Incorporating Feedback in Travel Forecasting: Methods, Pitfalls and Common Conerns

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Project Objectives

The most common method for producing regional or metropolitan area travel forecasts in the United States is to apply the following four modeling steps sequentially:

- *trip generation,*
- *trip distribution,*
- *mode choice, and*
- *route assignment.*

This traditional four-step process passes output from one step to the next as input, as illustrated in Figure 1. While the process has produced forecast results sufficiently accurate for many types of long range transportation planning, it is commonly found that some of the outputs of the process are not consistent with inputs to earlier steps. The research undertaken in this project focused on methods to ensure that link speeds used in each step of the travel forecasting process are consistent with the final speeds estimated in the final step of the process. As a product of this research, a final report was prepared to provide guidance in the application of feedback.

A variety of methods for introducing "feedback" into the process (reintroducing output of one step as input to a previous step) were explored and guidance was developed on when and how to incorporate feedback into the four-step modeling process. Figure 1 illustrates four possible ways in which feedback can be provided in the four-step process and one additional way that feedback can be provided to other modeling steps.

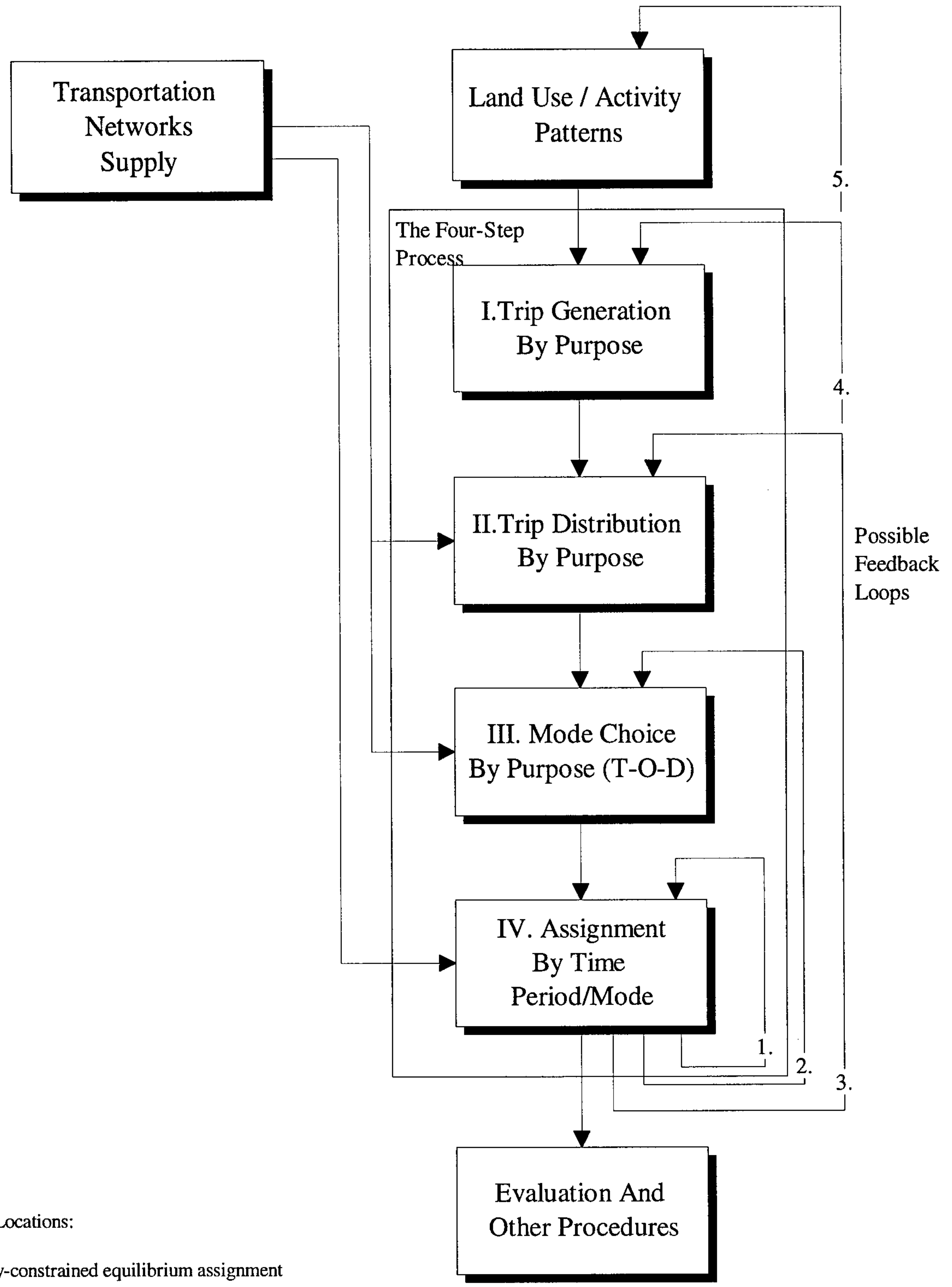
The exploration of methods for introducing feedback into the traditional four-step travel forecasting process is not new. Methods implementing feedback have been used for planning studies in major U.S. metropolitan area for at least twenty years. (Boyce et al, 1970; Boyce, et al 1994; Lawton and Walker, 1993; BMC, 1992; DRCOG, 1992; MWCOG, 1994; PBQD, 1992; Mann, 1993). But introducing feedback using currently available travel forecasting software is complex, is generally cumbersome, requires lengthy execution times, and is prone to significant pitfalls and errors. As a result, few modelers have chosen to pursue implementation of feedback in regional or metropolitan area models despite some of the theoretical and obvious intuitive justifications for doing so. This research effort was initiated because a recent increase in the use of regional and metropolitan area models for forecasting pollutant emissions has resulted in regulatory requirements that may force modelers for major metropolitan areas to incorporate feedback in a way that will produce consistent use of travel speeds throughout the modeling process. Another related motivation is the desire to better capture the effects of congestion on traveler choice behavior and the benefits of congestion management strategies in reducing delay.

The Federal Highway Administration (FHWA) sponsored this research effort to support states and metropolitan planning organizations in their responses to the new regulatory requirements for emissions modeling. To meet this basic objective, the research was designed to address the following questions:

- *Does feedback make a difference in the results of a four-step modeling process and if so, under what conditions?*
- *What methods are available for feedback and what are the advantages and disadvantages of each?*
- *What criteria should be used to determine when feedback has successfully resulted in consistency of speeds in the modeling process?*
- *What guidance can be provided to modelers who choose to undertake the introduction of feedback?*

Central to this research effort was the application of a variety of feedback methods within two case study model systems: the regional travel forecast systems for Memphis, Tennessee and Salt Lake City, Utah. These model systems were chosen because they were readily available to the research team and because they could be used to reflect a range of levels of congestion by manipulating the baseline conditions representing the input to the model system (the transportation network and the land use forecast). The performance of the alternative methods for implementing feedback and the variety of methods for assessing closure could be tested in response to the full range of travel

Figure 1: Feedback Locations within the Four-Step Process



Feedback Locations:

1. Capacity-constrained equilibrium assignment
2. Congested times as inputs to mode split
3. Congested times or composite impedances as inputs to trip distribution
4. Accessibility factors in trip generation
5. Land use - transportation interaction

conditions. From these tests, the research team was able to determine when feedback was likely to make a significant difference in forecast results, which methods are most likely to produce improved accuracy under different conditions, and the overall resource requirements of each method.

Addressing Regulatory Requirements for Feedback

The Clean Air Act Amendments (CAAA) of 1990 significantly increased the role of regional or metropolitan area travel forecasting models in the forecasting of pollutant emission levels for future years in non-attainment areas. The CAAA required the development of a "...comprehensive, accurate, and current..." emissions inventory for oxides of nitrogen (NOx), volatile organic compounds (VOC), carbon monoxide (CO), and small particulate matter (PM_{10}) for every non-attainment area (marginal and worse) as part of a state implementation plan (SIP) for air quality attainment (U.S. Congress, 1990 C.A.A.A.).

