# An Urban Intersection Model Based on Multi-commodity Kinematic Wave Theories


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***Abstract*-Traffic flow models of an urban intersection can be used to evaluate its performance and offer a base for traffic control strategies, road network design, and so on. The complex conflicts among traffic streams at an urban intersection, including zero conflict, merging conflict, diverging conflict and crossing conflict, have not been synthetically modeled by existing models. In this paper, we consider a four-leg urban intersection as a grid of 2×2 links (2×2 grid) and present a multi-commodity kinematic wave (MCKW) traffic flow model, in which all four conflicts are modeled. In this model, we use a fair merging rule and both strictly First-In-First-Out (FIFO) and non-strictly FIFO diverging rules. Moreover the model is used to simulate conflicts at a signalized urban intersection, where there are no crossing conflicts. As a result, the recurrence of gridlock occurred at un-signalized case is prevented.**


## I. INTRODUCTION

Urban intersections can be signalized or un-signalized. In an urban road network, intersections usually constitute major bottlenecks, due to complicated interactions between traffic streams in different directions. Thus it is important to understand traffic dynamics around urban intersections in order to understand the formation and dissipation of traffic congestion as well as to devise better signal and lane control strategies.

In literature, there have been many studies on modeling interactions among traffic streams at urban intersections and then measuring the static characteristics such as capacity, queue length and delay of specific traffic steams. Those models are mostly based on gap acceptance theory, in which priorities of all related streams have to be predefined. In [1] (Chapter 17), Un-signalized intersections are classified into three types: Two-Way Stop-Controlled intersection, All-Way Stop-Controlled intersection and Roundabouts, with capacities, queue length and delay computed. Each concerned traffic stream is given the priority of right-of-way, and different priority modes are used. Carlos F. Daganzo reviewed the traffic dynamics at an un-signalized intersection with only two traffic streams concerned in [2]. The minor-priority stream crosses the major-priority stream at an acceptable gap between two neighbored vehicles in major-priority stream. Thus only crossing conflict happened and was handled. In [3], a universal procedure is introduced to calculate the capacity at priority-controlled un-signalized intersections. In [4], a general queuing theory model for traffic flow at un-signalized intersections is described with critical gaps and merging times stochastically dependent. The model computes distributions of queue lengths, delays and capacities. For roundabouts — a cluster of transformed un-signalized intersections, only interactions of merging and diverging among traffic flows happen at the intersection due to their special network structure. In [5], limited priority was supplied for the major stream to accommodate the minor steam vehicles. In addition, a parsimonious macroscopic model for single-lane roundabout is presented in [6]. Two priority modes — absolute priority mode and limited priority mode — were introduced to account for the potential influence of inserting vehicles over circulating traffic.

However, the presented models can not help to better understand the time-dependent characteristics or dynamics associated with the formation of traffic congestion. Moreover they always considered simple scenarios, for instance a scenario with only a major stream and a minor stream, and thus modeled merely one or two kinds out of all interactions. Those models are mostly based on gap acceptance theory, in which priorities of all related streams have to be predefined.

In this paper, we attempt to model traffic dynamics at an urban intersection based on multi-commodity discrete kinematic wave models [7]. Different from an urban intersection model in [8], we consider the center of an urban intersection as a grid of 2×2 links. With this consideration, we can capture different types of conflict among traffic streams. At un-signalized case, traffic streams from all upstream links swarm into the intersection equivalently with the same priority. Four kinds of conflicts, including zero conflict, merging conflict, diverging conflict and crossing conflict, are jointly simulated. Several scenarios are adopted in the simulation to explore dynamics of the interactions and some significant results are obtained.

In the rest of the paper, an overview of the MCKW theories will be given in section II. In section III, we present the 2×2 grid model of urban intersections. In section IV, numerical simulation will be carried out. In section V, we make some conclusions and suggest future work to make this model more complete as well as other future work based on the model.

