

1 **A CONTROL THEORETIC FORMULATION OF GREEN DRIVING STRATEGY**
2 **BASED ON INTER-VEHICLE COMMUNICATIONS**

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17 **Word Count: 5000+250×10=7500**

18 **August 1, 2011**

19 **SUBMITTED TO 2012 TRB ANNUAL MEETING**

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20 **ABSTRACT**

21 The transportation sector generates a large percentage of local pollutants including hydrocarbons
22 (HC), carbon monoxide (CO), carbon dioxide (CO₂), and oxides of nitrogen (NO_x). Apart from
23 switching to alternative fuels, various green driving strategies that smooth traffic flow and reduce
24 congestion can be implemented to lower pollutant emissions and fuel consumption. In this paper,
25 we study constant and dynamic green driving strategies based on inter-vehicle communications.
26 With Newell's car-following model, a theoretical analysis demonstrates that optimal smoothing
27 effects can be achieved when the constant speed limit is close to but not smaller than the average
28 speed of traffic, which can guarantee a vehicle's speed profile be smooth while still following its
29 leader during a relative long time period. Then we consider a dynamic strategy in which, through
30 inter-vehicle communication, controlled vehicles share their location and speed information. By
31 simulations with Newell's car-following model and CMEM emission model for different market
32 penetration rates of IVC-equipped vehicles and communication delays, we can see that the pro-
33 posed dynamic green driving strategy can significantly reduce emissions and fuel consumption
34 without reducing average speed of traffic in a stop-and-go traffic stream. In the future, we will
35 investigate traffic and environmental benefits of green driving strategies for more realistic traffic
36 and communication scenarios.

37 **Keywords:** green driving, speed control, smooth traffic flow, emissions, fuel consumption,
38 Newell's car-following model, CMEM, inter-vehicle communication, market penetration rate, com-
39 munication delay

40 1 INTRODUCTION

41 According to Intergovernmental Panel on Climate Change, the transportation sector in the U.S. was
42 responsible for one third of greenhouse emissions in 2004, of which 80% was from passenger cars
43 and freight trucks on the roadway systems [1]. Globally, the situation was worsening with the rapid
44 development of motor vehicle transportation in developing countries [2]. Traffic oscillation, known
45 as stop-and-go wave, is one major cause of greenhouse emission in transportation systems [3]. The
46 frequent accelerations associated with stop-and-go states in traffic led to higher greenhouse
47 emissions [4]. Moreover, when vehicle traveled at the speed over 65 mph, i.e., excessive speed,
48 emissions and fuel consumption would increase significantly [3].

49 Speed control can offer a direct and effective way to control traffic and improve on-road per-
50 formance [5]. Some traditional speed control methods, such as speed bumps [6] and police en-
51 forcement [7], have been studied to control excessive speed and smooth traffic oscillations, but
52 they showed moderate effects on speed control. Recently, advanced technologies are introduced
53 to develop variable speed limit and realize speed control in transportation systems. Variable speed
54 sign is one method in which, advisory speed signals is displayed on road for driver to improve
55 traffic flow and safety. In [8], Smulders established a control strategy based on the mean of speed
56 on freeway, which showed significant improvement of the stability of the traffic stream. In [9],
57 Kuhne proposed a speed control scheme in which standard deviation of speed was regarded as one
58 decision variable to determine speed limit. Moreover, Papageorgiou implemented variable speed
59 limit in freeway to improve traffic flow efficiency [10].

60 Another method of speed limit control is applying telecommunication and information tech-
61 nologies to design in-car speed limiter [11]. Different from variable speed signs which provide
62 signals to all vehicles on roads, in-car speed limiter is able to enforce speed limit to individual
63 vehicle directly. Intelligent Speed Adaptation (ISA) is one common used advanced technology
64 for developing in-car speed limiter [12]. In the literature, there have been numerous studies on
65 ISA to smooth traffic. ISA systems used road congestion information provided by loop detec-
66 tors through wireless communication to adjust speed limits of individual vehicles on specific road

67 sections [13]. In [13, 14], a set of speed limits and appropriate actions, such as not accelerating
68 quickly, were provided to drivers through ISA systems. Experiments showed that ISA had potential
69 to mitigate congestion by smoothing the dynamics of congested traffic. ISA equipped vehicle had
70 a much smoother trajectory (with smaller speed variation), which led to low fuel consumption
71 and pollutant emissions. In addition, many field implementations of ISA systems had been done
72 to measure the influence of ISA on traffic safety and environment. ISA experiments in Tilburg
73 (Netherlands) showed that with speed limit control, driving behavior was much safer and more
74 environmentally friendly [15, 16, 17]. Moreover, in [5], with optimal speed limits adjustment,
75 freeway traffic conditions were more stable, which also benefited to driving safety and reduced air
76 pollutant emissions.

77 However, since ISA systems obtained aggregate traffic information from loop detectors, the
78 implementation was limited by the distribution of the detectors. Moreover, due to data processing
79 and internet access, the traffic information was delivered to the drivers for more than 30-second
80 delay [13], which might reduce the effect of green driving implementation. Recently, inter-vehicle
81 communication (IVC) technologies, which exchange individual vehicle information through wire-
82 less communication between vehicles, are investigated to develop advanced vehicle control system
83 [18]. A number of efforts, such as CarTalk [19] and FleetNet [20], are underway to investigate
84 inter-vehicle communication based on mobile ad hoc network technology as a means of develop-
85 ing "Internet on the road". Studies indicate that IVC systems could provide valuable, real-time
86 traffic information with smaller delay (< 0.1 second [21]) to the drivers than ISA systems. Apply-
87 ing IVC systems, people can drive more smoothly and safely with appropriate speed limit settings.
88 As the number of cars equipped with these technologies increases, we expect that drivers will adapt
89 their behaviors more accordingly [18, 22, 23]. Such collective behavior changes will result in dif-
90 ferent traffic flow characteristics, transportation system performance, and environmental impacts.
91 Therefore, IVC has even better potential to improve traffic flow, fuel consumption economy, and
92 reduce emissions. However, there have not been any systematic studies of potential environmental
93 effect of IVC system.

94 In this paper, we construct green driving strategies based on a feedback control system, with

95 which a set of appropriate speed limits is provided to individual vehicles to smooth stop-and-go
96 waves and decrease emissions and fuel consumption. In the study, vehicles equipped with IVC
97 systems share their individual traffic information. Based on these, we theoretically analyze effect
98 of a constant green driving strategy on microscopic car-following behaviors. We also propose one
99 dynamic green driving strategy to reduce traffic oscillations. The influence of characteristics of
100 IVC systems, such as market penetration rate (MPR) of equipped vehicles, communication delay,
101 on the strategy is investigated. To evaluate the potential benefits of such green driving strategies,
102 Newell's car-following model [24] and Comprehensive Modal Emission Model (CMEM) [25] are
103 integrated into a simulation platform. Comparing with other car-following models, such as Gipps's
104 model [26] and General Motor model [27], Newell's model is simple and straightforward to de-
105 scribe vehicle movement in traffic streams and is easy to be cooperated with speed limit control.
106 With the integrated simulation model we then study environmental benefits of green driving strate-
107 gies under various traffic conditions.