The U.S. Environmental Protection Agency's guidance on preparation of an emissions inventory (U.S. EPA, 1992) describes feedback as a necessary part of the travel forecasting process. It cites as support for this point a ruling by the Federal District Court of Northern California, in a suit brought by the Sierra Club against the Metropolitan Transportation Commission of the San Francisco Bay Area. The ruling stated, "where the model had the capability to incorporate feedback effects, the planning agency was obliged to project travel with those effects included." (U.S. District Court for Northern California, 1990). While the EPA did not state the conditions under which the network modeling approach should be used for emissions inventories, the discussion of the importance of feedback applies wherever the network models are to be used.

In a second area of guidance related to the Clean Air Act Amendments of 1990, each state must demonstrate that its transportation improvement programs (TIP), regional transportation plans (RTP), and projects of regional significance conform with the approved state implementation plan (SIP) for air quality attainment (U.S. EPA, 1993). In its final rule for determining conformity, the EPA calls explicitly for feedback in the transportation forecasting process for serious, severe, and extreme ozone non-attainment areas, and for serious carbon monoxide non-attainment areas. The guidance states that the models used in the preparation of transportation plans and programs must have the following elements:

- *The models must show a logical correspondence between an assumed land-use scenario and the future transportation system,*
- *Peak and off-peak travel demand and travel time must be provided,*
- *Methods to estimate traffic speeds and delays must be used that are sensitive to traffic volume in the network model,*
- *A capacity-sensitive assignment methodology must be used for peak-hour or peak-period traffic assignments (feedback within assignment),*
- *Zone-to-zone travel times used to distribute trips between origin-and-destination pairs must be in reasonable agreement with the travel times that result from the process of assignment of trips to network links (feedback between assignment and trip distribution), and*
- *Where use of transit is significant, the final zone-to-zone travel time should also be used for mode split (feedback between assignment and mode split).*

It is further recommended by the EPA that models used in the preparation of plans and programs include the following:

- *A dependence of trip generation on the accessibility of destinations (feedback between assignment and trip generation), and*
- *A dependence of regional economic and population growth on the accessibility of destinations (feedback between the transportation model and the land use model).*

With its guidance on conformity, the EPA significantly strengthened the regulatory requirements for use of feedback in the modeling processes of non-attainment areas.

The recent EPA guidance supporting the Clean Air Act Amendments of 1990 has provided the necessary motivation for modelers in most non-attainment areas to pursue the options for feedback despite the additional time and resource requirements and the potential pitfalls of inappropriate application of the procedures. Further motivation is also provided by a growing interest in management strategies that can achieve greater use of already existing transportation infrastructure for which greater sensitivity to speed differences is necessary if regional models are to be useful planning tools. The U.S. Department of Transportation's initiative for Intelligent Transportation Systems (ITS) funded under the Intermodal Surface Transportation Efficiency Act (ISTEA) has heightened the nation's interest in these management-oriented strategies and has increased the interest in more accurate model systems (Euler and Robertson, 1995). The introduction of feedback mechanisms would appear to be a major step forward in providing additional sensitivity and accuracy in the evaluation of management strategies.

Implementation of Feedback

The primary goal of the implementation of feedback in a traditional four-step modeling process is to provide a process for reaching an overall "equilibrium" within the forecasting system. Equilibrium can be defined as the state of balance in which all interactions have been accounted for and the inputs and outputs of each step of the process are reasonably consistent with one another. The most straightforward application of feedback would take the output from the assignment step in the modeling process and reintroduce it as input to a previous step. This is illustrated by direct feedback between trip assignment and trip distribution, the most common type of feedback currently pursued by modelers and the feedback mechanism of most concern to the EPA. Successful implementation of this basic feedback loop will result in a trip distribution model that determines the underlying pattern of trips within a region using zone-to-zone travel times that are consistent with the final loaded speeds of assignment: the last step in the process. The most common current modeling practice is to use a fixed set of travel times in trip distribution that may or may not reflect capacity-constrained conditions. This often results in significant differences between the speeds used for trip distribution and those that result from the assignment.

Feedback always involves the transfer of data from assignment to a previous point in the modeling process. This even includes an internal feedback within the assignment step that is necessary for equilibrium assignments. Feedback can include the reintroduction of assignment data at any point in the process, including the land use activity forecasting process that precedes the traditional four-step transportation model steps or the trip generation step. Regardless of the number of steps included within the feedback loop, the underlying concept remains the same: iterative transfer of data from the end of the process back to earlier steps until the difference between the values for input data and output data are within an acceptable range.

Convergence Criteria

In a well-designed feedback process, the values for input variables and output variables should converge toward common values. The development of that feedback mechanism requires the selection of appropriate convergence criteria to inform the modelers when the iterative application of the feedback loop can be ended and the final assignment results used.

The two most important variables for determining if equilibrium is achieved in the feedback process, especially for air quality analysis, are volumes on links and average operating speeds on links. Because of the way in which speeds are estimated in traditional travel forecasting models, volume and speed are directly related through a functional relationship, so convergence with respect to volume usually implies convergence with respect to speed and vice versa.

The selection of an appropriate convergence criterion is complicated, however, and a wide variety of measures can be constructed to reflect either travel volumes or travel speeds. For either, measures can be constructed to reflect the region as a whole, sub-groupings of links within the regional network (functional classifications, area-type classifications, sub-regional areas, screenlines, cordons or corridors) or specific links. In general, more fluctuation in values is experienced between iterations at the link level than for the more aggregate measures. But the choice of which measure is most appropriate for use as a convergence criterion depends upon the specific application. If the focus of the application is on a particular facility, the convergence criterion might be something like the following:

> *A change of five percent or less between iterations in the vehicle miles of travel assigned to the links on the facility.*

If an analysis is regionally oriented, as might be the case in modeling for the regional transportation plan for a conformity analysis, an appropriate convergence criterion might be the following:

> *At least ninety-five percent of the links in the system with a percent change in volume less than or equal to five percent.*

It is possible to see an apparent convergence with respect to one measure without achieving a true equilibrium or true convergence. If the feedback is implemented to include multiple steps in the four-step process, compensating change in the distribution of trips on the system can result in a low percentage changes in a particular measure while actually representing fairly significant changes in travel behavior. As an example, a feedback mechanism that can change the trip distribution may result in more trips in a corridor but with shorter length than in a previous iteration. If the convergence criterion is vehicle miles traveled on a facility, the assumption of equilibrium being achieved may be an error. For this reason, more than one convergence criterion might be needed to guarantee a higher probability of a true convergence when the criteria are met. Some of the other measures that might be used in addition to the volume and speed measures mentioned earlier include the following:

- *Maximum percent change in trips between an origin and destination,*
- *Percent change in origin-to-destination travel time,*
- *Percent change in mean trip length, or*
- *Percent change in vehicle delay.*

In addition to the specific concrete travel characteristics suggested above as measures of convergence, there are also at least two system-wide functions of travel times and delays common in travel assignment packages that might be used:

- *Percent change in the objective function of the equilibrium assignment (Sheffi, 1985; Florian, 1991; Boyce et al, 1994; Evans, 1976), or*
- *Percent change in the Gap or Normalized Gap functions (Boyce, 1995; Florian, 1991; Van Vuren, 1995).*