## II. MCKW MODELS OF HIGHWAY NETWORKS

### A. Godunov Method

Godunov method uses the discretized form of LWR model. Each link is cut into $N$ cells, with length $\Delta x$ each, and the time interval is cut into $K$ time steps, with duration $\Delta t$ each. Then the Godunov-type finite difference equation for total flow in cell $i$ from time step $j$ to time step $j+1$ is

$$\frac{\rho_i^{j+1}-\rho_i^j}{\Delta t}+\frac{f_{i+1/2}^{j^*}-f_{i-1/2}^{j^*}}{\Delta x}=0 \quad , \tag{1}$$

where $f_{i-1/2}^{j^*}$ ( $f_{i+1/2}^{j^*}$ ) denotes the flux through the upstream (downstream) boundary of cell $i$, which can be easily computed by the subsequent Supply-Demand method.

### B. Supply- Demand Method

Supply-demand method [10, 11] is used to computing fluxes through cell boundaries: $f_{i-1/2}^{j^*}, f_{i+1/2}^{j^*}$. Demand $D_i$ and supply $S_i$ that a cell $i$ produces is defined as

$$D_i=\begin{cases} Q(\rho_i) & when\ \rho_i\ is\ under-critical \\ Q_i^{\max} & when\ \rho_i\ is\ over-critical \end{cases},$$

and

$$S_i=\begin{cases} Q_i^{\max} & when\ \rho_i\ is\ under-critical \\ Q(\rho_i) & when\ \rho_i\ is\ over-critical \end{cases},$$

where $\rho_i$ denotes the density of cell $i$.

We discuss four junctions: link boundary, merge, diverge and intersection. However, we use two generic methods as follows to deal with those various junctions. They are called strictly FIFO method and non-strictly FIFO method [9]. For a junction with $U$ upstream cells and $D$ downstream cells, they are formulated as follows.

*1) Strictly FIFO method:*

$$\begin{aligned} f_{u,d} &= \min_{e\in D(u)}\left\{D_u\xi_{u,d}\,,\,S_e\frac{D_u\xi_{u,d}}{\sum_{v\in U(e)}D_v\xi_{v,d}}\right\} \\ &= \min_{e\in D(u)}\left\{1,\frac{S_e}{\sum_{v\in U(e)}D_v\xi_{v,d}}\right\}D_u\xi_{u,d} \end{aligned} \tag{2}$$

$$u=1,2,\cdots,U;\ d=1,2,\cdots D$$

*2) Non-strictly FIFO method:*

$$\begin{aligned} f_{u,d} &= \min\left\{D_u\xi_{u,d}\,,\,S_d\frac{D_u\xi_{u,d}}{\sum_{v\in U(d)}D_v\xi_{v,d}}\right\} \\ &= \min\left\{1,\frac{S_d}{\sum_{v\in U(d)}D_v\xi_{v,d}}\right\}D_u\xi_{u,d} \end{aligned} \tag{3}$$

$$u=1,2,\cdots,U;\ d=1,2,\cdots D$$

where, $f_{u,d}$ denotes flux from upstream cell u to downstream cell d, $\xi_{u,d}$ is the proportion of vehicles heading downstream cell $d$ in upstream cell $u$, D(u) is set of downstream links of upstream link u and U(e) is set of upstream links of downstream link e. Strictly FIFO method strictly meets the FIFO diverging rule and non-strictly FIFO method partly meets the FIFO diverging rule.

## III. MCKW MODEL OF URBAN INTERSECTIONS

### A. Structure of Urban Intersections and Conflicts among Traffic Streams

For a general four-leg urban intersection as shown in Fig. 1, there are eight links in total, with four links in and four links out. They are denoted by I1, I2, I3, I4, O1, O2, O3, O4. Each link can consist of a number of lanes. Up to 16 commodities are formed according to their paths or O-D pairs. They are commodities 11, 12, 13, 14, …, 41, 42, 43, 44.

For un-signalized case, there exist four kinds of conflicts between commodities at the center of the intersection: merge conflict, diverge conflict, crossing conflict and zero conflict (i.e., no conflict); for signalized case, there exist three kinds of conflicts: merge conflicts, diverge conflict and zero conflict, with crossing conflict eliminated. The four conflict patterns are described in Fig. 2, only one example for each pattern presented for simplification.

