$

where $(\vec{\mathbf{c}})_k = \mathbf{c}(m,k)$, $(\vec{P})_k = P(m,k)$, $(\vec{1})_k = 1$, A is a matrix with dimensions of $(n-m+1) \times (n-m+1)$

$$A(k,i) = \begin{cases} \lambda(k), & i = k, \\ 0, & \text{otherwise}, \end{cases}$$

and B is a matrix with dimensions of $(n-m+1) \times (n-m+1)$

$$B(k,i) = \begin{cases} 1, & i = m, \cdots, u(k)-1, \\ 0, & \text{otherwise}. \end{cases}$$

Let I be the identity matrix with dimensions of $(n-m+1)\times(n-m+1)$, we then have

$$(I+AB)\vec{P} = A\vec{1}, \tag{9}$$
$$(I+BA)\vec{\mathbf{c}} = \vec{1}, \tag{10}$$

which are equivalent to (7) and (8) respectively. From these equations, the connectivity and the end node probability can be solved with numerical methods for solving linear equations [30].

Since $\mathbf{c}(m,n) = 1-\sum_{i=m}^{u(n)-1} P(m,i)$, then (9) can be rewritten as

$$(I-AC)\vec{P} = A\vec{1}\mathbf{c}(m,n),$$

where C is a matrix with dimensions of $(n-m+1)\times(n-m+1)$

$$C(k,i) = \begin{cases} 1, & i = u(k),\cdots,u(n)-1, \\ 0, & \text{otherwise.} \end{cases}$$

From the equation, we can have $P(m,n) = \mathbf{c}(m,n)$ and $P(m,k) = 0$ for $k = u(n),\cdots,n-1$. Further we have

$$\vec{P} = (I-AC)^{-1}A\vec{1}\mathbf{c}(m,n) = \sum_{i=0}^{\infty}(AC)^i A\vec{1}.$$

Since $\vec{1}^T\vec{P} = 1$, we can directly compute $\mathbf{c}(m,n)$ as

$$\mathbf{c}(m,n) = \frac{1}{\vec{1}^T(I-AC)^{-1}A\vec{1}} = \frac{1}{\sum_{i=0}^{\infty}\vec{1}^T(AC)^i A\vec{1}}. \tag{11}$$

4 Properties and applications

4.1 Uniform vs non-uniform traffic

In this subsection, we consider a road section with a length of 10 km, on which a sender and a receiver are at the boundaries. The average density of the road section is ρ; i.e., there are $10\rho-1$ vehicles between the sender and the receiver. Here we assume that vehicles are uniformly distributed with density ρ_1 and ρ_2 on the first and second halves of the road, respectively. Thus, $\rho_1+\rho_2 = 2\rho$. When the probability for all vehicles inside the road section to be equipped is the same as $\mu = 0.1$, and the transmission range for all nodes is the same as $r = 1$ km, the connectivity between the sender and the receiver is shown in Figure 2 for different ρ and ρ_1. From the figure, we can see that, when the total number of vehicles inside the road section is the same, the distribution patterns of vehicles can significantly affect the connectivity. Further, the connectivity reaches its maximum when $\rho_1 = \rho$; i.e., when vehicles are uniformly distributed on the whole road section. In addition, we can see that the connectivity is symmetric in ρ_1 due to the symmetry in $\mathbf{c}(m,n) = \mathbf{c}(n,m)$.

Here, we make the following conjecture:

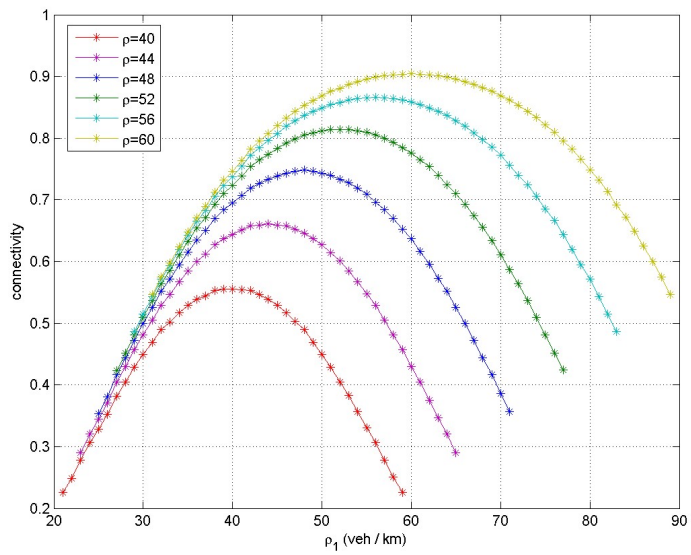

Figure 2: Impacts of the distribution patterns on connectivity

Conjecture 4.1 *For n vehicles between a sender and a receiver, when all n vehicles have the same probability to be equipped and all equipped vehicles have the same transmission range, then the connectivity between the sender and the receiver reaches its maximum when all n vehicles are uniformly distributed between the sender and the receiver.*

4.2 Road-side stations

Here we assume that road-side stations have the same transmission range as equipped vehicles and are not inter-connected through other wired or wireless networks. That is, road-side stations are considered as normal nodes, but stationary.

Theorem 4.2 *If k is a road-side station between the sender m and the receiver n with $\mu(k) = 1$ for $x(k) \in (x(m), x(n))$, then*

$$\mathbf{c}(m,n) = \mathbf{c}(m,k)\mathbf{c}(k,n). \tag{12}$$

That is, we can compute the total connectivity as the product of two connectivities.

Proof. For vehicle $i = u(k), \cdots, k-1$, since $k \leq d(i)$ and $\mu(k) = 1$, we have $\lambda(i) = 0$. From (8), we have for any $j > k$

$$\mathbf{c}(m,j) \;=\; 1 - \sum_{i=m}^{u(k)-1} \mathbf{c}(m,i)\lambda(i) - \sum_{i=k}^{u(j)-1} \mathbf{c}(m,i)\lambda(i) = \mathbf{c}(m,k) - \sum_{i=k}^{u(n)-1} \mathbf{c}(m,i)\lambda(i). \tag{13}$$

If $k+1 < j \leq d(k)$, then $\lambda(i) = 0$ for $i = k, \cdots, u(j)-1$, $\mathbf{c}(k,j) = 1$, and (13) leads to $\mathbf{c}(m,j) = \mathbf{c}(m,k)$. That is, (12) holds for $n = k+1, \cdots, d(k)$. Assuming that $\mathbf{c}(m,i) = \mathbf{c}(m,k)\mathbf{c}(k,i)$ for $i = k, \cdots, u(n)-1$, we then have

$$\begin{aligned}\mathbf{c}(m,n) &= \mathbf{c}(m,k) - \sum_{i=k}^{u(n)-1} \mathbf{c}(m,k)\mathbf{c}(k,i)\lambda(i) = \mathbf{c}(m,k)\left[1 - \sum_{i=k}^{u(n)-1} \mathbf{c}(k,i)\lambda(i)\right] \\ &= \mathbf{c}(m,k)\mathbf{c}(k,n).\end{aligned}$$

By mathematical induction, we conclude that (12) is always true. Note that another proof based on the definition of connectivity was given in [31]. ■

For a set of vehicles $\mathscr{S} = \{(x(k), \mu(k))_{k=m,\cdots,n}\}$ on a line $[a,b]$ with the sender at $x(m) = a$ and the receiver at $x(n) = b$, we denote the connectivity between the sender and the receiver by $\mathbf{c}(\mathscr{S})$. After deploying a road-side station, represented by $(x,1)$, along the line, the connectivity becomes $\mathbf{c}(\mathscr{S} \cup (x,1))$. Therefore, the best location to deploy the road-side station is the solution to the following optimization problem

$$\max_{x \in [a,b]} \mathbf{c}(\mathscr{S} \cup (x,1)). \tag{14}$$

From (12), $\mathbf{c}(\mathscr{S} \cup (x,1)) = \mathbf{c}(\mathscr{S}_1 \cup (x,1))\mathbf{c}(\mathscr{S}_2 \cup (x,1))$, where $\mathscr{S}_1$ is the set of vehicles in $[a,x)$, and $\mathscr{S}_2$ is the set of vehicles in $(x,b]$.