108 The rest of the paper is organized as follows. In section 2, we will describe Newell's car-
109 following model, CMEM emission model and a control scheme. In section 3, we will theoretically
110 analyze constant green driving strategy. Section 4 presents a dynamic green driving strategy and
111 its effect on environment. Conclusions and future work will be presented in section 5.

112 **2 Model Description**

113 **2.1 Newell's Car-following Model**

114 In an open road, if vehicle n follows an $(n-1)$ -th vehicle, the trajectory of the n -th vehicle $x_n(t)$
115 depends on $x_{n-1}(t)$ of the $(n-1)$ -th vehicle. This also indicates that the trajectory $x_{n+1}(t)$ of a
116 following $(n+1)$ -th vehicle has no effect of determining trajectory of $x_n(t)$. In Newell's model, if
117 vehicle n is able to advance its location where it keeps minimum distance (jam spacing or spacing
118 displacement) s_j away from vehicle $n-1$ without exceeding speed limit (or free flow speed) v_f , then
119 after a certain time gap (or time displacement) τ , vehicle n will arrive at $x_{n-1}(t) - s_j$. If vehicle

120 n cannot run to that location, then it will run with free flow speed. The two possible locations are
 121 combined to be Newell's car-following model.

$$x_n(t + \tau) = \min\{x_{n-1}(t) - s_j, x_n(t) + v_f \tau\} \quad (1)$$

122 We assume that a sampling interval of Newell's model is $\Delta t = \tau$ and $t = i\Delta t$. Then, the
 123 discrete-time expression of Newell's car-following model is obtained.

$$x_n(i + 1) = \min\{x_{n-1}(i) - s_j, x_n(i) + v_f \tau\} \quad (2)$$

124 **Equation 2** also implies that, traveling with Newell's car-following model, one vehicle always
 125 takes the most advanced position possible, limited by free flow speed and the position of its leader.
 126 Once a vehicle catches up with the car it follows, the two trajectories become translationally sym-
 127 metric with time and space displacement τ, s_j [28].

128 Moreover, Newell's car-following model is a stable model, with which the following vehicle
 129 will not have an amplified response of a change in the motion of the leading vehicle. Suppose that
 130 spacing between vehicle n and n-1 is constant for all $n = 1, 2, \dots, N$, i.e., $s(t) = x_{n-1}(t) - x_n(t) = s$,
 131 where $s < v_f \tau$, which indicates that the traffic is congested. From **Equation 1**, the relationship of
 132 speed between the leader and its follower in congestion is

$$v_n(t + \tau) = v_{n-1}(t). \quad (3)$$

133 We add a speed disturbance $y(t)$, which is generated by acceleration or deceleration, to the lead-
 134 ing vehicle at time t [29]. Assume that the disturbance $y(t)$ is very small, so that traffic is still
 135 congested. From **Equation 3**, we know that the disturbance $y(t)$ is preserved and translated to
 136 the follower. It indicates that disturbance $y(t)$ will not be amplified in the following vehicle by
 137 Newell's car-following model. But this also implies that without control of driving behavior, the
 138 disturbance cannot be released with Newell's car-following model. Study in [30] also shows that
 139 traffic oscillation will propagate from downstream to upstream without smoothing. Therefore, we

140 have to set control strategies to smooth the disturbances and satisfy our purpose of smoothing
141 traffic.

142 **2.2 Emission Model**

143 In August 1995, the College of Engineering-Center for Environmental Research and Technology
144 (CE-CERT) at the University of California, Riverside began a four-year research project to develop
145 a Comprehensive Modal Emissions Model (CMEM). This project was aimed to develop and verify
146 a modal emissions model that accurately reflects Light-Duty Vehicles emissions. In the developing
147 of CMEM, both engine-out and tailpipe emissions of over 300 vehicles, including more than 30
148 high emitters, were measured at a second-by-second level. CMEM can predict second-by-second
149 emissions and fuel consumption of individual vehicle or aggregated vehicles [25].

150 The CMEM is developed from a parameterized physical approach which breaks the entire emis-
151 sion process into two components: vehicle operation and emission production. Input of CMEM
152 contains two parts: 1) vehicle and operation variables, such as speed, acceleration, and road grade;
153 2) model calibrated parameters, such as cold start coefficients, engine friction factor. Output of
154 CMEM is either second-by-second or summarized emissions (HC, CO, CO₂, NO_x) and fuel con-
155 sumption [25, 4].

156 **2.3 Feedback Control**

157 Feedback control is a control mechanism that uses information from measurements to manipulate a
158 variable to achieve the desired result [31]. In a feedback control system, information about system
159 performance is measured and that information is used to correct how the system performs. There
160 are three basic components in a feedback control system: sensor, controller and plant. Sensor is
161 used to measure the system performance, controller reacts to information from sensor and applies
162 corrective action to a plant, and the plant will response to the action. Feedback control systems
163 vary in complexity, but generally fall into four categories: on-off, proportional (P), proportional
164 plus intergral (PI), proportional plus integral plus derivative (PID). The block diagram of feedback

control is illustrated in **Figure 1**

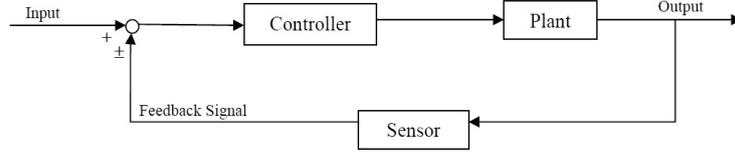


Figure 1: Block diagram of feedback control system

165
166 Feedback control system has potential on improving the robustness of system of reaction to
167 disturbance or noise. Therefore, introducing feedback control in green driving is able to smooth
168 traffic. In the implementation of feedback control on green driving, Newell’s car-following model
169 is chosen as the plant, and green driving strategy is designed as the controller. The goal of green
170 driving is smoothing traffic oscillations, i.e., stop-and-go traffic, and reducing emissions and fuel
171 consumption without decreasing the average speed. Therefore, the performance index in this con-
172 trol system is chosen as the standard deviation of speed [3, 4]. The control scheme is used in
173 section 4 to design a dynamic green driving strategy.

174 **3 Analysis of Simple Green Driving Strategies**

175 In this section, we will analyze some simple green driving strategies. In our study, we introduce
176 IVC technology in transportation system. One vehicle equipped with IVC system collects its own
177 traffic information, including location, spacing, velocity, acceleration, etc, from GPS device or
178 smartphone. It is also able to communicate with other vehicles equipped with IVC system either
179 from DSRC (Dedicated short range communication) [21] or from 3G network using smartphone
180 [32]. Hence, vehicles in transportation system can share their information, and based on these
181 information, drivers can decide their driving behaviors to smooth traffic.