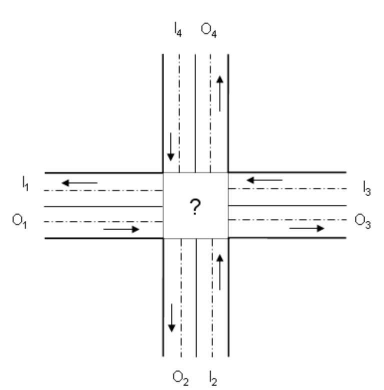


Fig. 1. Structure of a general four-leg urban intersection

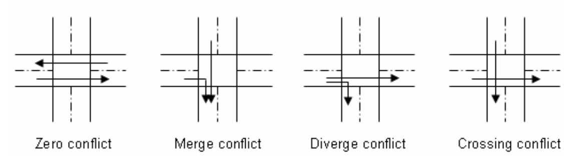


Fig. 2. Four conflict patterns

### B. Modeling of Un-signalized Intersection

In order to capture traffic dynamics at the macroscopic level, we can consider the center of an intersection as a point, a link, or a grid of 2×2 links. They correspond to three urban intersection models based on MCKW theory: point model, link model and the new 2×2 grid model.

In the point model [9], the center of an intersection takes no space and imposes no restriction on traffic flow. In this model, we have a junction of four upstream and four downstream links. We can see that all conflicts are roughly treated as merge and diverge conflicts.

In the link model [9], the center of an intersection is considered as a link of certain length. This model can be decomposed into a merge model and a diverge model. In this model, the capacity of the intermediate link will determine the overall capacity of the intersection. We can see that merging and diverging conflicts are reasonably captured, zero conflicts are not captured, and crossing conflicts are partially captured.

In the 2×2 grid model, the center of an intersection is considered as a grid of 2×2 links (each link is described in this model by a cell called central cell) shown in Fig. 3.

We can not just divide the 2x2 grid into four cells and then compute them respectively, because they are closely coupled together as Fig. 4 shows.

In Fig. 4, $D_{Ci}$ ($S_{Ci}$) denotes demand (supply) produced by central cell $C_i$, $D_{Ii}$ denotes demand produced by upstream link $I_i$ and $S_{Oi}$ denotes supply produced by downstream link $O_i$.

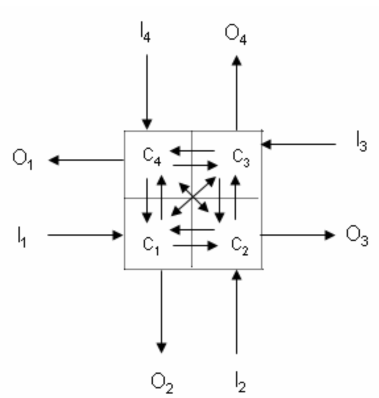

Fig. 3. A four-leg urban intersection with a 2x2 grid center

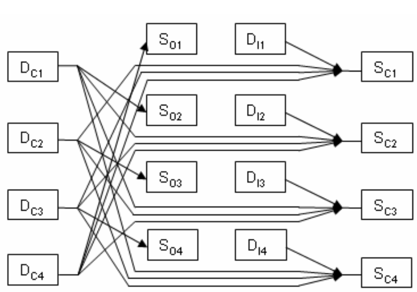

Fig. 4. Fluxes at a four-leg urban intersection with a 2x2 grid center

There are 20 boundary fluxes at the intersection which are denoted by arrows in Fig. 4 above. Here, strictly-FIFO method in Equation 2 and non-strictly FIFO method in Equation 3 are used to compute the model. One can use the Fig. 4 to easily find upstream cells (or links) and downstream cells (or links) of any cell or link. Here, when computing flux $f_{u,d}$, we can quickly index $D(u)$ and $U(e)$ of each $e \in D(u)$.

With this model, merge conflicts and diverge conflicts in real world are all considered; crossing conflicts are considered but underestimated; and zero conflicts overestimated. For example, the crossing conflict of commodity 14 with commodity 21 at the center is wrongly considered as zero conflict by the model. However, it is definitely true and progressive that 2×2 grid model can simulate the conflicts or interactions of commodities at the intersection more accurately compared to the point model and link model.