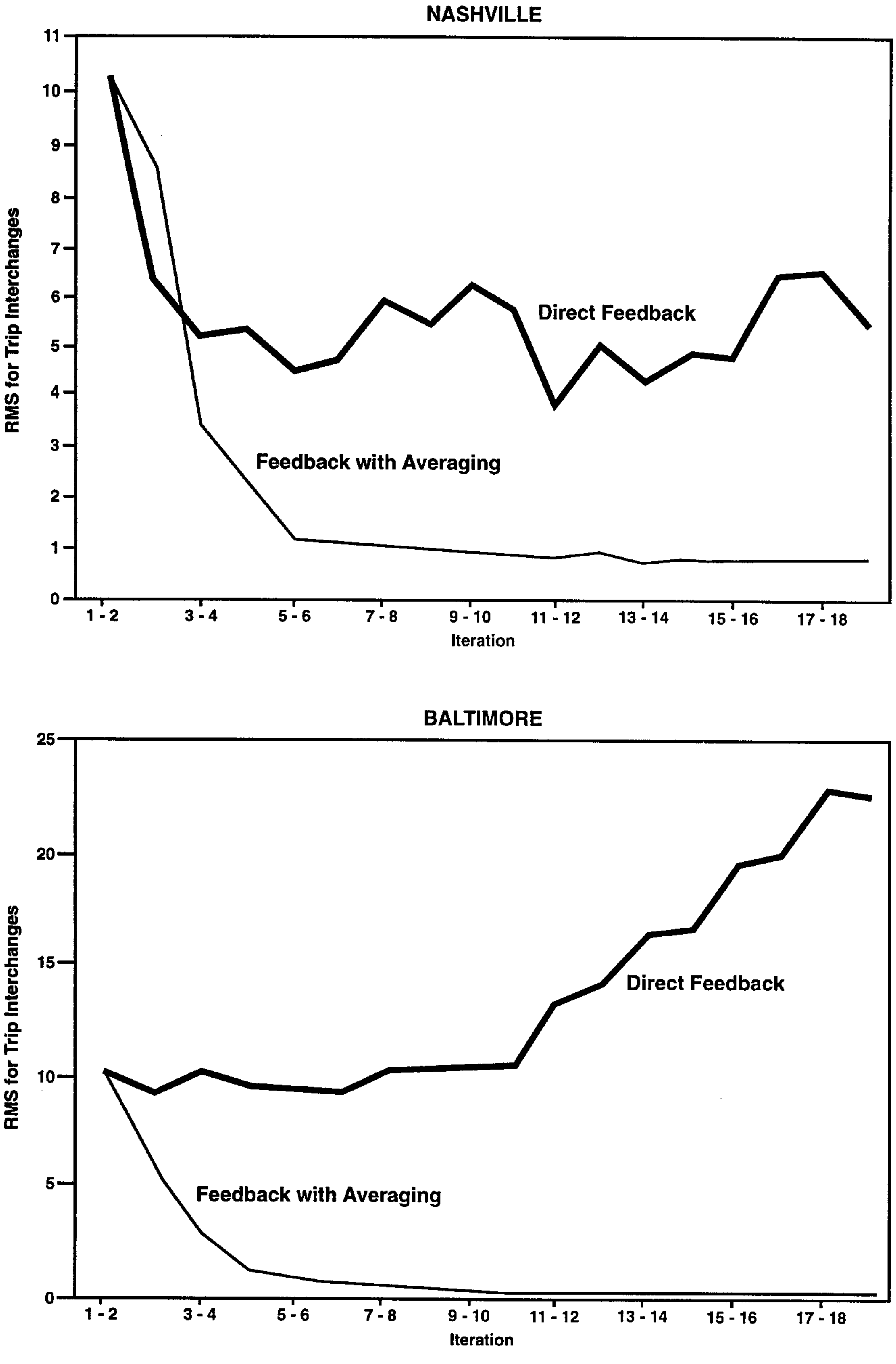
Both are used internally in traffic assignment to determine when a user equilibrium in the system has been achieved. As such, both represent system-wide measures that use link-specific assignment information as measures of changes in travel patterns or conditions. The measures have also been extended to incorporate the results of the other steps in the travel forecasting process.

Methods for Introducing Feedback

Numerous researchers and planning practitioners have experienced difficulty achieving convergence in a feedback process when a "Direct Method" of feedback is used: the output of assignment is used directly, unaltered, as input for a previous step in the modeling process. One example illustrated in Figure 2 is provided by a research effort undertaken internally by the Federal Highway Administration. Using data from Baltimore, Maryland and Nashville, Tennessee, the researchers found that there was instability in the approach from iteration to iteration and no sign of convergence to a consistent set of values. In most cases, convergence will occur using the Direct Methods but often only after many iterations and the consumption of considerable clock time and computer time. Florian et al. (1975) demonstrate mathematically and by example that the Direct Method will not always converge to the correct solution and may not converge at all.

A number of alternatives to the Direct Method have been identified by previous researchers and practitioners as ways to reduce processing time and assure convergence. All of the methods represent alternatives for using information from all previous iterations to move the next iteration toward a convergent solution. The methods use somewhat different approaches either in the assignment algorithm or in the method for combining results of previous iterations to produce new input values. The four alternative methods chosen for application in this research are as follows:

Figure 2: Comparison of Direct Feedback and Method of Successive Averages Root Mean Square for Trip Interchanges



Source: FHWA, 1994

- *Method of Successive Averages with Equilibrium Assignment (MSA-EQA) - provides equal weight to each previous iteration's equilibrium assignment results.*

- *Method of Successive Averages with All-or-Nothing Assignment (MSA-AON) - same as MSA-EQA but each assignment is made on single best-path basis.*

- *Method of Optimal Weighting with Equilibrium Assignment (MOW-EQA) - computes an optimal weighting for each iteration's equilibrium assignment.*

- *Method of Optimal Weighting with All-or-Nothing Assignment (MOW-AON) - same as MOW-EQA but each assignment is made on a single best-path basis.*

For all four of these alternative methods, the volumes from previous iterations are averaged with the volumes from the most recent assignment and new input speeds are determined based upon the averaged volumes. The speeds from previous iterations are not averaged directly because of the non-linear relationship between volume, capacity, and speed. All of these methods address the way in which output from assignment is manipulated prior to reintroduction as input to a previous step. The application of any one of the alternative approaches is basically the same regardless of where in the four-step process the assignment data are being fed back.

The Effects of Feedback

To test the applicability of feedback in the traditional four-step process and to provide an assessment of alternative methods for implementing feedback, a case-study approach was used. Model systems for two major metropolitan areas; Memphis, Tennessee and Salt Lake City; Utah; were selected for the case-study applications. The two sites provided a variety of land-use, network, and level-of-service characteristics and also represented two metropolitan areas for which the research team already had a significant amount of model data available.

The model system for the Memphis metropolitan area is maintained by the Memphis Metropolitan Planning Organization and has been validated for a 1988 base year. A regional population of slightly less than one million is represented in 365 zones. The highway network for the model has all major roads coded and the transit system has three types of modes: regular/local bus, blazers or express bus, and north/south or cross-town routes. The trip purposes of the model are home-based-work, home-based-other, non-home-based, trucks and taxis, and external trips. A gravity model is used for trip distribution and a multinomial logit model is used in the mode choice procedure.

The model for the Salt Lake City metropolitan area uses 556 zones to represent the regional land use which also supports a population of about one million. A base year of 1990 has been established and a validation has been performed for that year. The highway network also has most of the major roadways coded and includes local and premium bus services in the transit network. Trip purposes modeled include home-based-work, home-based-other, non-home-based, home-based college, commercial, and external trips. A gravity model is the basis for trip distribution, and a nested-logit model has been developed for the mode-choice process to model five modes: drive alone, two-person carpool, three-plus-person carpool, local bus, and premium bus.

The two case-study sites provided reasonable variation because the baseline conditions: Salt Lake City included considerable congestion while the Memphis base-year model had only a small amount of congestion in selected locations. The discernible difference between the two case-sites provided sufficient opportunity to use sensitivity testing with the two models to produce a wide variety of conditions. Sensitivity tests were conducted by testing the effects of twenty-five percent uniform growth throughout the area, twenty-five percent in radial growth along selected growth corridors and the effects of a major new facility being added into the highway network.