182 From Newell’s car-following model (**Equation 1**), we see that vehicle trajectories can be con-
183 trolled by speed limit (free flow speed) v_f . So, we will try to set an appropriate speed limit for

184 vehicles to control speed and smooth traffic. Our goal of designing this speed limit is smoothing
 185 vehicle trajectories, i.e., reducing standard deviation of velocities of the vehicles without decreas-
 186 ing the average speed.

187 Suppose that there are N vehicles traveling on a homogeneous road. We donate location, spac-
 188 ing, and velocity of vehicle n at time t by $x_n(t), s_n(t), v_n(t)$. Let the set of IVC equipped vehicles
 189 applying green driving strategy by \mathcal{G} , which is a subset of $\{1, 2, \dots, N\}$. We donate the number of
 190 green driving vehicles by G . v_f, s_j represent the free flow speed and the jam spacing.

191 3.1 Failure of Two Green Driving Strategies

192 With the information shared by IVC equipped vehicles, one of the most straightforward thinking
 193 of designing speed limit is using average speed. In [13, 14, 3], average speed of a road section is
 194 introduced as one major factor to set speed limit. Here, the first green driving strategy is

$$U_g(t) = U(t) = \frac{\sum_{g \in \mathcal{G}} v_g(t)}{G}. \quad (4)$$

195 Discretizing $U_g(t)$ as $U_g(i)$. Then, discrete Newell's car-following model (**Equation 2**) is modified.

$$x_g(i+1) = \min\{x_{g-1}(i) - s_j, x_g(i) + U_g(i)\tau\} \quad (5)$$

196 This strategy does not work in some traffic scenarios. For example, in one extreme solution, all
 197 controlled vehicles are stopped, i.e., $v_g(t) = 0$. Then, $U_g(t) = 0$. From **Equation 5**, $x_g(i+1) =$
 198 $x_g(i)$, and $v_g(i+1) = 0$. This is a steady-state solution of the control problem. It indicates that
 199 controlled vehicles will not move even the other vehicles is moving, i.e., the controlled vehicles
 200 fail to follow their leaders. From **Equation 4** and **Equation 5**, we can get multiple steady-state
 201 solutions if and only if $U_g(i)$ is smaller than the average speed of the leading vehicles.

202 Another green driving strategy is using predicted average speed of the controlled vehicles.

$$U_g(i) = U(i) = \frac{\sum_{g \in \mathcal{G}} x_{g-1}(i) - x_g(i) - s_j}{\tau G} \quad (6)$$

203 If the traffic is congested, **Equation 6** is equivalent to

$$U_g(i) = U(i) = \frac{\sum_{g \in \mathcal{G}} v_g(i+1)}{G}. \quad (7)$$

204 **Equation 7** is a predictive version of **Equation 4**. With **Equation 7**, the controlled vehicles would
 205 be able to follow the leading vehicles. But, the resulting trajectories may not be smoother. For
 206 example, if $\mathcal{G} = \{2\}$, i.e., only vehicle 2 is under control. Then,

$$U_2(i) = \frac{x_1(i) - x_2(i) - s_j}{\tau}. \quad (8)$$

Equation 5 becomes

$$x_2(i+1) = x_1(i) - s_j.$$

207 In this case, the following vehicle's trajectory is not smoother than that of the leading vehicle. Also
 208 the follower could have speed larger than v_f , if the spacing is too large.

209 **3.2 Analysis of Constant Green Driving Strategy**

210 From the analysis in the preceding section, we see that a successful green driving strategy should
 211 (1) enable the following vehicles to follow their leading vehicles, and (2) smooth the trajectories.

Theoretically, for the periodic stop-and-go traffic, if we set speed limit as the average speed, the controlled vehicles would have very smooth trajectories after a finite time and enable to track the leading vehicles on average; i.e.,

$$\|x_g(i) - x_0(i)\| \rightarrow \text{constant}.$$

212 Then, we will theoretically analyze effect of a constant speed limit on smoothing traffic. Firstly,
 213 average \bar{v}_g and standard deviation σ_g of speed of controlled vehicle g are set as the measurements
 214 of smoothness.

$$\bar{v}_g = \frac{1}{T} \int_{t=0}^T v_g(t) dt = \frac{\sum_{i=1}^K v_g(i)}{T} \quad (9)$$

$$\sigma_g = \sqrt{\frac{1}{T} \int_{t=0}^T (v_g(t) - \bar{v}_g)^2 dt} = \sqrt{\frac{\sum_{i=1}^K (v_g(i) - \bar{v}_g)^2}{k-1}} \quad (10)$$

215 Here, T is the trajectory period of stop-and-go traffic, and $K = T/\tau$. Since standard deviation
 216 measures the dispersion of speed, with larger σ_g , there are larger speed variation, and the traffic is
 217 less smoother.

218 Assume that a vehicle n follows a vehicle $n-1$, and vehicle n is IVC equipped vehicle. Speed
 219 limit of vehicle n is set as the average speed of its leader, vehicle $n-1$, i.e., $U_n = \bar{v}_{n-1}$. However, in
 220 reality, it is difficult for the following vehicle to estimate accurate value of the average speed of its
 221 leader. So, we put an additional term ε in the speed limit.

$$U_n = \bar{v}_{n-1} + \varepsilon$$

222 Considering safety issues, we let $0 \leq U_n \leq v_f$. We donate speed of vehicle $n-1$ by $v_{n-1}(t)$, and ini-
 223 tially, the road is congested. In [33], Li *et al* measured oscillation of traffic patterns and concluded
 224 that most trajectories were periodic with period from 2 to 4 minutes. So, We assume that $v_{n-1}(t)$
 225 is a sine function with period T .

$$v_{n-1}(t) = \frac{v_f(1 + \sin(\frac{2\pi t}{T}))}{2} \quad (11)$$

226 We let $x_{n-1}(0) = 0$. Trajectory of vehicle $n-1$ is

$$x_{n-1}(t) = \int_{\xi=0}^t \frac{v_f(1 + \sin(\frac{2\pi\xi}{T}))}{2} d\xi = \frac{v_f}{2} \left(t + \frac{T}{2\pi} - \frac{T}{2\pi} \cos(\frac{2\pi}{T}) \right). \quad (12)$$

227 Average speed of vehicle $n-1$ is $\bar{v}_{n-1} = v_f/2$. Initially, vehicle n stays at $x_n(0) = -\frac{v_f\tau}{2} - s_j$. With
 228 the constant speed limit adjustment, Newell's car-following model is revised below.

$$x_n(t + \tau) = \min\{x_{n-1}(t) - s_j, x_n(t) + U_n\tau\} \quad (13)$$

229 This indicates that if $x_{n-1}(t) - x_n(t) - (s_j + U_n\tau) < 0$, $x_n(t + \tau) = x_{n-1}(t) - s_j$; otherwise, $x_n(t +$

230 $\tau) = x_n(t) + U_n\tau$. Using this statement, we estimate position of vehicle n with constant green
 231 driving strategy. Then, without losing generality, we analyze effect of the strategy on smoothing
 232 trajectory in one period.

