CEE 123 Transport Systems 3: Planning & Forecasting

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Homework #4 -- Performance, Demand, Equilibration [S O L U T I O N S]

CEE123: Complete problems 1-3 (40 points). Each solution should include volumes and times on all links as well as total system travel time. Problem 4 is extra credit.

Problem 1 (10 points)
Three routes connect a suburban origin and a downtown destination (x in kvph; t in minutes):

Route #1: $t_1 = 4 + 2x_1$
Route #2: $t_2 = 8 + 1x_2$
Route #3: $t_3 = 9 + 2x_3$

a. If the total O/D flow is 5.0 kvph, find the User Equilibrium (UE) flow pattern {x,t}.
b. If the total O/D flow is 2.0 kvph, find the path flows that minimize total system travel time.
c. How does this System Optimal solution compare with the UE Solution in Part a?

Solutions:

*(a) If the total O/D flow is **5.0 kvph**, find the equilibrium (UE) flow pattern {x,t}*

• Test if all three routes are used at the specified total flow under UE assumptions:

1. At T=0, t_1=4, t_2=8, and t_3=9 , thus Route #1 is used first, Route #2 second, Route #3 last.
2. At t_1=8, x_1=2.0 kvph. Until this volume is exceeded (when $t_1 = t_2$), all traffic uses Route #1.
3. At t_2=9, x_2=1.0 kvph & x_1=2.5 kvph. Until demand exceeds 3.5 kvph, Route #3 is not used.
4. At T_{od}=5 kvph, all routes are used.

• UE Solution

1. $t_1 = 4 + 2x_1$ [Link Performance Function #1]
2. $t_2 = 8 + 1x_2$ [Link Performance Function #2]
3. $t_3 = 9 + 2x_3$ [Link Performance Function #3]
4. T_{od}= 5 [Total O/D Demand]
5. $T_{od} = x_1 + x_2 + x_3$ [Flow Conservation]
6. $t_2 = t_3$ [UE Condition]
7. $t_1 = t_3$ [UE Condition]
8. Solve (1-5) simultaneously: t_1 = 9.75 min
9. From 6 and 7: $t_1 = t_2 = t_3$ = 9.75 min
10. From 1: x_1 = 2.875 kvph
11. From 2: x_2 = 1.750 kvph
12. From 3: x_3 = 0.375 kvph
13. From 4 and 5: $T_{od} = x_1 + x_2 + x_3$ = 5.0 kvph (checks)
14. $C_{tot} = \Sigma\, x_a\, t_a$ ...thus... C_{tot} = 48.75 k-min at UE

*(b) If the total O/D flow is **2.0 kvph**, find the path flows that minimize total system travel time.*

There are at least two possible approaches:

(b1) In the first approach, substitute the flow constraint into the objective function and take derivatives with respect to the remaining variables.

Min $C = \Sigma t_a \cdot x_a(t_a)$ ...subject to: $T_{od} = x_1 + x_2 + x_3 = 2$

Min $C = [4x_1+2x_1^2] + [8x_2+ x_2^2] + [9x_3+2x_3^2]$ ...subject to: $x_1 + x_2 + x_3 = 2$ kvph

Thus: $x_2 = 2 - x_1 - x_3$

Thus: $C = [4x_1+2x_1^2] + [8(2-x_1-x_3)+(2-x_1-x_3)^2] + [9x_3+2x_3^2]$

so : $\delta C/\delta x_1 = 9 + 4x_1 - 8 - 2(2-x_1-x_3) = 0$ ...thus... $6x_1+2x_3-3=0$

and: $\delta C/\delta x_3 =-8 - 2(2-x_1-x_3) + 4 + 4x_3 = 0$ ...thus: $6x_3+2x_1-8=0$

Solving these results simultaneously (then computing travel times) yields:

- $x_1 = 1.3125$ and $t_1 = 6.625$
- $x_2 = 0.6250$ and $t_2 = 8.625$
- $x_3 = 0.0625$ and $t_3 = 9.125$

(b2) In the second approach, marginal travel times may be equated in the same manner that average travel times are equated in the UE-solution. This is because we are trying to minimize the contribution to total travel time that each additional trip makes. In this case, the trip maker selects the route which has this minimal contribution (marginal travel time, the derivative of total travel time). When these contributions are equal (equal marginal travel times), then the system is in equilibrium.