We consider the same system as in the previous subsection: the sender is at 0, the receiver at 10 km, traffic densities in $[0,5]$ km and $[5,10]$ km are ρ_1 and ρ_2 respectively, and $\rho_1 + \rho_2 = 100$ veh/km. For $\rho_1 = 30$, 40, 50, 60, and 70 veh/km, the connectivity between the sender and the receiver for different locations of a road-side station is shown in Figure 3. From the figure, we can see that the better positions to deploy a road-side station are in the regions with lower densities, e.g., downstream to a bottleneck or a surface street.

4.3 Time-dependent instantaneous connectivity on a ring road with bottlenecks

In this subsection we consider an inhomogeneous ring road with a length of $L = 600l$ (l = 0.028 km) in Figure 4. The ring road composes of two homogeneous parts: the number of lanes $a(x) = 1$ for part 1 with $x \in [0, L_1]$, and $a(x) = 2$ for part 2 with $x \in (L_1, L)$, where $L_1 = 100l$. There are totally $N_0 = 801$ vehicles on the ring road, and they are labeled as $k = 0, \cdots, 800$. The initial distribution of the vehicles follows the following density function: $\rho(x,0) = a(x)(\rho_0 + 3\sin\frac{2\pi x}{L})$, where $\rho_0 = 26.1367$. Thus, the total number of vehicles on the ring road is

$$N_0 = 2\rho_0 L - \int_0^{100l} (\rho_0 + 3\sin\frac{2\pi x}{L})dx = 1100l\rho_0 - \frac{450}{\pi}l = 801.$$

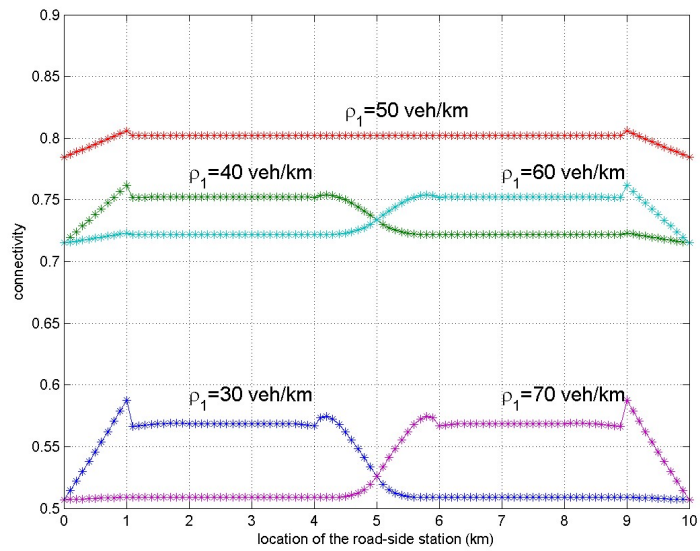

Figure 3: Impacts of the location of a road-side station on connectivity

Here we assume that traffic dynamics on the ring road is described by an inhomogeneous Lighthill-Whitham-Richards model [24, 32], and the location-dependent speed-density relationships are revised based on [33, 34]

$$V(\rho, a(x)) = 5.0461 \left[\left(1 + \exp\{[\frac{\rho}{a(x)\rho_j} - 0.25]/0.06\} \right)^{-1} - 3.72 \times 10^{-6} \right] l/\tau,$$

where τ = 5 s; the free flow speed $v_f = 27.8$ m/s; the jam density of a single lane ρ_j = 180 veh/km/lane. The corresponding fundamental diagram $q = Q(\rho, a(x)) \equiv \rho V(\rho, a(x))$ is non-convex but unimodal in density ρ. By dividing the time duration $[0, T]$ into a number of time intervals with a period of Δt and splitting the ring road into S sections with a length of Δx with $S\Delta x = L$, where $v_f \frac{\Delta t}{\Delta x} < 1$ according to [35], we can solve traffic dynamics with the Godunov-type discrete version of the LWR model in the following:

$$\rho_s^{t+1} = \rho_s^t - \frac{\Delta t}{\Delta x} \left(q_{s+1/2}^{t*} - q_{s-1/2}^{t*} \right), \tag{15}$$

where ρ_s^t is the average density in section s between $[(s-1)\Delta x, s\Delta x]$ at time step t, and the boundary flux from section $s-1$ to section s can be computed by the so-called supply-demand method [36, 37, 38]