Effects on System-wide Travel Characteristics

Table 1 presents the baseline results from application of the Direct Method and the two MSA options for an "assignment-to-trip-distribution" feedback loop in the two test-case cities[1]. Using system-wide average speed as a measure of effect, the results of

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[1] Because of the specific characteristics of the test case models, the Method of Optimal Weighing could not be tested for its effect on system-wide travel characteristics; however, a later section of this report compares its convergence characteristics with those of the Direct Method and the Method of Successive Averages.

the two case studies indicate that feedback can produce significantly different results when congested conditions occur, but has very little effect where there is little or no congestion. In the Salt Lake City model, in which the average baseline speed without feedback was roughly 22 miles per hour, all three of the feedback methods produced system-wide speed increases between 21 and 23 percent for the baseline year. But for the Memphis metropolitan area, where the system-wide model average baseline speed without feedback was roughly 42 miles per hour, feedback produced a system-wide increase of less than two percent. Even when 25 percent uniform growth was added in the Memphis model, the increase in system-wide average speed over the no-feedback baseline was less than 3 percent. In the more congested Salt Lake City model system, 25 percent growth produced a difference of roughly 50 percent in system-wide speed between the no-feedback baseline and the three alternative feedback mechanisms. When the growth was concentrated radially, there was an even greater difference between the no-feedback baseline and the three alternative feedback methods.

Table 2 reflects a somewhat similar pattern of change in results from feedback where the system-wide vehicle miles traveled is the measure. Because the feedback loop tested in Memphis and Salt Lake City allowed for the use of an equilibrium set of travel times in the trip distribution step, a different trip-length distribution could result for a fixed number of total vehicle trips. In both cases, feedback resulted in a reduction of system-wide vehicle miles traveled reflecting shorter mean travel lengths. Again, the change produced by feedback is significantly greater in the more congested Salt Lake City model (a reduction ranging from 11.5 percent to 12.5 percent) than in the Memphis model (where the change ranged from 2.2 percent to 2.5 percent).

The sensitivity testing with the two test case models demonstrated a consistency in the nature and direction of change produced by the introduction of feedback. Although not all of the impacts of feedback on system characteristics are reported here, the tests indicated that feedback produced the following changes in assignment results:

- *Average link speeds are increased,*
- *Average travel time is decreased,*
- *Average travel distance is decreased,*
- *Average Volume/Capacity ratio is decreased, and*
- *Total vehicle miles travel is decreased.*

While the direction of change was consistent in the observed results, the magnitude of the change for each of the above measures varied significantly and was almost always directly related to the amount of congestion in the network being modeled: the greater the level of congestion, the greater the change introduced by feedback. The systematic changes in results produced by the introduction of feedback have two significant implications. The first is the need for recalibration of a baseline model after feedback has been introduced into the modeling system. The second is the need for the use of feedback modeling to accurately reflect the level of impact of increasing congestion on trip distribution and travel speeds.

Convergence Characteristics of Alternative Feedback Methods

The test case results also clearly demonstrate the value of using one of the averaging methods over the direct feedback method. The averaging methods each produced faster and more complete convergence. Figure 3 provides a comparison of the Direct Method results with the MSA-EQA results for Salt Lake City. Both methods produced roughly the same change in speeds from about 18 miles per hour to about 22 miles per hour, but the MSA-EQA shows virtually complete convergence after the sixth or seventh iteration while the Direct Method is still oscillating at a level of one percent to two percent in the ninth and tenth iterations. Both methods that used successive averages produced almost identical convergence results. The MSA-AON required only fifteen all-or-nothing assignments, however, while the MSA-EQA required seventy all-or-nothing assignments.

Table 3 provides a comparison of the execution times for the three feedback methods and for the no-feedback baseline. For the test cases, feedback resulted in execution times roughly five to eight times that of no feedback for the Memphis model and 1.5 to 1.9 times that of no feedback for the Salt Lake City model. Of the three feedback methods tested, the one method using all-or-nothing assignments had a noticeably shorter execution time but also took considerably more iterations to converge. It should be noted that the different applications did not terminate in relation to a specific convergence criterion but were instead set to run for a fixed number of iterations.

Table 1: Effects of Feedback on Model Systemwide Average Network Speed

Feedback		Test Scenario			
Method	Number of Iterations	Base	25% Uniform Growth	25% Radial Growth	New Facility
Memphis		Percent Change From "No Feedback" Speed	Percent Change From "No Feedback" Speed	Percent Change From "No Feedback" Speed	Percent Change From "No Feedback" Speed
No Feedback	1	41.6 mph	39.4 mph	39.1 mph	43.3 mph
Direct	10	1.1%	2.4%	2.9%	0.6%
MSA (Equilibrium)	10	1.1%	2.4%	2.1%	0.6%
MSA (A-O-N)	15	1.3%	2.5%	2.1%	0.8%
Salt Lake					
No Feedback	1	22.0 mph	14.4 mph	12.9 mph	23.7 mph
Direct	10	22.8%	47.8%	62.6%	13.1%
MSA (Equilibrium)	10	21.3%	48.2%	66.6%	12.6%
MSA (A-O-N)	15	23.0%	50.2%	68.1%	14.4%

Note: For No Feedback Case the Average Network Speed is shown. For the Direct and Method of Successive Averages cases the % Change from No Feedback is shown.

Table 2: Effects of Feedback on Model Systemwide Vehicle Miles Traveled

Feedback		Test Scenario			
Method	Number of Iterations	Base	25% Uniform Growth	25% Radial Growth	New Facility
Memphis		Percent Change from "No Feedback" VMT	Percent Change from "No Feedback" VMT	Percent Change from "No Feedback" VMT	Percent Change from "No Feedback" VMT
No Feedback (VMT)	1	15,824,577	19,559,521	19,849,273	16,273,549
Direct	10	-3.5%	-5.0%	-5.5%	-3.2%
MSA (Equilibrium)	10	-3.2%	-4.7%	-5.1%	-2.9%
MSA (A-O-N)	15	-3.6%	-5.0%	-5.4%	-3.1%
Salt Lake					
No Feedback (VMT)	1	17,796,907	22,668,632	22,794,044	18,004,694
Direct	10	-12.5%	-12.0%	-12.1%	-6.7%
MSA (Equilibrium)	10	-11.6%	-11.1%	-11.5%	-6.1%
MSA (A-O-N)	15	-11.5%	-10.8%	-11.2%	-6.0%

Note: For No Feedback Case the Vehicle Miles Traveled is shown. For the Direct and Method of Successive Averaging cases the % Change from No Feedback is shown.

Figure 3: Comparison of Convergence Characteristics for Alternative Feedback Methods in the Salt Lake City Model