### C. *Modeling of Signalized Intersection*

For signalized case, conflicts merely exist among those upstream link flows which are permitted to enter the intersection at certain phases. As a result, here exist merge conflicts, diverge conflicts and zero conflicts except for crossing conflicts. At each phase during a signal cycle, we find the permitted commodities that can pass through the intersection and thus obtain a partial intersection, which is afterwards studied with the un-signalized 2×2 grid model in section 5. Then, by integrating the four partial intersection models at four corresponding phases, we obtain model of the intersection during the whole signal cycle which is a piecewise model. To extend the un-signalized case to the signalized case, we just need to revise the upstream link demand $D_{u,d}$ as the following way and then use strictly FIFO method (3) and non-strictly FIFO method (4) to compute out-fluxes of upstream links.

$$\overline{D_{u,d}} = D_{u,d} \cdot \pi_{u,d}(t), \quad u = 1,2,3,4; d = 1,2,3,4 \quad ,$$

where, $\pi_{u,d}(t)$ is a piecewise function of time $t$. The value of $\pi_{u,d}(t)$ is initialized as follows.

$$\pi_{u,d}(t) = \begin{cases} 1 \ \ if\ \ commodity\ (u,d)\ is\ permitted\ at\ time\ t \\ 0 \ \ else \end{cases} .$$

Here, we take the common signal control strategy in china, four-phase signal control (Fig. 5), as an example.

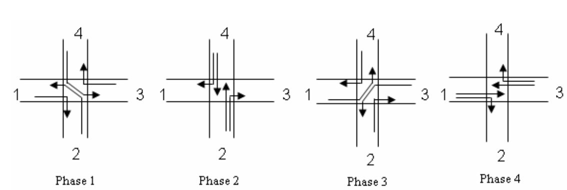

Fig. 5. Four-phase signal control

Vehicles making U-turn are mixed into vehicles making left-turn. Their start-stop movements are synchronous with each other.

## IV. NUMERICAL SIMULATIONS

The 2×2 grid model (Fig. 3) is used here to simulate the interactions of traffic commodities at urban intersections. The parameters' value of the intersection structure, initial condition, boundary condition and some others are as follows. All links are 2 miles long, partitioned into 200 cells with each 0.01 miles long, and 2 lanes wide. Namely $\Delta x$ =0.01 miles. The simulation time is T=0.2 hours, which is divided into K=1000 time steps, with each time step $\Delta t$ =0.0002 hours long. Therefore the maximum CFL number is $v_f \Delta t / \Delta x = 0.9 < 1$, meeting the requirements of Godunov method. As for each lane, jam density is $\rho_j$ =180 vpmpl, critical density is $\rho_c$ =36 vpmpl, free flow speed is $v_f$ =45 mpl, and lane capacity is $q_c = \rho_c v_f = 1620$ vphpl.

Initially, all links are empty. As for boundary conditions, the origin demand arrays (4 elements each array) vary at different levels in different scenarios (TABLE I) for link I1, I2, I3, I4, and the destination supply arrays are always at capacity level 2$q_c$ for link O1, O2, O3, O4. We obtain 16 commodities by their path. An U-D (Upstream link-Downstream link) pair represents a commodity. Corresponding to the 16 commodities, the turning proportions of the four upstream flows to the four downstream links are all predefined in different scenarios (TABLE I). All links have the same triangular fundamental diagram as (1) shows. By experience, $\alpha$ is set to 0.6 for vehicles in the U-turn commodity, 0.7 for the left-turn commodity, 0.8 for the right-turn commodity, and 1 for straight commodity. However, in the simulations here, we fix $\alpha$ to 1 for simplification as if $\alpha$ is not introduced.

We compute the throughput as an indicator of the performance of the intersection, because it's simple, feasible and significant for the intersection network. Throughput of intersection is usually a bottleneck of traffic network. When we design an intersection, we hope it can get more vehicles through in less time. We implement the model by programming and then do a convergence test to validate the model.

### *A. Simulation for Un-signalized Intersections*

Using strictly FIFO method and non-strictly FIFO method respectively, we draw out Fig. 6 and Fig. 6, in which we demonstrate that throughputs of an urban intersection varied with time for the first five different scenarios in TABLE I.