$$q_{s-1/2}^{t*} \quad = \quad \min\{D(\rho_{s-1}^t), S(\rho_s^t)\}, \tag{16}$$

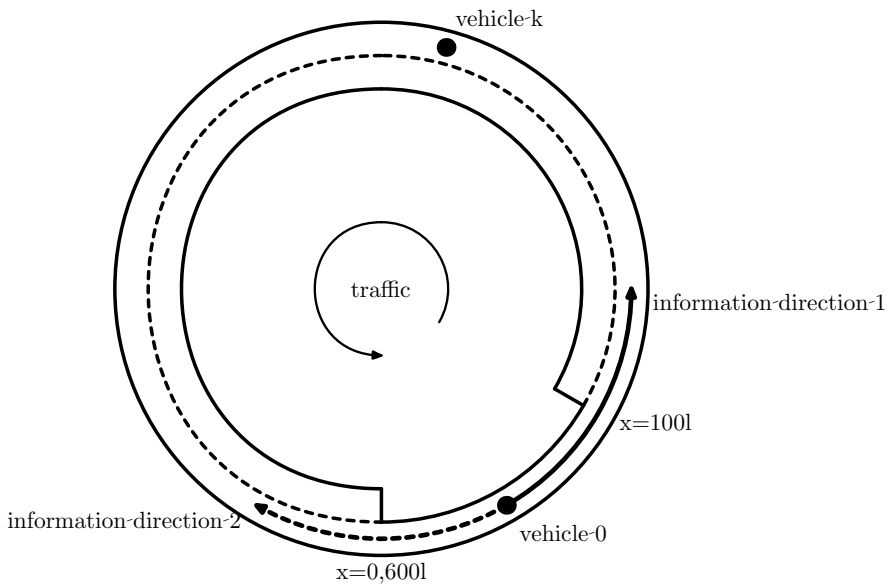

Figure 4: A ring road with a bottleneck

where the local traffic demand and supply functions are given by $D(\rho) = Q(\min\{\rho, \rho_c\})$ and $S(\rho) = Q(\max\{\rho, \rho_c\})$ with the critical density ρ_c satisfying $Q(\rho_c) \geq Q(\rho)$.

With $S = 2400$, $\Delta x = 7$ m, $\Delta t = 0.2$ s, and $T = 3000$ s, Figure 5 shows the contour plot of the density in $x - t$ space. In the figure, the (blue) thick solid line is the trajectory of vehicle 0, and the (red) thick dashed line is the trajectory of vehicle 400. From the figure, we can see that, after a sufficient amount of time, traffic on the ring road approaches stationary states [39]: the flux or flow-rate at any location is the same as the capacity of the bottleneck, and traffic density is 35.8944 veh/km for $x \in [0, 100l]$, 26.4162 veh/km for $x \in (100l, 471.5493l]$, and 118.3550 veh/km for $x \in (471.5493l, 600l)$.

Denoting the cumulative number of vehicles passing $x = 0$ during $[0, t\Delta t]$ by $A(0,t)$, from the numerical solutions of boundary fluxes we then have $A(0,t) = \sum_{i=0}^{t-1} q_{-1/2}^{t*} \Delta t$. At any time instant $t\Delta t$, the cumulative number of vehicles on the ring road is approximated by a piece-wise linear equation for $x \in [(s-1)\Delta x, s\Delta x]$ $(s = 0, \cdots, S)$

$$N(x,t) \quad = \quad N(s-1,t) + (\frac{x}{\Delta x} - (s-1))(N(s,t) - N(s-1,t)),$$

where $N(s,t) = \sum_{i=1}^{s} \rho_s^t \Delta x$ can also be computed from numerical solutions of densities. We assume that, at $t = 0$, the location of vehicle k $(k = 0, \cdots, N_0 - 1)$, $x(k,0)$, is determined by