233 1. $\varepsilon \leq 0$, i.e., $U_n \leq \bar{v}_{n-1}$.

234 In this scenario, the initial spacing of vehicle n is $x_{n-1}(0) - x_n(0) = \frac{v_f\tau}{2} + s_j \geq U_n\tau + s_j$. From
 235 **Equation 13**, we know that $x_n(\tau) = x_n(0) + U_n\tau$, i.e., $v_n(t) = U_n, \forall t \in [0, \tau)$. It indicates that
 236 vehicle n runs in free flow traffic and does not follow its leader. Assume that after $t = \tau$,
 237 there is a point at $t = t_1$ when vehicle n starts to run in congested traffic (**Equation 14**), so
 238 that vehicle n follows its leader.

$$x_{n-1}(t) - x_n(t) < s_j + U_n\tau \quad (14)$$

During $[\tau, t_1)$, vehicle n runs with speed limit U_n . So, $x_n(t) = x_n(\tau) + U_n(t - \tau) = x_n(0) + U_n t, \forall t < t_1$. Substituting $x_n(t)$ and **Equation 12** into the left side of **Equation 14** lead to

$$\frac{v_f}{2} \left(t + \frac{T}{2\pi} - \frac{T}{2\pi} \cos\left(\frac{2\pi t}{T}\right) \right) - (x_n(0) + U_n t) = (s_j + U_n\tau) - \varepsilon t + \frac{v_f}{2} \left(\frac{T}{2\pi} - \frac{T}{2\pi} \cos\left(\frac{2\pi t}{T}\right) \right) \geq s_j + U_n\tau$$

239 It implies that there is no solution for t_1 in this scenario, and vehicle n always runs with the
 240 adjusted speed limit U_n , i.e.,

$$x_n(t) = x_n(0) + U_n t, \forall t \geq 0 \quad (15)$$

241 Speed of vehicle n is obvious.

$$v_n(t) = x'_n(t) = U_n, \forall t \geq 0 \quad (16)$$

242 **Figure 2** shows speed solution of vehicle n under conditions: $\varepsilon < 0$ (purple dashed line with
 243 speed limit $U_{\varepsilon < 0} = \bar{v}_{n-1} + \varepsilon$), $\varepsilon = 0$ (yellow dashed line with speed limit $U_{\varepsilon = 0} = \bar{v}_{n-1}$).
 244 Hence, we obtain standard deviation of vehicle n's speed.

$$s.d.(v_n(t)) = 0$$

245 When $\varepsilon < 0$, $v_n(t) = U_n < \bar{v}_{n-1}$, which indicates that during one period, average speed of
 246 the controlled vehicle is smaller than that of the leading vehicle. While since with Newell's
 247 model, the leader's speed is translated to the follower, the average speed of an uncontrolled
 248 follower in one period is the same to that of the leader. Hence, $\varepsilon < 0$ is not acceptable in
 249 green driving strategies, since it reduces the average speed of the controlled vehicle.

250 2. $\varepsilon > 0$, i.e., $U_n > \bar{v}_{n-1}$.

251 Under this condition, initially, spacing of vehicle n is smaller than or equal to $s_j + U_n\tau$, i.e.,
 252 vehicle n travels in congested traffic. $x_n(\tau) = x_{n-1}(0) - s_j = x_n(0) + \frac{v_f}{2}\tau$. After time $t = \tau$,
 253 vehicle n will follow its leader until its spacing is larger than or equal to $s_j + U_n\tau$. Assume
 254 that at $t = t_1$,

$$x_{n-1}(t_1) - x_n(t_1) = s_j + U_n\tau \quad (17)$$

255 From **Equation 13**, vehicle n travels in free flow traffic with the speed limit U_n after $t =$
 256 $t_1 + \tau$. Vehicle n will not follow its leader until its spacing is smaller than $s_j + U_n\tau$ again.
 257 So, we get

$$x_{n-1}(t_2) - x_n(t_2) = s_j + U_n\tau, \quad (18)$$

258 where $t_2 > t_1$. $t_1 + \tau$ is the starting time that vehicle n fails to follow its leader, and $t_2 + \tau$ is
 259 the time that vehicle n follows its leader again after $t_1 + \tau$. From **Equation 17** and **Equation**
 260 **18**, there exists

$$x_{n-1}(t_2) - x_{n-1}(t_1) = x_n(t_2) - x_n(t_1) \quad (19)$$

261 **Equation 19** indicates that if a green driving vehicle wants to follow its leader, the distance
 262 it travels with the designed speed limit should be equal to that the leader travels in the same

263 period. **Equation 19** is equivalent to

$$\int_{t_1}^{t_2} \frac{v_f(1 + \sin(\frac{2\pi t}{T}))}{2} dt = U_n(t_2 - t_1). \quad (20)$$

264 After $t = t_2 + \tau$, vehicle n will follow its leader again. The situation is similar to that of $t < t_1$.
 265 Solving **Equation 17** and **Equation 20**, we can get $t_1 = \frac{T}{2\pi} \arcsin(\frac{2\varepsilon}{v_f})$, but there is no close
 266 form for t_2 . If ε tends to 0, cosine and sine function can be linearized: $\lim_{\delta \rightarrow 0} \sin(\delta) =$
 267 $\delta, \lim_{\delta \rightarrow 0} \cos(\delta) = 1 - 2\delta^2$. Substituting these linearizations into **Equation 18** and **Equation**
 268 **20**, we solve t_1 and t_2 .

$$t_1 = \frac{\varepsilon T}{\pi v_f} = aT \quad (21)$$

$$t_2 = T + \frac{\varepsilon T}{2\pi v_f} - \sqrt{\frac{\varepsilon T^2}{\pi v_f} + \frac{\varepsilon^2 T^2}{4\pi^2 v_f^2}} = bT \quad (22)$$

269 where, $a = \frac{\varepsilon}{\pi v_f}, b = 1 + \frac{\varepsilon}{2\pi v_f} - \sqrt{\frac{\varepsilon}{\pi v_f} + \frac{\varepsilon^2}{4\pi^2 v_f^2}}$. From the preceding analysis, we conclude that

$$x_n(t + \tau) = \begin{cases} x_{n-1}(t_1) - s_j + U_n(t - t_1) & t_1 \leq t < t_2 \\ x_{n-1}(t) - s_j & t_2 \leq t < T + t_1 \end{cases}. \quad (23)$$