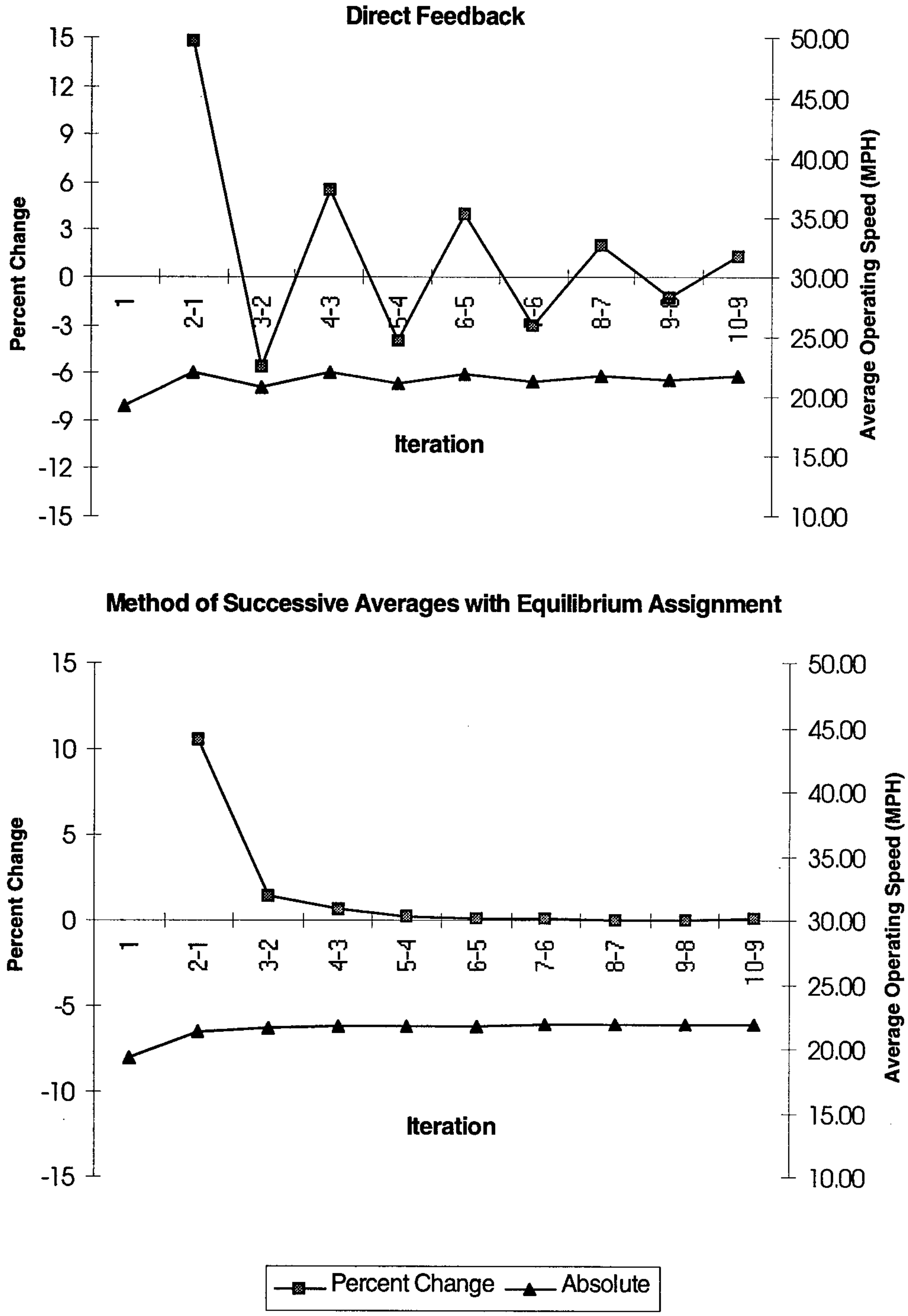


Table 3: Effects of Feedback on Model Systemwide Execution Time

Feedback		Execution Time (in minutes)						
Mechanism	Number of Loops or Iterations	Distribution	Transit	Mode Choice	Assignment	Updates	Evaluation	Total
Memphis								
No Feedback	1	1.27	2.17	0.65	2.28	0.00	0.00	6.37
Direct	10	13.83	3.35	0.68	22.83	4.67	18.83	63.20
MSA (Equil)	10	12.33	3.35	0.68	22.83	13.67	18.67	71.03
MSA (A-O-N)	15	17.00	3.35	0.68	7.25	18.25	26.00	72.53
Salt Lake								
No Feedback	1	2.27	58.03	42.40	5.48	0.00	0.00	108.18
Direct	10	23.00	58.03	42.40	55.00	16.50	40.67	235.60
MSA (Equil)	10	21.00	58.03	42.40	50.67	34.00	37.67	243.77
MSA (A-O-N)	15	33.50	58.03	42.40	11.00	52.75	61.25	258.93

Methods Using Optimal Weighting

Extensive research has been conducted on methods for feedback that use an optimal weighting for each iteration rather than a fixed weighting as used in the MSA-EQA and MSA-AON previously discussed (Evans, 1976; Horowitz, 1991; Florian et al., 1975; Boyce et al., 1994; Boyce et al., 1988; Walker and Peng, 1995; Sheffi, 1985; Ortuzar and Willumsen, 1990). Evans first proposed one of the most widely used methods for optimal weighting. Evans' algorithm finds the weight for averaging the most recent assignment and trip distribution results of an iteration with the results from previous iterations that will minimize an objective function that includes a representation of the volume-delay function and the zone-to-zone friction factors used in trip distribution. Because of the characteristics of the objective function in Evans' algorithm, the method produces the best convergence results when the trip distribution model uses an exponential friction function that matches that in Evans' algorithm. Neither of the two test-case model systems had such a trip distribution model and so one model (Salt Lake City) was adapted for application of Evans' algorithm. The adaptation produced a different distribution of trips between zones in the baseline (no feedback) application and so the results cannot be compared to the previous results of the sensitivity tests for the two test-case models. This section provides a comparison of the convergence characteristics of Evans's algorithm with that of the other feedback methods.

When tested in feedback in the Salt Lake City model, the Method of Optimal Weighting (MOW), as represented by Evans' algorithm, demonstrated convergence characteristics almost identical to the Method of Successive Averages. As indicated by Figure 4, MOW-EQA demonstrated convergence characteristics similar to MSA-EQA and the performance of MOW-AON was similar to that of MSA-AON. Similar results were obtained using a comparison of the percent change in volume and speed. The applications in the Salt Lake City model system showed no significant improvement in convergence characteristics with MOW and the execution time was considerably greater than MSA as illustrated by Figure 5. While the results for Salt Lake City do not indicate that the additional complexity of the MOW produces better or faster convergence, other researchers have suggested that for large networks with extreme congestion, MOW-AON may produce more efficient convergence than either of the MSA options (Walker and Peng, 1995).

Inclusion of Mode Choice in the Feedback Process

In areas where significant transit service exists or is planned, the 1990 CAAA Conformity Guidance suggests the use of the

Figure 4: Comparison of Convergence Characteristics of Feedback Processes for the Salt Lake City Model