Min $C = \Sigma t_a \bullet x_a(t_a)$ ...subject to: $T_{od} = x_1 + x_2 + x_3 = 2$

Min $C = [4x_1+2x_1^2] + [8x_2+x_2^2] + [9x_3+2x_3^2]$ ...subject to: $T_{od} = x_1 + x_2 + x_3 = 2$

$m = \delta C/\delta x$, thus: $m_1=4+4x_1$ ...and... $m_2=8+2x_2$ ...and... $m_3=9+4x_3$

• Test if all three routes are used at the specified total flow:

1. At T=0, $m_1=4$, $m_2=8$, and $m_3=9$, thus Route#1 is used 1st; Route#2 2nd; Route#3 last.
2. At $m_1=8$, $x_1=1.00$ kvph. Until this volume is exceeded, all traffic uses Route#1.
3. At $m_2=9$, $x_2=0.50$ & $x_1=1.25$ kvph. Until total demand > 1.75 kvph, R#3 is not used.
4. At $T_{od}=2$ kvph, all three routes are used.
5. $m_3=m_2 \Rightarrow 9+4x_1=8+2x_2 \Rightarrow x_3 =-0.25+0.5x_2$
6. $m_2=m_1 \Rightarrow 8+2x_2=4+4x_1 \Rightarrow x_1 = 1.00+0.5x_2$
7. thus: $x_1+x_3 = 0.75+x_2$
8. Demand: $x_1+x_2+x_3 = 2$ or $x_1+x_3 = 2-x_2$
9. thus: $0.75+x_2 = 2-x_2$
10. So: $x_2 = 0.625$ and $x_3 = 0.0625$ and $x_1 = 1.3125$
11. and: $t_2 = 8.625$ and $t_3 = 9.125$ and $t_1 = 6.625$
12. Check: $m_1 = 9.25$; $m_2 = 9.25$; $m_3 = 9.25$
13. Total time: $C_1 = 8.70$; $C_2 = 5.39$; $C_3 = 0.57$; $C = 14.66$

Note: The first approach essentially assumed that all routes would be used. Had not all three routes been used, the resultant flows would have shown negative volumes on at least one route.

(b) How does the System Optimal (SO) solution compare with the UE Solution (Problem 1B)?

There are different volumes, so the tewo solutions are NOT directly comparable. However, for either volume, the UE solution will always be as least as great as the SO solution.

Problem 2 (10 points)
Mr. Dunphy has two alternate routes when he drives from the desert (where he lives) to the sea (where he works). Route 1 has a higher base travel time but it's less sensitive to traffic congestion. There are linear performance functions (with travel time in minutes and volume in 1000s of VPH, or kvph).

Performance Parameter	Route 1	Route 2
Intercept (Free Flow Time)	2.0	1.0
Slope (Route Sensitivity)	1.0	2.0

The current travel demand function is linear: base demand is 15 (1000s of trips) but is reduced by 2 (1000s of trips) for each added minute of travel time. Solve algebraically or graphically for the user equilibrium flows.

SOLUTION: Graphical solution not shown.

a. Linear Demand Function: $T_{AB} = 15 - 2t_{AB}$
b. Performance Function 1: $t_1 = 2 + x_1$
c. Performance Function 2: $t_2 = 1 + 2x_2$
d. User Equilibrium (UE) : $t_1 = t_2 = t_{AB}$
   Assume both paths used: $2+x_1 = 1+2x_2 \Rightarrow x_1 = 2x_2-1$
e. Conservation of Flows : $T_{AB} = x_1 + x_2$
   thus: $15-2[1+2x_2] = [2x_2-1] + x_2$
f. Solving yields => $x_2 = 2$ kvph
g. Performance Function 2: $t_2 = 5$ min
h. User Equilibrium (UE) : $t_1 = 5$ min
i. Performance Function 2: $x_1 = 3$ kvph
j. Linear Demand Function: $T_{AB} = 5$ kvph

Problem 3 (20 points)
This problem is part of the Miasma Beach Project and provides you the opportunity to estimate trip generation models using Excel (as was done in HW 1). using the data provided in Task 3, Tables 2 and 3, estimate a home-based work (HBW) trip production and a home-based work (HBW) trip attraction model for the six internal TAZs.

1. Pick an explanatory variable from Table 2 that you think would be most strongly related to HBW productions in a TAZ. Estimate this model.
2. If this model gives acceptable results, add a second variable that you think would also be strongly related to HBW productions (if results are not acceptable, choose another single variable). Estimate this model.
3. Compare the two models. Which would you choose and why?
4. Repeat parts 1-3 for HBW attractions.

Solution Approach:
HBW trip production models will be estimated in Task 3 of the CEE123 course project. The goal is to begin the process of model development. Attached is a page of a spreadsheet with the Miasma Beach data set and results of 5 bivariate production models using 5 residential variables), a sample multivariate model (adding CARS to the POP model), and 2 sample HBW attraction models (with total and retail employment, but other employment variables may be viable too). *Note that the database is slightly different each year to introduce variability. Results shown may not match yours but they should be relatively close*. The bottom line is that building models is an art. There's no right answer, although there are many wrong answers (based on statistical amd/or logical problems). Note also that models estimated on 6 data points are virtually never acceptable.

Problem EC (10 points) [Extra Credit for 123; Required for 223]
Solution available to those who submit extra credit attempt.

Last Updated: 5 May 2024