A KINEMATIC WAVE MODEL FOR EMERGENCY EVACUATION PLANNING

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ABSTRACT

Emergency evacuation plans are critical to reducing damages caused by natural or human-made disasters. In this paper, we present a framework for developing and evaluating evacuation strategies based on a kinematic wave model of network vehicular traffic. We then discuss a measure of the effectiveness of evacuation plans and apply the model to study a simple road network. This model is suitable for emergency evacuations, since road networks can be easily set up and calibrated, computational cost is independent of the number of vehicles, and traffic congestions caused by capacities of road links and network structure can be effectively simulated.

KEY WORDS: Emergency evacuation planning, kinematic wave model of network vehicular traffic, measure of effectiveness

INTRODUCTION

To reduce damages caused by natural or human-made disasters, it is necessary to evacuate people out of the dangerous zone. Emergency evacuation plans are critical in these situations, especially for massive evacuations involving as many as millions of vehicles on a regional road network. None or inefficient evacuation plans could make evacuation itself a disaster. For example, during the massive evacuation for Hurricane Rita [1], millions of people got stuck in traffic jams as long as 100 miles, and many vehicles ran out of gasoline after being on road for hours.

When evaluating the effectiveness of different evacuation strategies, it is important to simulate or predict time-dependent traffic patterns under different evacuation strategies, since heavy congestion usually occur with a sudden surge of traffic demand. In literature, various traffic flow models have been used when developing and evaluating evacuation plans. For examples, a macroscopic model was used in [2], DYNASMART-P in [3], and VISSIM in [4]. In this study, we propose a new platform for emergency evacuation planning based on a kinematic wave model of network vehicular traffic. This model is based on the Lighthill-Whitham-Richards [5,6] model and, different from microscopic models, only considers capacities of road links and network structure but ignores heterogeneity in drivers' behaviors, vehicle types, lanes, etc. It is simpler to build a road network, since only a limited number of parameters such as number of lanes and speed limit are needed in this model. In addition, the computational cost of the model is independent of the number of vehicles. All these features make this model very suitable for emergency evacuation planning.

In the rest of the paper, we first introduce a kinematic wave model of network vehicular traffic in Section 2. In Section 3, we discuss how to model evacuation with the model and present a measure of effectiveness of different evacuation plans with cumulative flow. In Section 4, we study evacuation plans for a simple road network. We conclude our study with some future studies in the conclusion part.

A COMMODITY-BASED KINEMATIC WAVE MODEL OF NETWORK VEHICULAR TRAFFIC

For each road link, we use the following triangular fundamental diagram to model the relationship between density ρ (veh/km) and flow-rate q (veh/hr):

$$Q(\rho)=\begin{cases}\rho v_f & 0\le\rho\le\rho_c\\ \rho_c v_f\left(\dfrac{\rho_j-\rho}{\rho_j-\rho_c}\right) & \rho_c<\rho\le\rho_j\end{cases}\qquad(1)$$

where v_f is free flow speed, ρ_c critical density, and ρ_j jam density. Here we only need three parameters for each road link: v_f is usually determined by the speed limit of a road, ρ_j is proportional to the number of lanes of a link, and ρ_c is the traffic density when a road link reaches its capacity flow and also proportional to the number of lanes. In the fundamental diagram (1), traffic capacity is $\rho_c v_f$.

The evolution of traffic dynamics on a unidirectional road link can be modeled by the LWR model [5, 6]:

$$\rho_t+(Q(\rho))_x=0\qquad(2)$$

When LWR model is used to simulate the density wave at link boundary with a step function as initial condition, it is called Riemann Problem. Numerically, we can use Godunov method [7] to solve the LWR model. In the Godunov method, each link is split into N cells, with cell length of Δx, and the time interval is divided into K time steps, with a time step of Δt. Then the Godunov-type finite difference equation for total flow in cell i from time step j to time step $j+1$ is

$$\frac{\rho_i^{j+1}-\rho_i^j}{\Delta t}+\frac{f_{i+1/2}^{j^*}-f_{i-1/2}^{j^*}}{\Delta x}=0\qquad(3)$$

where $f_{i-1/2}^{j^*}$ ($f_{i+1/2}^{j^*}$) denotes the flux through the upstream (downstream) boundary of cell i. Boundary fluxes can be computed by the following supply-demand method [8] [9]. Here we consider four types of boundaries: link boundaries, merging junctions, diverging junctions, and general intersections.