TABLE I
SCENARIOS WITH DIFFERENET DEMAND PATTERNS

| Sce. | Demand level | Demand distribution | Sce. | Demand level | Demand distribution |
|---|---|---|---|---|---|
| 1 | 1 | 0, 0, 1, 0 | 4 | 0.5 | 0, 0, 1, 0 |
| | 0 | 0, 0, 0, 1 | | 0.5 | 0, 0, 0, 1 |
| | 1 | 1, 0, 0, 0 | | 0.5 | 1, 0, 0, 0 |
| | 0 | 0, 1, 0, 0 | | 0.5 | 0, 1, 0, 0 |
| 2 | 1 | 0, 0, 1, 0 | 5 | 1 | 0.1,0.2,0.5,0.2 |
| | 1 | 0, 0, 0, 1 | | 1 | 0.2,0.1,0.2,0.5 |
| | 0 | 1, 0, 0, 0 | | 1 | 0.5,0.2,0.1,0.2 |
| | 0 | 0, 1, 0, 0 | | 1 | 0.2,0.5,0.2,0.1 |
| 3 | 1 | 0, 0, 1, 0 | 6 | 1/8 | 0, 0, 0.8, 0.2 |
| | 1 | 0, 0, 0, 1 | | 1/8 | 0.2, 0, 0, 0.8 |
| | 1 | 1, 0, 0, 0 | | 1/8 | 0.8, 0.2, 0, 0 |
| | 1 | 0, 1, 0, 0 | | 1/8 | 0, 0.8, 0.2, 0 |

From Fig. 6, we can observe the following phenomena explained subsequently. The capacity of each central cell is 2$q_c$. In scenario 1, there exists no conflicts between the two commodities, traffic is free and the steady value of throughput is 4$q_c$ that is two times of a central cell's capacity. In scenario 2, there exists crossing conflict in one central cell $C_2$, traffic flow is congested and the steady value of throughput is a central cell's capacity. In scenario 3, the four commodities form a loop at the intersection center and traffic is jammed due to high demand level. In scenario 4, a loop is also formed like in scenario 3, but traffic is free and steady value of throughput is 4$q_c$. Through simulations in many more scenarios, we jump into the heuristic conclusion that, in the case that there exists a loop formed by commodities, a gridlock happens as long as one of central cells is congested, and the total throughput decreases to zero quickly. Base on this point, 2×2 grid model can be used to demonstrate that signal control at the intersection is very necessary. Moreover, we can find that intersection throughput is limited to a maximum value for certain group of commodities passing the intersection, whatever we raise the commodities' upstream demand to. For instance, the maximum value is 4$q_c$ for passing commodities 13 and 31 in scenario 1. Based on this point, 2×2 grid model can be used to demonstrate that the bottleneck size of an urban intersection is decided by both central cell's capacity and combination of passing commodities.

From Fig. 7, we can observe that the results are more exciting than in Fig. 6. The steady values of throughputs in all five scenarios are no less than that in corresponding scenarios in Fig. 6. The same with in Fig. 6, it is also demonstrated that the bottleneck size of an urban intersection is decided by both central cells' capacity and combination of passing commodities. However, the gridlock happens less frequently, which is demonstrated by the two curves in scenario 3 and scenario 5 in Fig. 7 compared to those in Fig. 6.

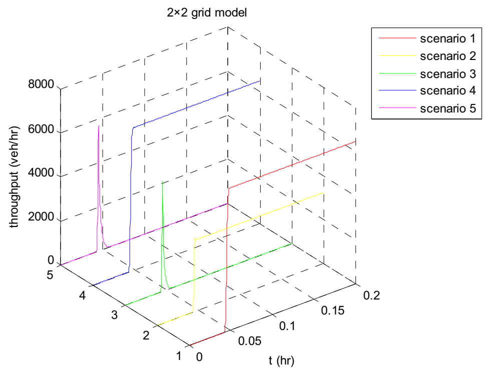


Fig. 6. Throughput of an un-signalized intersection (strictly FIFO method)