Root Mean Square Change of Volume

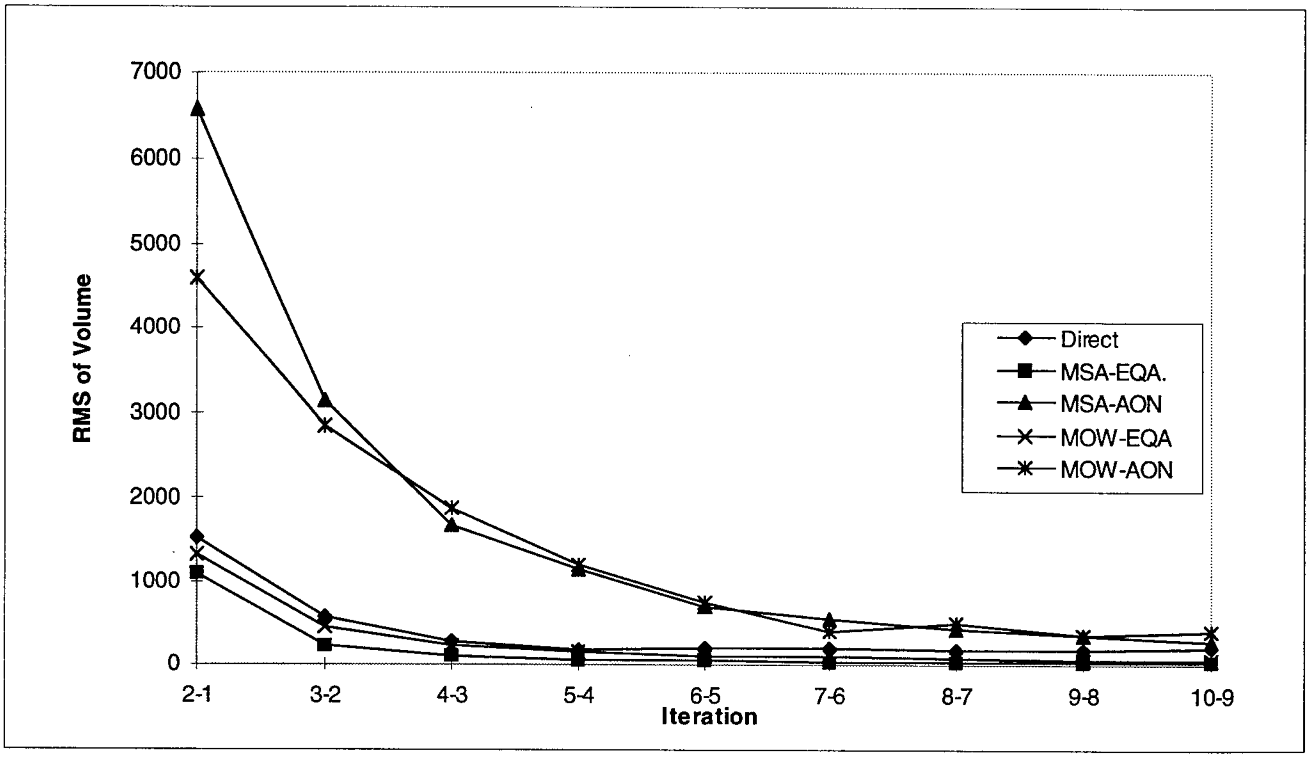
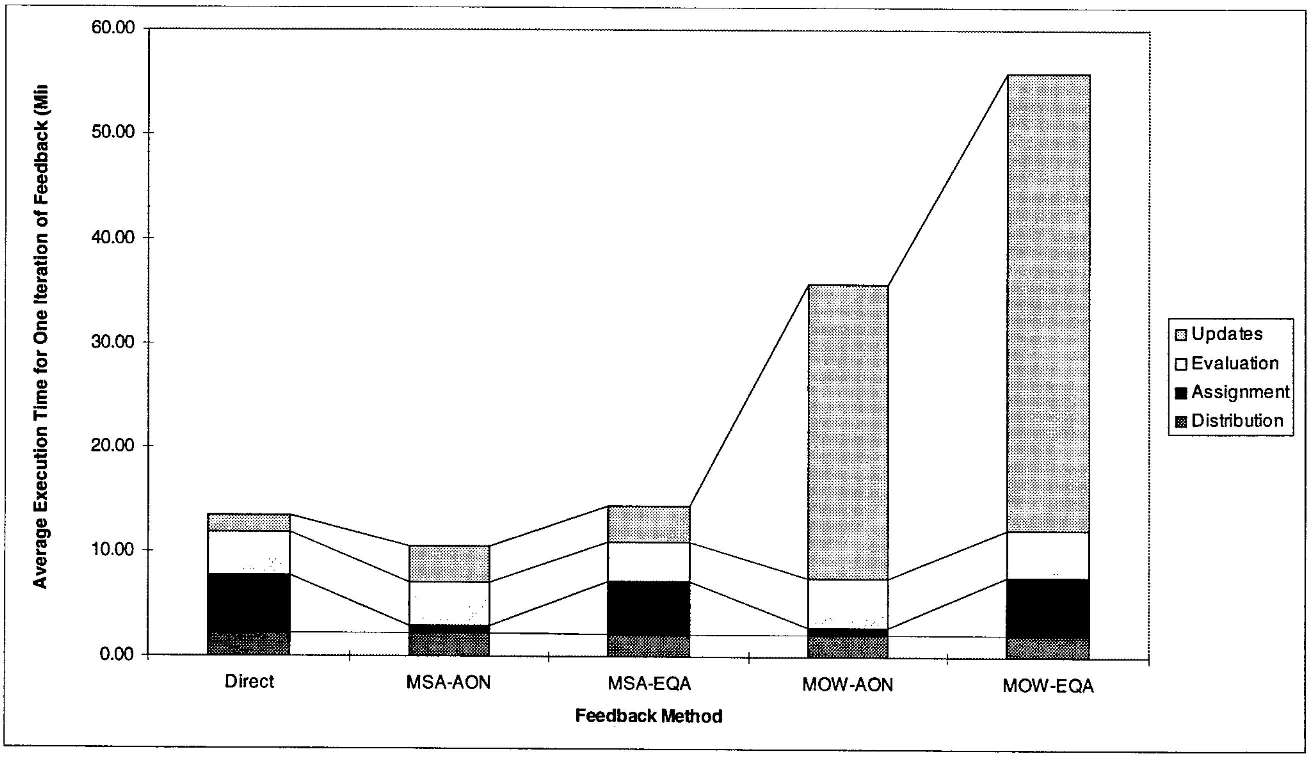


Figure 5: Comparison of Execution Times for Feedback Methods for Salt Lake City Model

Average Execution Time for an Iteration (Minutes)



final "equilibrated" highway times (trip distribution input times in agreement with output times from assignment) in estimating mode shares.

> Where use of transit currently is anticipated to be a significant factor in satisfying transportation demand, these times (the final highway times) should also be used for modeling mode choice (U.S. EPA, November 1993).

There are two basic options for including mode choice in a four-step modeling process that includes feedback. The options are as follows:

- *Post Feedback Mode Choice - The simplest option applies the mode choice model after feedback is applied between trip distribution and assignment. Default factors to convert person trips to vehicle trips are used within each iteration of feedback.*
- *Integrated Mode Choice Within Feedback - In this option mode choice is applied within each iteration of feedback. It replaces the use of default person-to-vehicle-trip conversion factors with a full mode choice run for each iteration of the feedback process.*

These options can be implemented using only highway travel times (impedances) for trip distribution or using a composite impedance to reflect the level of service by all modes.

The two options for incorporating mode choice in feedback were implemented in the two test-case model systems to assess the effects of the options on modeling results and on execution time. In both test-case models, the MSA-EQA was used as the basic feedback mechanism and only highway travel times were used in trip distribution. The model systems for Memphis and Salt Lake City could not be adapted for use of a composite impedance measure in trip distribution within the resources of the project.

When mode choice was included in the feedback process, there was little change in the results for either of the two test case models. As indicated in Tables 4 and 5, the number of transit trips changed less than one percent in the Memphis model. In the Salt Lake City model, the number of transit trips decreased by roughly four percent but that represented a shift in total travel of only about one-twentieth of one percent. Despite the small change in the number of transit trips in Salt Lake City, the inclusion of mode choice in the feedback loop did result in an additional increase in average system-wide speed of 21.3 percent when mode choice was run using equilibrated speeds from an assignment-to-distribution feedback (Option 1) and a 37.0 percent increase in speed when mode choice was integrated into each feedback iteration (Option 2).

Although the change in the number of transit trips in the two test case models was small, the change probably has significant implications for the test-case models where the total share of travel by transit was small (0.8 percent in Memphis and 1.5 percent in Salt Lake City). Application of feedback with mode choice incorporated into the process may very well produce more significant changes in a model for a metropolitan area with a more significant share of travel by transit. Such an outcome is suggested by the nature of the difference in results between Memphis and Salt Lake City. The additional transit use and congestion in the Salt Lake City model resulted in more significant changes in results when feedback with mode choice incorporated was introduced.

The specific nature of the change in the Salt Lake City results when feedback with mode choice was introduced is also significant. Most of the change results from a shift in the distribution of trips along congested routes, which are most often the routes where transit services exist. By linking origin zones with destination zones that are closer and whose linkage avoids congested routes, the feedback process reduces the number of trips in corridors where transit is more competitive. This result is illustrated by the reduction in average trip length, the increase in average system-wide speed and the decrease in transit trips.

The results from Salt Lake City, illustrated in Table 4, indicate that integrating mode choice into the feedback process can significantly increase the execution time for the model system. In the case of the Salt Lake City model, execution time increased from 108 minutes (1.8 hours) without feedback to 1085 minutes (18.1 hours) when feedback was introduced with mode choice fully integrated. By contrast, the execution time for feedback with post-feedback application of mode choice was only 243 minutes (4.1 hours). The incorporation of mode choice into the feedback process on each iteration significantly increased execution time, adding fourteen hours in the case of the Salt Lake City model. As indicated in Table 5, the increase in execution time was not as great in the Memphis model but

Table 4: Comparison of Mode Choice Feedback Options for Salt Lake City

	Mode Choice with No Feedback	Post-Feedback Mode Choice	Feedback with Integrated Mode Choice
Person Trips by Mode			
Auto	3,188,247	3,188,727	3,189,991
Transit	43,734	42,254	41,990
Total	3,231,981	3,231,981	3,231,981
Statistics			
Average Speed (mph)	22.02	26.72	30.17
Average Trip Time (minutes)	18.15	16.23	17.20
Average Trip Length (miles)	6.40	5.92	6.07
Execution Time (minutes)	108	244	1,085

[1] The feedback method was MSA with equilibrium assignment.