1. Link boundaries. Given $x_{i-1/2}$ as a link boundary, its upstream cell is denoted by u and downstream cell is denoted by d. ρ_u,ρ_d is the density of cells u and d respectively. We define the upstream demand as

$$D_u=\begin{cases}q(\rho_u) & when\ \rho_u\ is\ under-critical\\ q_u^{\max} & when\ \rho_u\ is\ over-critical\end{cases}$$

and define the downstream supply as

$$S_d=\begin{cases}q_d^{\max} & when\ \rho_d\ is\ under-critical\\ q(\rho_d) & when\ \rho_d\ is\ over-critical\end{cases}$$

then the boundary flux can be simply computed as

$$
\begin{aligned}
f_{i-1/2,d}^{j^*} &= \min\{D_u, S_d\} \\
f_{i-1/2,u}^{j^*} &= \min\{D_u, S_d\}
\end{aligned}, \tag{4}
$$

where $f_{i-1/2,\,d}^{j^*}$ is the inflow of downstream cell d, and $f_{i-1/2,\,u}^{j^*}$ is the outflow of upstream cell u.

2. Merging junctions. Given $x_{i-1/2}$ as a merging junction with P upstream cells and a downstream cell, its upstream cell are denoted by u_p $(p = 1, 2, \cdots, P)$, downstream cell by d, upstream demand by D_p, and downstream supply by S_d. The boundary fluxes are computed as

$$
\begin{aligned}
f_{i-1/2,\,d}^{j^*} &= \min\{\textstyle\sum_{p=1}^{P} D_p, S_d\} \\
f_{i-1/2,\,u_p}^{j^*} &= f_{i-1/2,\,d}^{j^*} \frac{D_p}{\sum_{p=1}^{P} D_p} \qquad p = 1, 2, \cdots, P
\end{aligned} \tag{5}
$$

where $f_{i-1/2,\,d}^{j^*}$ is the inflow of downstream cell d, and $f_{i-1/2,\,u_p}^{j^*}$ is the outflow of upstream cell u_p

3. Diverging junctions. Given $x_{i-1/2}$ as a diverging junction with P downstream cells and an upstream cell, its upstream cell is denoted by u, downstream cell by d_p $(p = 1, 2, \cdots, P)$, upstream demand by D_u, downstream supply by S_p, and proportion of commodity p in total traffic by ξ_p. The boundary fluxes are computed as

$$
\begin{aligned}
f_{i-1/2,\,u}^{j^*} &= \min_{p=1}^{P}\{D_u, S_p / \xi_p\} \\
f_{i-1/2,\,d_p}^{j^*} &= f_{i-1/2,\,u}^{j^*} \xi_p \qquad p = 1, 2, \cdots, P
\end{aligned} \tag{6}
$$

4. General intersections. For a general intersection with U upstream cells and D downstream cells, we combine merge and diverge models together.

$$
\begin{aligned}
&f_{i-1/2}^{j^*} = \min_{d=1}^{D}\{\sum\nolimits_{u=1}^{U} D_u,\ S_d \Big/ \left(\frac{\sum\nolimits_{u=1}^{U} D_u \xi_{u,d}}{\sum\nolimits_{u=1}^{U} D_u} \right)\} \\
&f_{i-1/2,\ d}^{j^*} = \frac{\sum\nolimits_{u=1}^{U} D_u \xi_{u,d}}{\sum\nolimits_{u=1}^{U} D_u} f_{i-1/2}^{j^*} \qquad d = 1, 2, \cdots, D \\
&f_{i-1/2,\ u}^{j^*} = \frac{D_u}{\sum\nolimits_{u=1}^{U} D_u} f_{i-1/2}^{j^*} \qquad u = 1, 2, \cdots, U
\end{aligned} \tag{7}
$$

where $\xi_{u,d}$ is the proportion of vehicles heading downstream cell d in upstream cell u.