$$N(x(k),0) \quad = \quad k + \frac{1}{2}. \tag{17}$$

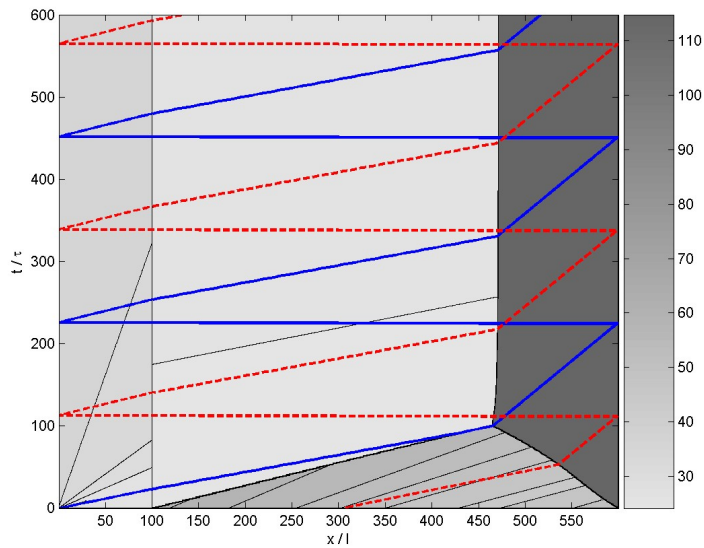

Figure 5: Solutions of density (veh/km) in the $x-t$ space: thick solid and dashed lines are the trajectories of vehicles 0 and 400

Then at a time instant $t\Delta t$, the location of vehicle k, $x(k,t)$, is a solution of

$$N(x(k,t),t) \quad = \quad A(0,t)+k+\frac{1}{2} \bmod N_0. \tag{18}$$

Therefore, from the numerical solutions of the LWR model of the ring road, we are able to determine the locations of all vehicles at any time instant as $x(k,t)$.

For the traffic stream on the ring road, we consider the instantaneous information propagation from the sender $k=0$ every 5 seconds. We assume the probability for other vehicles to be equipped is $\mu = 0.1$, and the transmission range is $r = 1.0$ km. As we know, on a ring road, a piece of information can reach a receiver through a communication chain in the same or opposite directions as the traffic. For information propagation in the same direction as the traffic, the connectivity between vehicle k and the sender is denoted by $\mathbf{c}_1(0,k)$; for information propagation in the opposite direction, the connectivity is denoted by $\mathbf{c}_2(0,k)$. Since the communication chains in the two directions are independent, then the final connectivity is

$$\mathbf{c}(0,k) \quad = \quad \mathbf{c}_1(0,k)+\mathbf{c}_2(0,k)-\mathbf{c}_1(0,k)\mathbf{c}_2(0,k). \tag{19}$$

The three connectivity values at the initial time $t=0$ are shown in Figure 6. The contour plot of the instantaneous connectivity between sender 0 and vehicle k at different times are shown in Figure 7. The connectivity between the sender at $x(0,t)$ and vehicle k at $x(k,t)$ at different times are shown in Figure 8. From all the figures, we can see that traffic dynamics on a road network can yield significantly different connectivity profile of an IVC network.