Table 5: Comparison of Mode Choice Feedback Options for Memphis

	Mode Choice with No Feedback	Post-Feedback Mode Choice	Feedback with Integrated Mode Choice
Person Trips by Mode			
Auto	3,138,128	3,138,301	3,138,328
Transit	24,661	24,488	24,461
Total	3,162,789	3,162,789	3,162,789
Average Statistics			
Average Speed (mph)	41.60	42.07	42.07
Average Trip Time (minutes)	8.92	8.76	8.75
Average Trip Length (miles)	6.13	6.04	6.09
Execution Time (minutes)	6	71	103

[1] The feedback method was MSA with equilibrium assignment.

the mode-choice procedure was a far less complex algorithm than would be used in most modeling efforts where mode choice was of specific interest.

The results from the test cases suggest that incorporation of mode choice in feedback can result in a significant change in results, but only when there is congestion in the network and when transit carries a significant share of regional trips. Because the full integration of mode choice into feedback dramatically increases execution time, incorporation of mode choice <u>after</u> feedback should be considered whenever transit is not a major regional mode.

The Effect of Feedback on Model Sensitivity

Previous sections of this chapter have demonstrated how feedback can affect the results of a model when there is congestion in the network being modeled. The comparisons made in the previous sections were for prescribed baseline conditions, however, and merely suggest the need to recalibrate a model to match observed travel characteristics for those baseline conditions. Once recalibrated, the model with feedback would generally produce travel characteristics for the baseline condition similar if not identical to those of the model without feedback. The true test of the effect of feedback on the output of a modeling system must be based on the difference in results from forecasting with a recalibrated model with feedback and a calibrated model system without feedback.

The resources of this research project did not allow for a full recalibration of either of the test case models with feedback incorporated. As a result, a true test of the model sensitivity and forecasting is not possible. A reexamination of the result of the sensitivity test for the Memphis and Salt Lake City models can provide a useful indication of the effect of feedback on sensitivity of forecasts, however, by comparing the percent change from baseline to the conditions of the sensitivity tests (25 percent uniform growth, 25 percent radially concentrated growth and a new facility). For the model without feedback and for each of the models with feedback, the differences in percentage change provide an indication of how sensitive the model alternatives are to changes that would produce more or less congestion in the network.

Table 6 provides a comparison for three system-wide performance measures: average speed, vehicle miles traveled and average V/C ratio. The results presented in the table suggest that the model with feedback is less sensitive to growth or to strategies designed to reduce congestion. In both model systems, the test of the high-growth scenarios produced less decrease in average speed and less increase in average V/C ratio in the models with feedback than in the models without feedback. This was true for both the uniform growth scenario and the radially-concentrated growth scenario. The test of a major new roadway facility produced a decrease in congestion (as reflected in the reduction in average V/C ratio and the increase in average speed) for all of the models, but the decrease in congestion was less in the models with feedback than in the models without feedback.

A similar conclusion about model sensitivity does not seem appropriate with respect to changes in VMT. The models with feedback did not consistently produce less change in VMT when alternative growth scenarios were tested as was the case for average speed and V/C ratio. Unique location-specific characteristics can, in some cases, result in a greater change in VMT with feedback. As illustrated in Table 6, the addition of a major new roadway facility produced an increase in VMT in all of the models but the increase in VMT in the models with feedback was slightly more than in the models without feedback.

The conclusions on the sensitivity of models with feedback suggest that a recalibrated model with feedback may provide a better representation of speeds and travel times but may show less benefit from projects designed to reduce congestion or improve speeds. Likewise, models with feedback will probably show less deterioration of speed and less overall congestion as a result of growth in trips and VMT.

Guidance in the Application of Feedback

When Feedback Should Be Considered

The application of a variety of feedback methods for the two test case model systems clearly demonstrated that feedback only produces a change in modeling results when there is congestion predicted in a baseline run of the model without feedback. Feedback produced significant changes in the Salt Lake City model where congestion existed in the baseline application but produced only very slight change in the results for Memphis where there was no significant congestion. Some impact of

Table 6: Effects of Feedback on Model Sensitivity

Change in Model Systemwide Average Speed from Baseline

Method	25% Uniform Growth	25% Radial Growth	New Faciltiy
Memphis			
No Feedback	-5.2%	-5.9%	4.1%
Direct	-4.0%	-4.3%	3.5%
MSA (Equilibrium)	-4.0%	-5.0%	3.5%
Salt Lake			
No Feedback	-34.5%	-41.5%	7.6%
Direct	-18.1%	-19.5%	2.9%
MSA (Equilibrium)	-16.8%	-16.5%	3.7%

Change in Model Systemwide V/C Ratio from Baseline

Method	25% Uniform Growth	25% Radial Growth	New Faciltiy
Memphis			
No Feedback	23.7%	25.4%	-8.0%
Direct	21.7%	22.7%	-6.6%
MSA (Equilibrium)	21.8%	22.9%	-6.8%
Salt Lake			
No Feedback	27.2%	27.0%	-2.6%
Direct	20.8%	21.1%	-1.1%
MSA (Equilibrium)	22.1%	22.2%	-1.1%

Change in Model Systemwide Vehicle Miles of Travel from Baseline

Method	25% Uniform Growth	25% Radial Growth	New Faciltiy
Memphis			
No Feedback	23.6%	25.4%	2.8%
Direct	21.7%	22.8%	3.2%
MSA (Equilibrium)	21.8%	23.0%	3.2%
Salt Lake			
No Feedback	27.4%	28.1%	1.2%
Direct	21.4%	21.9%	2.2%
MSA (Equilibrium)	21.1%	21.1%	1.6%

feedback was noted in Memphis, however, after congestion was artificially introduced into the model network through representation of significant growth in the number of trips.

The test applications in Salt Lake City demonstrated that the effects of introducing feedback were not uniform throughout the network but were specifically correlated with the location of the congestion in the network. The feedback process changed the paring of trip origins and destinations in the trip-distribution process and resulted in fewer trips being made between zones connected by congested links and increased the trips between zones connected by uncongested links. The research clearly demonstrated that feedback affects not only the aggregate travel characteristics, such as trip length and average speed, but also the geographic variance in travel characteristics within a region. The research demonstrated that wherever significant congestion might exist in a baseline or future year network, feedback can produce significantly different results. These findings would suggest inclusion of feedback to the trip distribution step wherever congested is expected in the network and inclusion of feedback to mode choice when transit or HOV use is significant. The research was not sufficient to determine whether feedback should also include other steps in the process (trip generation or land use). The model systems used in the test case did not permit sufficient testing of feedback to these other steps.

Appropriate Feedback Methods

The test applications of feedback in sample networks demonstrated that the use of a method that averages previous iterations when calculating new input provides significantly better performance in reaching convergence than a Direct Method that reintroduces only the results of the last iteration. Of the methods tested that included averaging, the Method of Successive Averaging with Equilibrium Assignment (MSA-EQA) demonstrated superior performance characteristics without an appreciable increase in resource requirements over the Direct Method. Others have reported that in extremely large systems with significant congestion there may be advantages to using the more complex processes of mathematical optimization of the weights used to average the results from previous iteration. This occurs when the time required to reach equilibrium in assignment far exceeds the time required for a trip-distribution run.