EVACUATION PLAN AND A MEASURE OF EFFECTIVENESS

Under non-emergency situations, people would selfishly choose their routes, and a road network approaches the state of user equilibrium [10]. For non-emergency traffic management, various means, such as signal control and congestion pricing, are applied to minimize total travel time of all drivers so that a road network is in the state of system optimal. To better evacuate people from dangerous zones in emergency situations (e.g. hurricane, hazard material leakage, or dirty bomb attack), however, all vehicles would be strictly guided with their departure times and routes, and the objective of traffic management would be to evacuate people out of the dangerous zones as fast as possible, but not to minimize the total time for them to arrive their destinations. If a transportation network has sufficient large capacity or the number of people to be evacuated is small, the task would be relatively easy. However, for massive evacuation, an extraordinary level of traffic demands can cause serious congestion or even totally clog the whole transportation network. Without careful planning, the evacuation process can be put into halt as in the aforementioned case of Hurricane Rita. Thus it is important to understand the effectiveness of different evacuation plans.

Many factors can influence the effectiveness of different evacuation plans, such as the topology of a road network, weather, and traffic management and control mechanisms. Since travel demand usually surpasses the capacity of a road network in emergency situations, it is important to simulate the formation and propagation of traffic queues at various network bottlenecks. In this study, we use the kinematic wave model to simulate traffic dynamics for different evacuation plans. In addition, we propose a measure of the effectiveness of an evacuation plan by evacuated number of vehicles during a time interval, which equals the number of vehicles that can be loaded into a road network at its origin. After finding the maximum evacuated number of vehicles which are restricted by capacity and structure of a road network with the kinematic wave model of network vehicular traffic, we can arrange the evacuation times and routes of individual vehicles accordingly.

The evacuated number of vehicles is the cumulative flow, N, at the origin. Since cumulative flow is the total number of cars passing the origin during the time interval t_0 to t, then the cumulative flow at origin is

$$N([t_0, t]) = \int_{s=t_0}^{t} f(s)ds, \tag{8}$$

where $f(t)$ is the boundary flux out of origin at time t. In the discrete form, cumulative flow can be computed by $N([j_0, j]) = \sum_{s=j_0}^{j} f(s)\Delta t$, where $j_0\Delta t$ is the start of the evacuation period, and $j\Delta t$ the end. Figure 1 shows a typical cumulative flow curve, which is non-decreasing in time and strictly increasing with positive boundary flux.

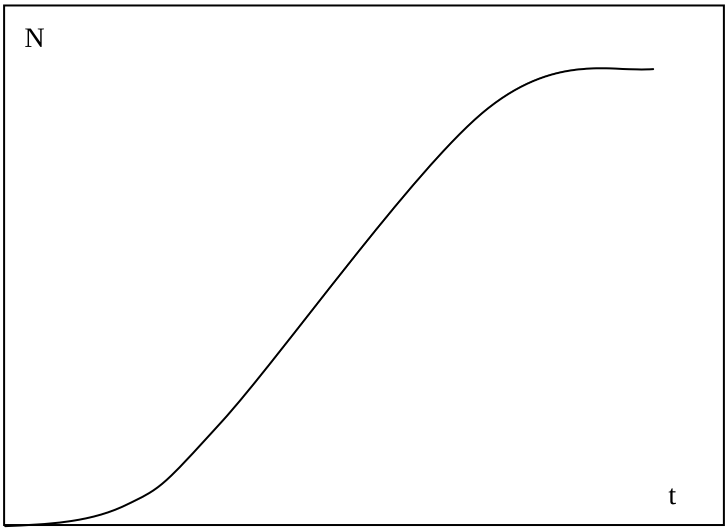

Figure 1. Cumulative flow

SIMULATION RESULTS

In this section, we demonstrate the feasibility of the evacuation model in the preceding section with a simple road network. Here we assume a sufficient large travel demand at the origin during the whole evacuation period.

A ROAD NETWORK

We study a simple road network shown in Figure 2. In this network, links 2, 3, and 5 have the same length of 1000 meters, and the length of link 4 is 2000 meters. Link 3 has only one lane, links 4, and 5 have two lanes, and link 2 has three lanes. The fundamental diagrams for all links are triangular in the following form:

$$Q(a,\rho)=\begin{cases} v_f\rho, 0\le\rho\le a\rho_c \\ \dfrac{\rho_c}{\rho_j-\rho_c}v_f(a\rho_j-\rho), a\rho_c\le\rho\le a\rho_j \end{cases}$$

where ρ is the total density of all lanes, a the number of lanes, ρ_j =180 veh/mile=0.112 veh/m the jam density of each lane, $\rho_c=36$ veh/mile=0.022 veh/m the critical density of each lane, and v_f =65 mph=29.1 m/s the free flow speed. That is, all lanes have the same fundamental diagram with capacity of $q_c=\rho_c v_f$. At the origin, travel demand is always $3q_c$, among which a proportion, ξ, of all vehicles take link 3, and 1-ξ take link 4. At the destination, traffic supply is always $2q_c$. Here we simulate traffic dynamics of the road network during a time interval from 0 to 2000s with time step $\Delta t=2.5$ s and the cell length $\Delta x=100m$.