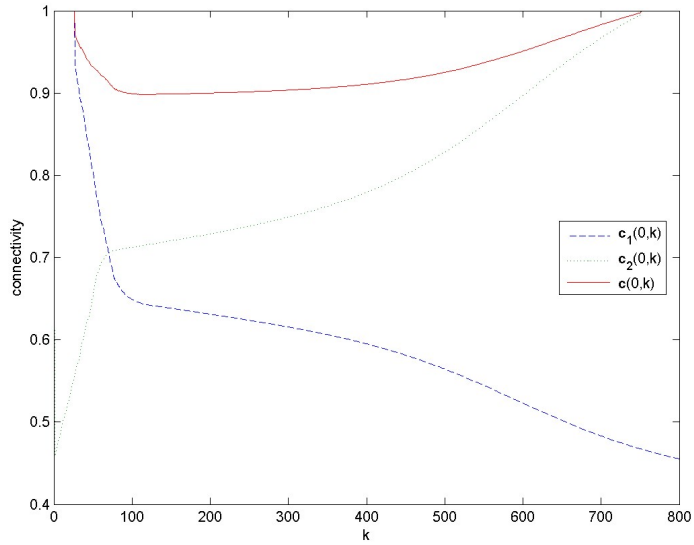

Figure 6: Connectivity between vehicles 0 and k at time $t = 0$

5 Conclusion

In this paper, we presented a new model for computing the instantaneous connectivity and end node probability for vehicles on a line in a transportation network. With given locations of all vehicles, the proposed model can be used to estimate the connectivity when densities of vehicles vary along the line and vehicles have different probabilities to be equipped. With the new model, we studied the impacts of the distribution patterns of vehicles on connectivity, formulated the choice of the location of a road-side station as an optimization problem, and discussed the time-dependent connectivity when traffic evolves on an inhomogeneous ring road. Similar as the model in [20], the new model can be used for estimating connectivity in a general traffic stream when traffic density is not uniform and vehicles' positions are dependent on each other. However, this model is much simpler in formulation and computation, but does not provide hop-related information. In addition, the model is more general and capable of treating different probabilities for vehicles to be equipped.

In this study we assume no capacity constraint, which could negatively impact the connectivity. Arguably, however, this assumption is reasonable for transportation related applications, in which a piece of traffic message can be very small, e.g., 73 bytes in [40]. In addition, we do not consider the impact of possible communication routing protocols. However, such connectivity could be realized with protocols in a most forwarded within-range fashion as discussed in [19]. These arguments are added in the Conclusion part. In the future, we will be interested in extending the results for studying information propagation in a general road network, in which several communication

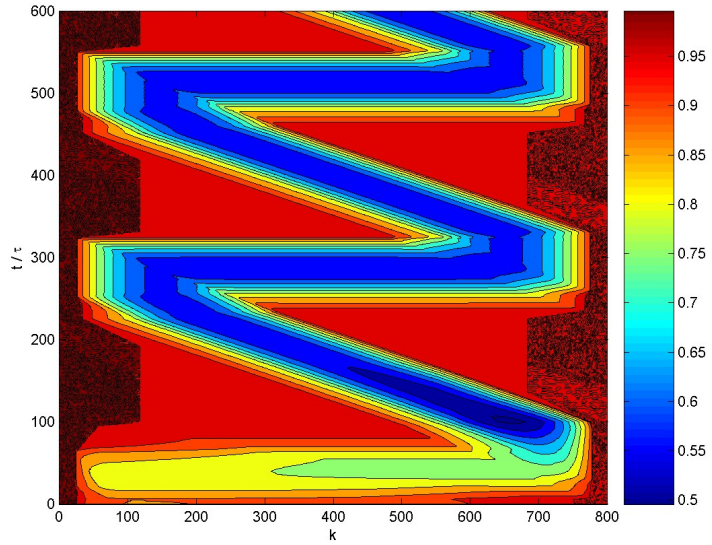

Figure 7: Time-dependent connectivity between vehicles 0 and k

paths may not be independent as in the ring road. Also we will attempt to study the connectivity properties for an IVC system when vehicles are moving in a transportation network. Yet another interesting topic would be to analyze the sensitivity in the impacts of transmission range, market penetration rate, and traffic density.

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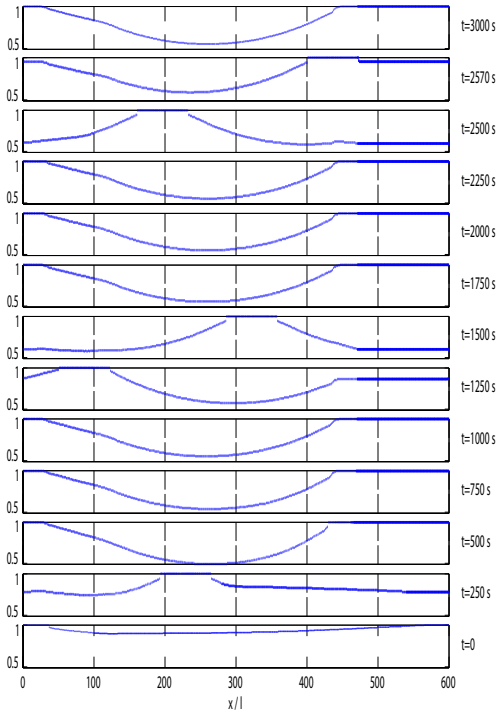

Figure 8: Connectivity profile at different locations and times

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