270 Controlled velocity of vehicle n is

$$v_n(t + \tau) = x'_n(t + \tau) = \begin{cases} U_n & t_1 \leq t < t_2 \\ \frac{v_f(1 + \sin(\frac{2\pi t}{T}))}{2} & t_2 \leq t < T + t_1 \end{cases}. \quad (24)$$

271 **Figure 2** shows the solution of the velocity in this scenario (green dashed line with speed
 272 limit $U_{\varepsilon > 0} = \bar{v}_{n-1} + \varepsilon$). And standard deviation of the velocity is obtained below.

$$\begin{aligned} s.d.(v_n) &= \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} (v_n(t + \tau) - \bar{v}_n)^2 dt} \\ &= \sqrt{\frac{\varepsilon^2(t_2 - t_1) + \frac{v_f^2}{4} \{t_1 + T - t_2 + \frac{T}{4\pi} [\sin(\frac{4\pi t_2}{T}) - \sin(\frac{4\pi t_1}{T})]\}}{T}} \end{aligned} \quad (25)$$

273 We derive that $s.d.(v_n) = \sqrt{\varepsilon^2(b-a) + \frac{v_f^2}{4}\{a+1-b + \frac{1}{4\pi}[\sin(4\pi b) - \sin(4\pi a)]\}}$ when ε is
274 very small. This indicates that the standard deviation of speed is not related to the period T .
275 **Figure 3** shows the relationship between standard deviation of the velocity and ε with $T =$
276 200 seconds and $v_f = 65$ mph. It indicates that when ε becomes larger, standard deviation
277 of velocity tends to be larger. In order to smooth traffic without reducing average speed, the
278 speed limit should be larger than but close to the average speed of uncontrolled traffic.

279 From previous analysis of constant green driving strategy, we obtain several tips of setting
280 speed limit for green driving vehicles.

- 281 1. Speed limit should not be smaller than the average speed of the traffic; otherwise, it will
282 increase travel time;
- 283 2. Speed limit could be set close to but larger than the average speed of the traffic, which can
284 smooth traffic waves with small standard deviation of speed.

285 In reality, the average speed of the traffic cannot be predicted precisely due to the complicate
286 traffic conditions and noises. Hence, constant green driving strategies are not sufficient to smooth
287 traffic well. We will investigate a dynamic green driving strategy in section 4.

288 **4 Development and Evaluation of a Dynamic Green Driving** 289 **Strategy**

290 In section 3, we have theoretically analyzed the underlying mechanism that the constant green
291 driving strategy adjusted vehicle speed to smooth traffic. However, in reality, period of traffic
292 oscillation and average speed cannot be measured accurately [33]. Moreover, the average speed of
293 the traffic is not constant in real transportation system. For example, when a shock wave occurs,
294 the average speed of the traffic will decrease significantly. Hence, in real world, the constant green
295 driving strategy is not sufficient to obtain a better control of traffic. Targeting on processing green

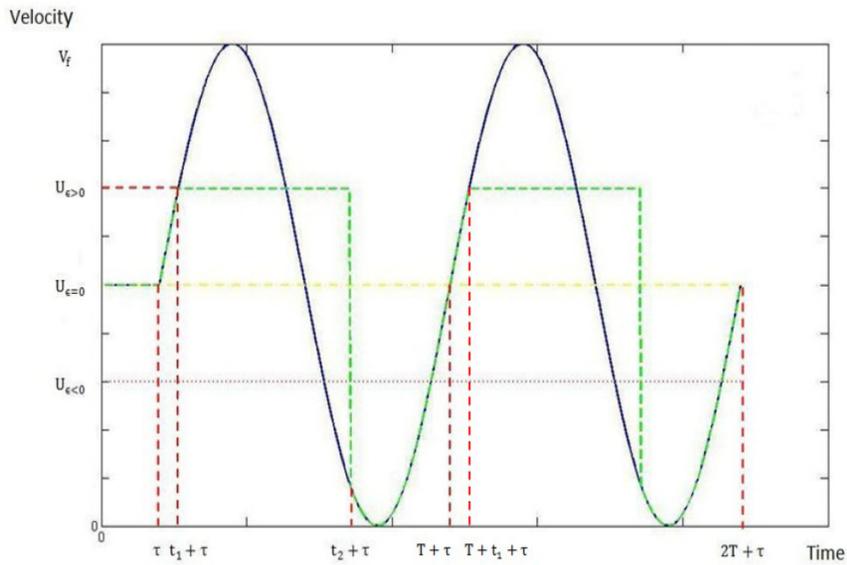


Figure 2: Controlled speed with different speed limits: $\epsilon < 0$ (dotted), $\epsilon = 0$ (dashdot), $\epsilon > 0$ (dashed)

296 driving problem in reality, we propose one dynamic green driving strategy based on feedback
 297 control. In this section, we will firstly describe the dynamic green driving strategy. Then, effect of
 298 this strategy, including traffic smoothness and emission savings, is evaluated under various market
 299 penetration rates and communication delays by simulations.

300 4.1 Model Description

301 In this part, we present a dynamic green driving strategy based on feedback control system. Design
 302 of dynamic speed limit should satisfy the goals and requirements of green driving.

- 303 1. Vehicle doing green driving should travel smoother without too much excessive speed and
 304 quick accelerations;
- 305 2. Average speed of controlled vehicles should not decrease;
- 306 3. The strategy should work even only one vehicle is doing green driving;

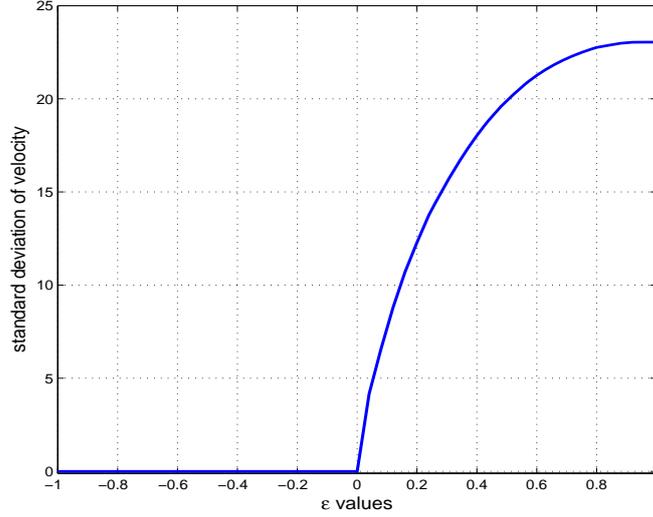


Figure 3: Standard deviation of controlled speed with different ϵ values (T=200 seconds, $v_f=65$ mph)

307 4. More vehicles doing green driving would smooth traffic better.

308 In that sense, we use mean and standard deviation of speed as the control objective. A PI control
 309 is applied in the feedback control system to realize green driving. The PI control components
 310 integrated with a processor is designed for a dynamic green driving strategy to set speed limits.
 311 The control system is illustrated in **Figure 4**.