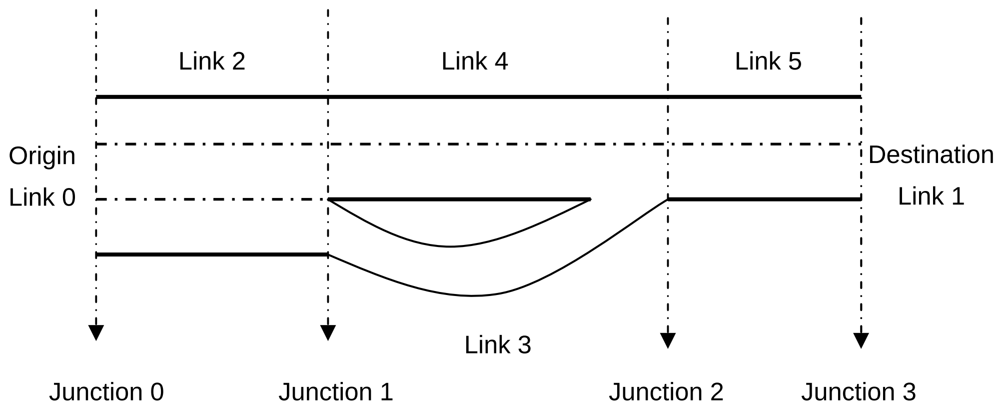

Figure 2. The traffic network

In this network, the evacuation plan is determined by the proportions of vehicles using different routes. Since link 5 has lower capacity than link 2, traffic queues will form after some time and prevent vehicles entering the network at the origin at the capacity of link 2. Here we are interested in how the proportion ξ, affects the total number of evacuated vehicles during a time period.

IMPACTS OF DIFFERENT EVACUATION PLANS

In Figure 3, we demonstrate the cumulative flow at the origin with two different proportions: ξ=0.2 or 0.6. From the figure, we can see that the cumulative flow is increasing with time, and, for different evacuation plan, cumulative curves are different. That is, the two evacuation plans have different effectiveness.