Recalibrating After Feedback Has Been Introduced

Virtually all four-step modeling processes that are in use today have been calibrated to produce results for a baseline year that reasonably match observed travel characteristics: screenline volumes, volumes on specific roadway facilities, speeds on specific facilities or ridership on existing transit services. The process of calibration usually includes the adjustment of model parameters in one or more of the steps in the four-step process. If the introduction of feedback results in a significant change in any of the travel characteristics, recalibration of the baseline model will be required before the model can be used for forecasting future travel conditions. The most significant need for recalibration will almost certainly be in the trip distribution step where friction factors are used to ensure that the trip distribution model produces a trip-length distribution similar to that reported in a home interview survey, the Census Journey to Work tables or from some other observed data source on trip length. Recalibration may also be required in the mode choice model if model parameters were adjusted to compensate for previously biased estimates of roadway operating speeds.

Common Pitfalls

Most modelers who have attempted to introduce feedback into the traditional four-step modeling process have been the victims of one or more of the common pitfalls of introducing feedback. The process is complex and should be undertaken with thorough checks and reviews to ensure that proper caution has been taken and the system is operating correctly.

Excessive Storage Requirements: One of the most common pitfalls of feedback processes is excessive storage use. As previously indicated, the feedback method that uses information from all previous iterations will generally result in the fastest convergence of the process. Retaining the full information from each iteration throughout the process until closure is achieved will quickly consume the available storage of most microcomputer systems. An efficient method of averaging that uses the information from each iteration as it is completed and then deletes the results of the run, as in the case for MSA, is necessary to avoid the storage problem.

Errors due to rounding: Even a modeling system with a relatively small number of zones (300 to 400) produces an enormously large number of zone-to-zone interchanges in the trip-distribution process for each trip purpose modeled. For efficiency in the modeling process and conservation of both memory and storage, integer representation is frequently used in trip distribution. Rounding errors can be significant if an appropriate method for rounding is not introduced into the process. This is particularly relevant when changes in the trip table are being examined between iterations or between the beginning and the end of a feedback process. Some travel forecasting packages now have capabilities for smaller zone systems to represent tables as fractional (or real) values without sacrificing resource efficiency For large zone systems or where fractional values cannot be retained, a "bucket rounding" method that keeps each fractional element and adds it back in when accumulated to a whole number can preserve the total number of trips in the trip table.

Hypersensitivity: A basic assumption underlying the application of feedback is that travelers base all travel decisions on differences in times and costs between the choice options. It is generally recognized that there are other determinants of travel choice, but these other influences are often not incorporated into the modeling process. This is particularly relevant in the case of feedback because a number of factors besides time or cost are known to affect an individual's choice of where to travel. An individual may choose a job location because of the salary offered or the nature of the work. Similarly the location of a place to live might be based on characteristics of the neighborhood, characteristics of a specific house, and the price of housing in the neighborhood. The linkage of the home and job location for that individual is likely to be only partly determined by the travel time and cost between the two. The introduction of feedback into trip distribution assumes that a change in travel time from the routes connecting the home and the job relative to the routes connecting other zone pairs may induce the traveler to change either home or job location.

A significant change in the level of congestion in a network between a base year and a forecast year can result in a significant shift in trip distribution when feedback is being implemented. This predicted change might be significantly greater than would be expected in reality if the forecast time period is not sufficiently long to expect all of the location decisions to be made or if there are significant factors inhibiting this shift such as differences in income level or other characteristics of the residential population and employment in "competing" zones. Unfortunately, little is known about how quickly location-choice decisions are made or reevaluated in response to changes in congestion. Similarly, the degree to which travel time and cost determine trip distribution rather than other less tangible characteristics is largely unknown. Careful monitoring of the reasonableness of changes in trip distribution resulting from feedback is therefore recommended.

Research Conclusions and Directions for Future Research

The research conducted in this project identified clear theoretical justification for inclusion of feedback in the traditional four-step modeling process. The research also identified specific regulatory requirements for the incorporation of feedback under certain categories of non-attainment according to the Clean Air Act Amendments of 1990. While designed to represent a series of traveler choices made on the basis of travel times and costs, many sequential modeling processes without feedback produce speeds and travel times as outputs that are not consistent with the speeds and travel times used as inputs to steps earlier in the sequential process. As a result of the research in this study, the value of feedback in the four-step process was recognized not only as necessary to satisfy regulatory requirements, but as a means of representing speeds and travel times more accurately and consistently in the modeling system.

Based on numerous tests of alternative feedback methods using two test-cases, the following specific conclusions were drawn:

- *The implementation of feedback is possible within the existing travel forecasting packages used today.*
- *The implementation of the assignment-distribution feedback can produce different system-wide travel characteristics such as the average speeds and average trip length when there is congestion in the modeled networks. This result suggests that feedback may be essential to accurate forecasts when congestion exists. It also suggests that a recalibration of a model system to observed baseline data is necessary after introduction of feedback.*

- *A recalibrated model system with feedback will generally show less sensitivity of speed to growth in travel than a model system without feedback. The model system with feedback will shift trips away from congested links to the extent possible given the constraints imposed by the modeler.*

- *The Direct Method for feedback, which uses the results of the last iteration directly as input to next iteration, takes significantly longer to converge with greater fluctuations than the other methods that average the results between successive iterations.*

- *Among the feedback methods that average results of each iteration with the results of previous iterations, the tests that were performed showed the methods that use a fixed weight from the results of each iteration (Methods of Successive Averages) have almost the same convergence characteristics as methods that calculate an optimal weight for each iteration (Methods of Optimal Weighting), but with considerably less complexity and with faster execution.*

- *Integrating mode choice in the feedback process can lead to substantial increases in computing time, and in the test cases did not change the transit trips significantly beyond what was produced by an assignment-distribution feedback with assumed person-to-vehicle conversion factors and a full mode choice execution after convergence. If realistic assumptions about the final mode shares and auto occupancies can be obtained prior to execution, a feedback process with integrated mode choice may not be necessary except where transit use is very high and the transit networks are complex.*

- *In all cases, incorporating feedback in the process increased the computational time and storage requirements to produce a forecast. Feedback is complicated and each additional feedback iteration increases the execution time. It also increases the difficulty of understanding/explaining the interrelationships between the transportation improvements and forecasts that result from their implementation.*

The research in this project examined the feasibility and impacts of introducing feedback into the four-step forecasting process. It focused primarily on the assignment-distribution feedback loop required by the 1990 Clean Air Act Amendments and its implementation within existing travel forecasting software. There are a number of other potential feedback options that were not addressed in this effort, and a number of issues arose during the research that should be explored to gain a better understanding of feedback in travel forecasting. These potential research topics include:

- *The impacts of feedback to trip-generation and land-use forecasting,*

- *The incorporation of time-of-day into the feedback process,*

- *The use of impedance functions that include costs and composite impedances,*

- *Further exploration of the Method of Optimal Weighting in complex model systems, and*

- *The accuracy of the feedback process in predicting changes over different lengths of time.*

Another FHWA-sponsored research project is examining the changes in travel characteristics (network speeds, trips, etc.) that arise from incorporating feedback from the transportation models to the land-use forecasting models. The results from this parallel research effort will be available in 1996. Others have examined ways to incorporate measures of forecasting of accessibility into trip generation and time of travel, but the incorporation of accessibility measures should be explored more fully. The test cases chosen for this project did not lend themselves to a detailed analysis of composite impedance functions that include cost, or modes other than auto. More analysis in this area is warranted.

The Method of Optimal Weighting (MOW) also warrants further exploration. This research tested the implementation of MOW in a standard forecasting package (MINUTP) without custom programs. The tests indicated that the MOW method can be implemented using current software, but because of the computational requirements of MOW, special custom programming is required to maximize efficiency and reduce run time. MOW was not explored under a full range of conditions. Further research is warranted to determine when MOW provides clear benefits with respect to convergence and execution time.

The development and validation of both the traditional four-step process and processes with feedback have to date been based upon cross-sectional data for a particular validation year. The true test of feedback is its performance in predicting future conditions when the congestion and other variables change in different proportions throughout a region causing new travel patterns to develop. Examining the stability of the feedback relationships over time should consequently be investigated further by applying a validated feedback process to two base years (1980 and 1990 for example) to see if the feedback process captures the changes in travel patterns that actually occurred over time.

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