Here, we introduce a dynamic control variable $U_g(t)$ as the speed limit for green driving vehicle $g(g \in \mathcal{G})$ at time t . This speed limit is only applied when it is safe, since we cannot control vehicle's speed directly without considering safety. In a traffic stream, we can apply the same dynamic speed limit for all controlled vehicles, i.e., $U_g(i) = U(i)$, where $U_g(i)$ is discretization of $U_g(t)$. In this case, communications among vehicles are necessary to share related information, including vehicle location $x(t)$, speed $v(t)$ and spacing $s(t)$. Once the strategy is applied, variance of the speed should tend to zero, and controlled vehicle g should follow its leading vehicle when its spacing is the smallest during a time period T_1 . That is, for any i , there exists $k \in \{i - T_1 + 1, \dots, i\}$,

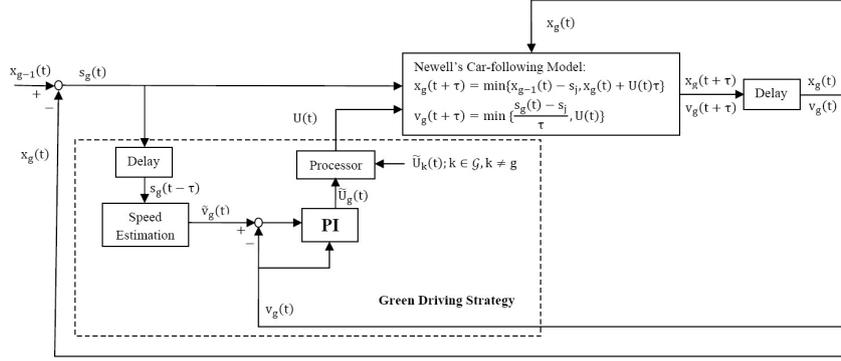


Figure 4: Block diagram of a dynamic green driving Strategy

such that

$$v_g(k) = \frac{x_{g-1}(k-1) - x_g(k-1) - s_j}{\tau}.$$

312 Here, T_1 should be larger than half of the period of the leading trajectory. From **Equation 2**,
 313 $v_g(k) \leq \frac{x_{g-1}(k-1) - x_g(k-1) - s_j}{\tau} = \tilde{v}_g(k), \forall k$, where $\tilde{v}_g(k)$ is predictive speed of vehicle g at time k ,
 314 which is calculated by speed estimator. Based on these, we use a PI control to design a intermediate
 315 speed limti of vehicle g at time i , i.e., $\tilde{U}_g(i)$. Speed of vehicle g and the error between actual speed
 316 v_g and estimated speed \tilde{v}_g are used in PI control.

$$\tilde{U}_g(i) = \frac{\sum_{k=i-T_1+1}^i v_g(k)}{T_1} + K_p \cdot \min_{k=i-T_1+1}^i \left\{ \frac{x_{g-1}(k-1) - x_g(k-1) - s_j}{\tau} - v_g(k) \right\} \quad (26)$$

317 As we know that when speed limit is closer to average speed of the traffic, controlled vehicle
 318 can travel much smoother. So, the first term $\frac{\sum_{k=i-T_1+1}^i v_g(k)}{T_1}$ is added in the speed control strat-
 319 egy. This term is designed to smooth traffic. The second term $\min_{k=i-T_1+1}^i \left\{ \frac{x_{g-1}(k-1) - x_g(k-1) - s_j}{\tau} - \right.$
 320 $v_g(k) \left. \right\}$ is introduced to guarantee that the controlled vehicles can follow its leader.

321 IVC systems provide both historical and other vehicles' information, which could be imple-
 322 mented to make the strategy more robust and efficient. Therefore, a processor is introduced to
 323 apply these information to design speed limit $U(i)$. Intuitively, the most recent information are
 324 more useful in determining the speed limits. We set different weights for historical information

325 and get

$$U_g(i) = w_1 \cdot \frac{\sum_{k=T_1-1}^{i-T_1} \tilde{U}_g(k)}{i - 2(T_1 - 1)} + w_2 \cdot \frac{\sum_{k=i-T_1+1}^i \tilde{U}_g(k)}{T_1}. \quad (27)$$

326 Here, $w_1 + w_2 = 1$. w_2 is the weight of the most recent traffic information, so we set w_2 be
 327 larger than w_1 . Then thinking about information from the other IVC equipped vehicles, we set
 328 speed limit for all equipped vehicles as

$$U(i) = \frac{\sum_{g \in \mathcal{G}} U_g(i)}{G}. \quad (28)$$

329 Considering communication delay in IVC system, the strategy should be modified. Since with
 330 communication delay, different equipped vehicles can receive different traffic information, the
 331 speed limits of different vehicles will be different. Assume that the communication delays are the
 332 same and constant for all equipped vehicles. We donate the communicate delay by D . Hence,
 333 speed limit of one equipped vehicle g at time i is

$$U_g^D(i) = \frac{U_g(i) + \sum_{k \neq g, k \in \mathcal{G}} U_k(i - D)}{G}. \quad (29)$$

334 Here, we start to apply the green driving strategy at $2(T_1 - 1) + D$, so that there is sufficient
 335 information to obtain speed limits. $U_g^D(i)$ is donated as the speed limit of vehicle g with delay.

336 4.2 Effect of Dynamic Green Driving Strategy

337 In this part, one freeway traffic scenario is analyzed with MATLAB simulation. As a starting point,
 338 a straight open roadway section is considered. On this road section, several vehicles runs in the
 339 same lane without lane-changing. The trajectory of the leading vehicle is provided, and several
 340 following vehicles are controlled with dynamic green driving strategy. Smoothness of traffic is
 341 compared between controlled traffic and unadjusted traffic to estimate effect of the green driving
 342 strategy. Moreover, CMEM emission model is implemented to measure the total emission and fuel
 343 consumption savings.

Table 1 Setting of Simulation and Green Driving Strategy

Parameters	Values
Free flow speed v_f	65 mph
Jam spacing s_j	23.83 ft
Time gap τ	1 sec
Trajectory Period I T_{p1}	50 sec
Trajectory Period II T_{p2}	150 sec
Trajectory Period III T_{p3}	300 sec
Duration T_1	2.5 min
K_p	0.01
w_1	0.25

344 Assume there are only two vehicles $\{1, 2\}$ running with Newell's car-following model on the
345 road. The speed of the leading vehicle 1 is known.

$$v_1(t) = v_f \left(\frac{1 + \sin\left(\frac{2\pi t}{T_{p1}}\right)}{6} + \frac{1 + \sin\left(\frac{2\pi t}{T_{p2}}\right)}{6} + \frac{1 + \sin\left(\frac{2\pi t}{T_{p3}}\right)}{6} \right), t \geq 0 \quad (30)$$

346 where, T_{p1}, T_{p2}, T_{p3} are the trajectory periods varying from 50 to 150 seconds. **Table 1** gives the
347 settings of this simulation. The result is shown in **Figure 5**. The solid line shows speed of vehicle
348 2 without applying green driving strategy; while the dashed line indicates the speed of vehicle
349 2 applying green driving strategy. In both scenarios, trajectory of the leading vehicle 1 is the
350 same. The two velocities have the approximately the same average values, but vehicle applying
351 the dynamic green driving strategy has a much smoother speed trajectory. This will also result in
352 lower emissions and fuel consumption. **Table 2** lists the statistical comparison of the two simulated
353 speeds of vehicle 2 when the green driving strategy is used. We further use CMEM to estimate
354 emissions and fuel consumption of these two trajectories. **Table 3** lists the total emissions, fuel
355 consumption and relative savings. It implies that there exist significant savings of emissions and
356 fuel consumption, and at the same time, the average speed does not change too much.