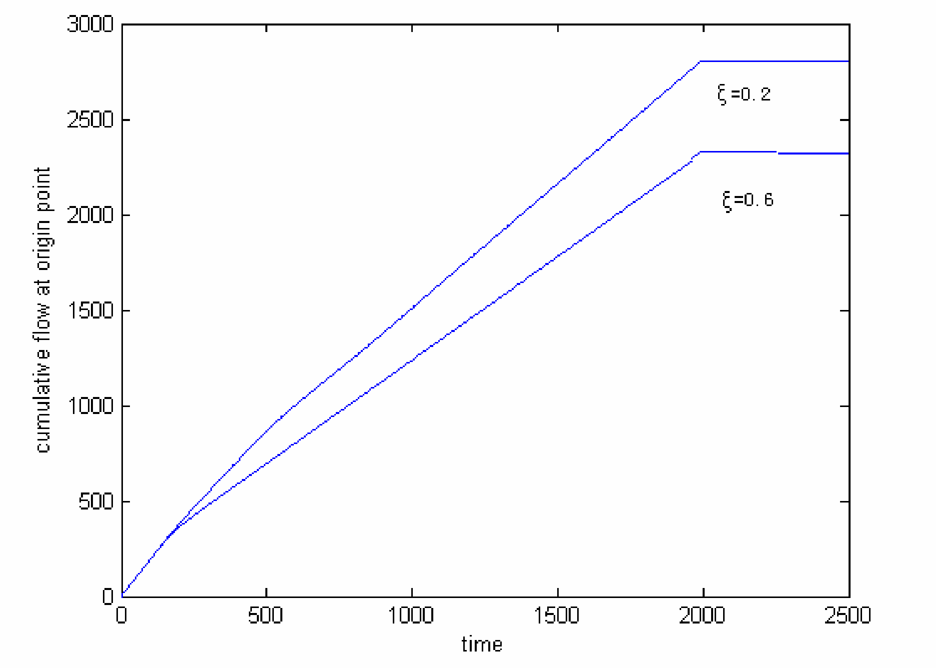

Figure 3. Cumulative flows at the origin for two evacuation plans

In Figure 4, we demonstrate the total number of evacuated vehicles during the interested time interval of 2000 s with different values of ξ. From the figure, we can see that there is an optimal evacuation strategy at $\xi = 0.3250$, even for a very small cell size of $\Delta x = 3.125\,\text{m}$. That is, when 32.5% of vehicles take link 3, we can evacuate the maximum number of vehicles out of the origin during a time period of 2000 s. Actually, we expected the result to be 1/3, which is the ratio of the number of lanes of link 3 to the total number of lanes of both links 3 and 4. We also solve the problem for different time periods and obtain different optimal proportions. As an extreme case, when the time interval is very small, congestion initiated at the diverge or the merge does not propagate back to the origin, and the evacuation flow-rate can be as large as the capacity of link 2. In this case, the percentage will not affect the result, and, for any percentage between 0 and 1, we can have the same number of evacuated vehicles.

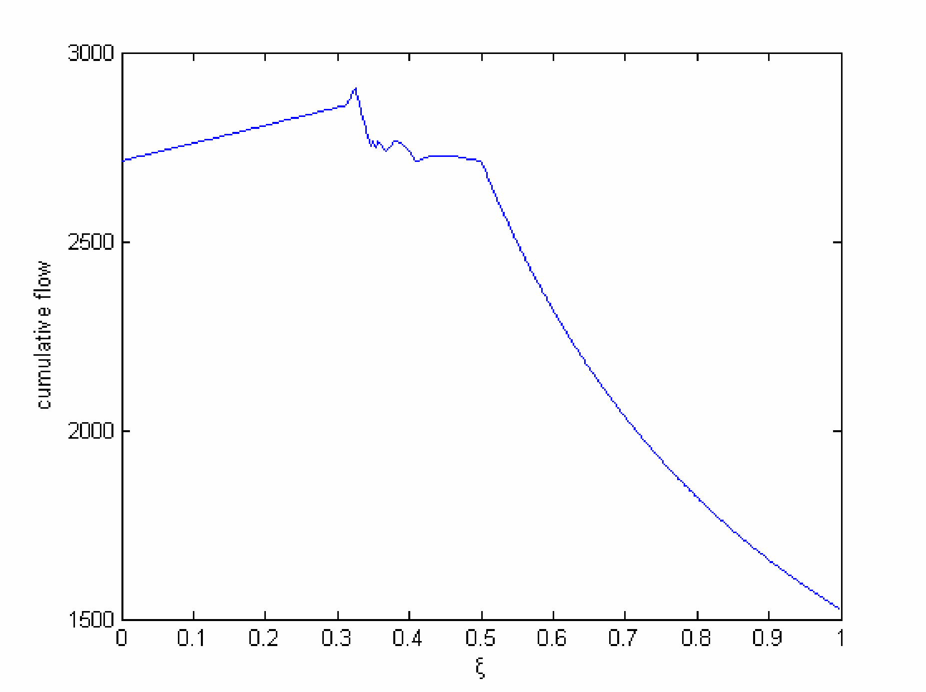

Figure 4. Number of evacuated vehicles for different ξ

CONCLUSION

In this paper, we proposed a model for evacuation planning based a commodity-based kinematic wave model of network vehicular traffic. We also discussed the measure of the effectiveness of an evacuation plan and its computation with the traffic flow model. We applied this model to study evacuation problem of a simple road network and demonstrated the feasibility of the model. That is, we can apply the evacuation model to find an optimal evacuation strategy when a disaster strikes, and vehicles can be advised to evacuate following the strategy. Attractive features of this model include easy set-up of road networks, efficient simulation of traffic dynamics, and effective capture of constraints of road capacities and network structure. Therefore, we expect this model to be an ideal platform for emergency evacuation planning.

In the future, we will be interested in studying properties of different evacuation plans for more complicated road networks and comparing this model with others regarding its computational efficiency and other merits. Indeed, this study is only the first step toward building a comprehensive platform for evacuation planning, and such a platform will be of our research interest in the longer term. In reality, with constraints in familiarity, quality and timeliness of information dissemination mechanism, we might not be able to obtain the optimal evacuation result. In the future, we will investigate how these factors affect the results. In addition, we will be interested in validating the proposed model by comparing results from the platform with those observed in real-world situations.

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