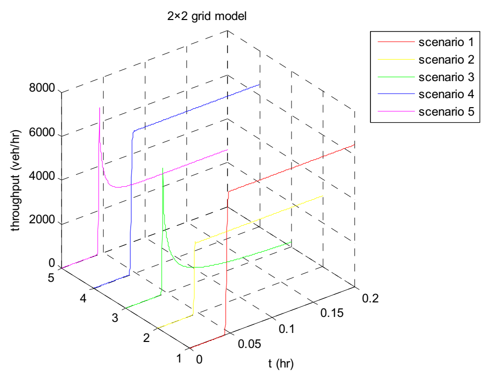


Fig. 7. Throughput of an un-signalized intersection (non-strictly FIFO method)

### *B. Simulation for Signalized Intersections*

Different from un-signalized intersection, the last cell of each upstream link can be treated as a four-commodity diverge cell from the term of commodity. Here we simulate signalized intersection with the 2×2 grid model by only non-strictly FIFO method since strictly FIFO method does not make significance. We set cycle length of signal control to 144 seconds with each phase of 36 seconds.

In Fig. 8, we compute the intersection throughput with time in scenario 6. Observing the steady cycle [0.16hr, 0.20hr), commodities 21 and 43 are discharged into intersection at phase 1. The throughput decreases soon to a valley after increase to a peak since the stopped commodities 24 and 42 are suppressing commodities 21 and 43 respectively. After cumulated for a cycle, commodities 24 and 42 are thrust into the intersection at the beginning of phase 2. The throughput decreases soon to a valley after increase to a peak since the stopped commodities 21 and 43 are suppressing commodities 24 and 42 respectively. The variation of the throughput trajectory over the latter two phases can be inferred in the same way.

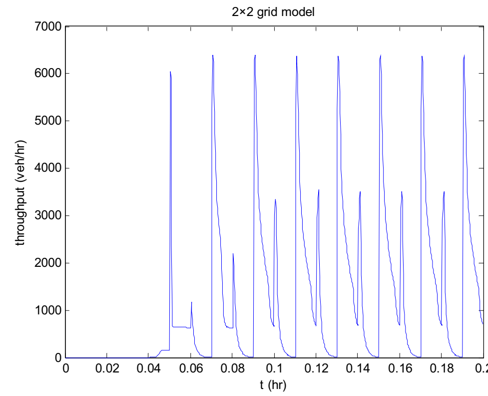


Fig. 8. Throughput of a signalized intersection

## V. CONCLUSION

In this paper we presented a multi-commodity kinematic wave model of a general urban intersection, in which a more complex 2×2 structure is used to better capture the interactions or conflicts at the intersection center. This model is able to distinguish all types of conflicts among traffic streams: zero conflict, merge conflict, diverge conflict and crossing conflict. In the model, two general supply-demand methods, strictly FIFO method and non-strictly FIFO method, are introduced to compute the fluxes through boundaries between cells. Compared with point or link models, this model can describe various conflicts more accurately. Through numerical simulations, we demonstrated that the model is able to capture traffic gridlock at an un-signalized urban intersection when the demand level is relatively high for a number of directions. With the simple model we also demonstrate the effectiveness of traffic signals for an isolated intersection. Namely, by removing crossing conflicts among traffic streams, we are able to avoid serious traffic gridlock.

Compared with microscopic models, kinematic wave models are able to simulate traffic dynamics at the macroscopic level more efficiently. Therefore, we expect that the urban intersection model developed in this study could be applied to study traffic dynamics in a large urban road network. Also as demonstrated in the study, this model can be applied to evaluate and develop different control strategies with traffic signals. In the future, we will be interested in using the 2×2 grid model for further research in traffic flow modeling of a road network with multiple intersections. We will also be interested in applying the model for road network design and signal control strategy design for different O-D matrices.

The kinematic wave model developed in this study, however, is subject to further extensions for other types of junctions in an urban road network. For example, for an intersection with dedicated left-turn or u-turn lanes, we have to consider the diverging of traffic streams before entering the intersection and expect to have more complicated interactions in the center of the intersection. The kinematic wave model is also not appropriate for an intersection with stop-signs. In addition, the model developed here is also subject to calibration and validation with observed data [12]. Then we will research more its application to real-world urban intersections.

## Acknowledgment

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