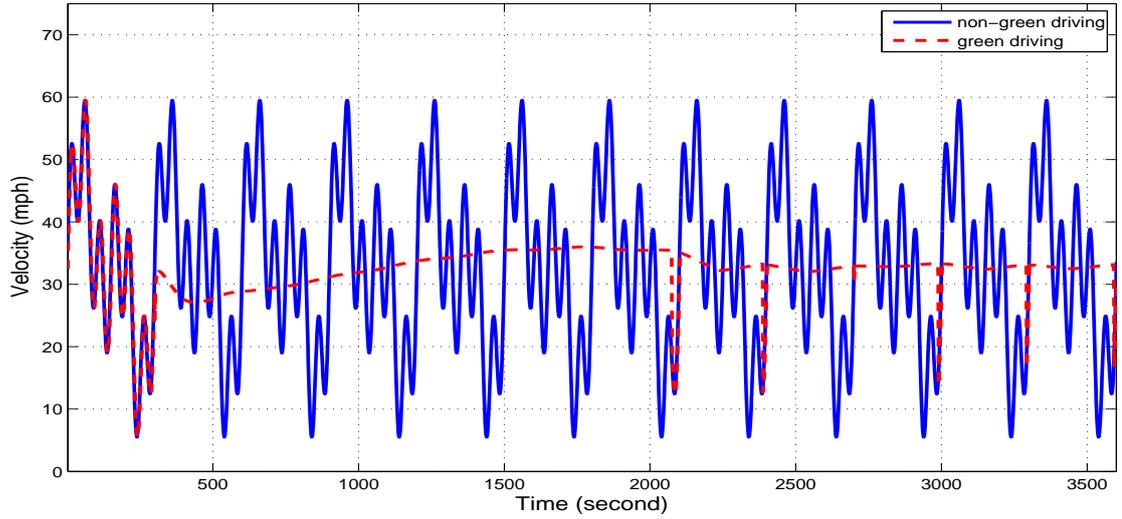


Figure 5: Speed comparison of green driving and non-green driving traffic

Table 2 Statistics of Velocity

Velocity	Non-green driving	Green driving	Difference (%)
Min (mph)	5.49	12.41	125.87
Max (mph)	59.51	36.00	-39.51
Mean (mph)	32.50	32.50	0
std. dev. (mph)	13.27	2.95	-77.80

357 **4.3 Effect of Market Penetration rates of IVC Equipped Vehicles and Com-** 358 **munication Delay on Traffic and Emissions**

359 In order to investigate the effect of the characters of IVC systems on smoothing traffic and reducing
360 emissions and fuel consumption, we simulate traffic in an one way ring road with length $L = 1.3$
361 miles, where $N = 100$ vehicles are traveling in the same direction. The initial locations of all
362 vehicles are set in **Equation 31**.

$$x_n(0) = \sum_{k=1}^n (L/N - s_j) \left(\sin\left(\frac{2\pi k}{N}\right) + 1 \right) + s_j, \forall n = 1, 2, \dots, N \quad (31)$$

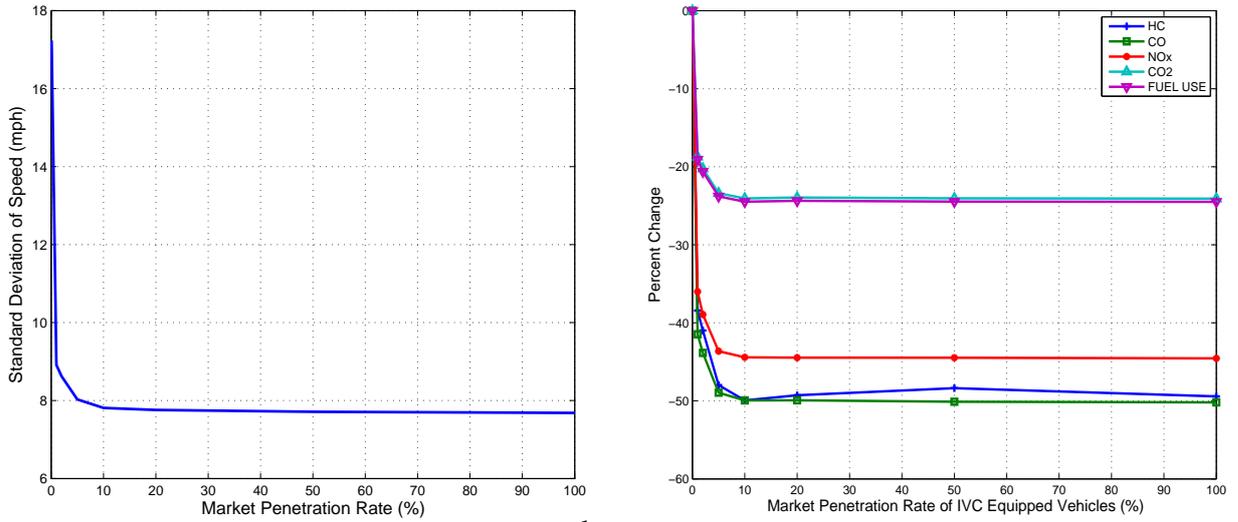
Table 3 Emissions and Fuel Consumption of Two Sample Speed Trajectories

	Non-green driving	Green driving	Difference (%)
CO ₂ (g/mi)	276.897	210.693	-23.91
CO (g/mi)	3.053	1.373	-55.03
HC (g/mi)	0.141	0.060	-57.45
NO _x (g/mi)	0.420	0.086	-79.52
Fuel Use (g/mi)	88.948	67.160	-24.50

363 In the simulation, the duration time T_1 is set as 1 minutes, and the total simulation time is 10
364 minutes. All the other parameters are the same to section 4.2.

365 Considering Newell's car-following model in a ring road, we see that even only one vehicle is
366 applying the green driving strategy, traffic can still be smoothed, since the green driving vehicle
367 will affect the driving behaviors of its followers and make their trajectories smoother. Moreover,
368 with higher MPR of IVC equipped vehicles, more vehicles will exchange information, which is
369 helpful on smoothing traffic and reducing emissions. Therefore, it is expected that higher MPR's
370 of IVC equipped vehicles lead to higher savings of emission and fuel consumption, and this saving
371 is significant even fewer vehicles are equipped. To determine the effectiveness of MPR of IVC
372 equipped vehicles on emissions and fuel consumption, one simulation was constructed to imple-
373 ment the dynamic green driving strategy under different MPR's. **Figure 6** shows the results. The
374 results indicates that the savings increase gradually and the standard deviation of speed decrease
375 with MPR's. For HC, its reduction increases from 38.4% at 1% MPR to 49.4% at 100% MPR; for
376 CO, it increases from 41.5% to 50.2%; for NO_x, it is from 36.0% to 44.6%; for CO₂, it is from
377 18.7% to 24.1%; and for fuel consumption, it is from 19.1% to 24.5%. Moreover, the reduction
378 of speed limit is smaller than 1.5% for any market penetration rates. Another observation is that
379 when MPR is smaller than 10%, the influence of MPR on saving emission and smoothing traffic
380 is also very significant, i.e., the dynamic green driving strategy works for smaller IVC equipped
381 vehicles.

382 Another issue is studying the effect of communication delay on the dynamic green driving
383 strategy. When communication delay is larger, the traffic information exchanged between IVC



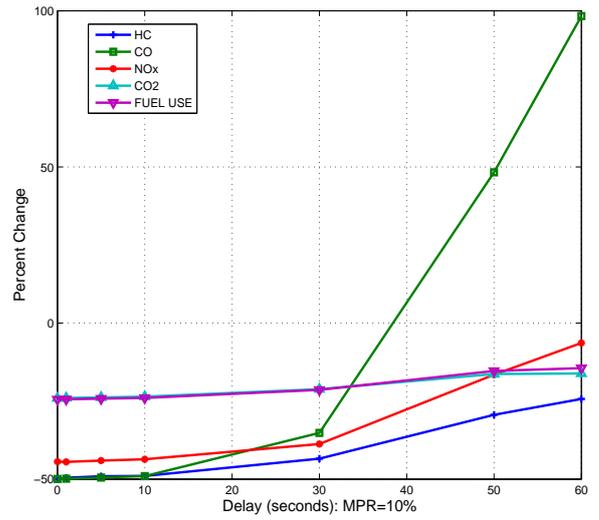
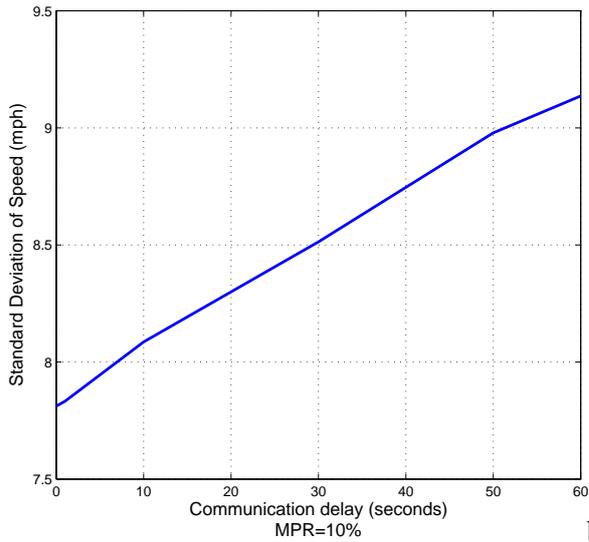
a. b.

Figure 6: Effect of the dynamic green driving strategy under different market penetration rates of IVC equipped vehicles: a. standard deviation of speed; b. emission and fuel consumption reductions.

384 equipped vehicles are less useful to determine the speed limits. It implies that communication
 385 delay will decrease the effect of the green driving strategy. Since the strategy can work for small
 386 market penetration rate, we arbitrarily set MPR be 10% to investigate the effect of communication
 387 delay. In the simulation, communication delay varies from 0 to 60 seconds. **Figure 7** verifies our
 388 prediction that with higher communication delay, savings of emissions and fuel consumption are
 389 smaller and traffic is less smooth, i.e., standard deviation of speed tends to be larger. However, this
 390 reduction is not significant when communication delay is smaller than 10 seconds.

391 5 Conclusion and Future Work

392 In this paper, we developed a set of green driving strategies through a feedback control system.
 393 The green driving strategies were implemented via eliminating the stop-and-go waves to reduce
 394 the number and severity of individual accelerations and decelerations, and result in lower emis-
 395 sions and fuel consumption. We first investigated the failure of two green driving strategy using



a. b.

Figure 7: Effect of the dynamic green driving strategy under different communication delays of IVC equipped vehicles: a. standard deviation of speed; b. emission and fuel consumption reductions.

396 the average speed and the predictive average speed as speed limits theoretically. The limitation
 397 existed in these two strategies was that they might reduce the average speed of the traffic. We
 398 then theoretically analyzed effect of one constant green driving strategy on smoothing traffic and
 399 investigated the effect of a dynamic strategy.

400 One important contribution of this paper was that a significant insight was obtained through
 401 theoretical discussions on a constant green driving strategies: a better and executable green driving
 402 strategy was setting speed limit close to the average speed of the traffic, but it should not be smaller
 403 than the average value. The larger the speed limit, the less smoother the traffic. The principle of
 404 a successful green driving strategy was that the distance that a controlled vehicle traveled with
 405 designed speed limit to follow its leader should be equal to that the leader traveled during the same
 406 period without control.

407 Another contribution of this paper was that a robust dynamic green driving strategy was pro-
 408 posed with consideration of smoothing traffic flow and guaranteeing that a following vehicle fol-
 409 lowed its leader during a certain long period, i.e., reducing speed variation without decreasing the

410 average speed of the traffic. The strategy was built from a feedback control system with PI con-
411 trol, and provided dynamic speed limits to IVC equipped vehicles. With experiment, this strategy
412 could reduce standard deviation of speed as much as 77.8% with smaller reduction of the average
413 speed. Emissions and fuel consumption were reduced significantly without decreasing the aver-
414 age speed of the traffic even only one vehicle was controlled. Moreover, market penetration rates
415 of IVC equipped vehicles and communication delay were investigated to show their influences
416 on the dynamic green driving strategy. Results indicated that higher MPR could lead to higher
417 emissions and fuel consumption savings, and this savings were significant even MPR was smaller
418 than 10%. Communication delay could reduce the effect of the green driving strategy. With larger
419 communication delay, emissions and fuel consumption savings were smaller and the traffic was
420 less smooth.

421 In the future, the dynamic green driving strategy will be further developed and applied in non-
422 homogeneous traffic and evaluated using simulation modeling tools. Properties of communication,
423 such as dynamic communication delays, communication connectivity [18, 23], will be in consider-
424 ation. Further, real world experiments would be helpful for future studies on the effect of dynamic
425 green driving strategy for various traffic scenarios.

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