Macroscopic Models of Lane-Changing, Bounded Acceleration, and Capacity Drop

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Outline

• Introduction
• Lane-changing
  • Phenomenological model
  • Behavioral model
• Bounded acceleration
• Capacity drop
  • Behavioral model
  • Phenomenological model
• Conclusion
Introduction
Freeway network
Arterial network
Stop-and-go traffic

SHOCKWAVE TRAFFIC JAMS RECREATED FOR FIRST TIME

Footage courtesy of University of Nagoya, Nagoya, Japan
Inter-vehicle communication
CAV emulation
A control system view

Desired performance

Control, Management, Planning, Design, ...

Transportation systems

Detection, Estimation, Communication

Safety
Mobility
Costs
Emissions
Land use
Driver vs network behaviors

- Social
  - house/job
  - trip/destination
  - mode/vehicle
  - departure time
  - route
  - speed/lane
  - maneuvering

- Economics

- Physics

Bottlenecks
- Network congestion

Centralized control

Distributed cooperative control
Lighthill-Whitham-Richards model: Macroscopic

• variables
  • $k(t, x)$: density
  • $v(t, x)$: speed
  • $q(t, x)$: flow-rate

• Five rules
  R1. constitutive law: $q = kv$
  R2. fundamental diagram: $v = \eta(k)$, $q = \phi(k)$
  R3. continuity equation: $\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0$
  R4. weak solutions: shock waves
  R5. unique solution: entropy condition

• The LWR model: R1+R2+R3
  $$\frac{\partial k}{\partial t} + \frac{\partial \phi(k)}{\partial x} = 0$$
Other models

• Microscopic models
  • car-following: Newell, Pipes, Optimal Velocity, GM
  • lane-changing: >40 parameters

• Simulators
  • Paramics, Transmodeler, Aimsun, VISSIM

• Advantages
  • detailed, individual characteristics

• Disadvantages
  • macroscopic characteristics
  • computational cost
Lane changing
Lane-changing bottleneck
First principle of lane changes

- First principle: One car occupies two lanes
- Effective total density: \( \text{Total vehicle} \times \text{lane} \times \text{miles} \)
- Lane-changing intensity: \( \epsilon (t, x) \)
  - effective total density = actual total density \( \times \) \( (1 + \epsilon) \)
- Fundamental diagram with lane changes
  - \( v = V \left( \frac{k(1+\epsilon)}{l} \right) \)
  - \( q = k V \left( \frac{k(1+\epsilon)}{l} \right) \)
  - \( l \): number of lanes
Calibration and validation

• NGSim: I-80 in Emeryville (San Francisco), CA
  • 2:35-3:05pm, Dec 3, 2003 (1/15 sec)
  • 4-4:15pm, 5-5:30pm, Apr 13, 2005 (1/10 sec)
Kinematic wave model

• The LWR model: \( \frac{\partial k}{\partial t} + \frac{\partial k \cdot V \left( \frac{k(1+\epsilon)}{l} \right)}{\partial x} = 0 \)

• Inhomogeneous LWR model: \( \epsilon(t, x) \)

• Multi-lane fundamental diagram and LWR model
  • \( a(t, x) \): number of lanes
    \( \frac{\partial k}{\partial t} + \frac{\partial k \cdot V \left( \frac{k}{a} \right)}{\partial x} = 0 \)
  • lane changes = lane reduction (lane drop) \( \Rightarrow \) capacity reduction
    \( a = \frac{l}{1+\epsilon} \)

• References:
  • WL Jin, A kinematic wave theory of lane-changing traffic flow, Transportation Research Part B 44 (8-9), 1001-1021.
Behavioral model

• Total #/LC: \( N_{LC} = \frac{\phi}{n} T \cdot 1 + \ldots + \frac{\phi}{n} T \cdot (n - 1) = \frac{n-1}{2} \phi T \)
  - from right to left: #/LC linearly decreases
  - total #/LC proportional to on-ramp flow

• Constant lane-changing duration: \( \pi \)

• Intensity: \( \epsilon = \alpha \frac{\phi}{k} \)
  - \( \alpha = \frac{n-1}{2L} \pi \)

• Calibration and validation with NGSIM data
Kinematic wave model

• $k$: total density; $\rho$: weaving density
  • $\frac{\partial k}{\partial t} + \frac{\partial k \eta(k, \rho)}{\partial x} = 0$
  • $\frac{\partial \rho}{\partial t} + \frac{\partial \rho \eta(k, \rho)}{\partial x} = 0$

• Impacts of HOV lane at a lane-drop bottleneck
  • Increased throughput $\approx Q_{HOV} - 1400$

• References
  • Gan, Q.J. and Jin, W.L., 2015. Left-Lane Changes in Laterally Unbalanced Traffic: Estimating Number of Lane Changes with Data from Lane-Based Loop Detectors. Transportation Research Record: Journal of the Transportation Research Board, (2490), pp.106-115.
  • Gan, Q.J. and Jin, W.L., 2013. Validation of a macroscopic lane-changing model. Transportation Research Record: Journal of the Transportation Research Board, (2391), pp.113-123.
Bounded acceleration
Bounded acceleration model

• maximum acceleration rate without vehicles in the front: \( a = A(v) \)
  • \( v \in [0, u], 0 \leq A(v) \leq a_0 \)
  • \( A'(v) \leq 0 \)

• Examples:
  • constant: \( A(v) = a_0 \)
  • TWOPAS: \( A(v) = a_0 \left( 1 - \frac{v}{u} \right) - gG \)
  • Gipps: \( A(v) = 2.5 \ a_1 \left( 1 - \frac{v}{u} \right) \sqrt{0.025 + \frac{v}{u}} \)
  • no bound: \( A(v) = \infty \)
Stationary states inside a lane-drop zone
Capacity drop
Capacity drop
Cause of capacity drop?

• Acceleration?
  • the reduced flow is a consequence of the way drivers accelerate away from the queue (Hall and Agyemang-Duah, 1991)

• Lane changes?
  • lane changes are the main cause of the drop in discharge rate (Laval and Daganzo, 2006)
Optimization formulation of entropy condition

- max $q^*$
  - s.t. $q^* \leq d_1; q^* \leq s_2$;
  - bounded acceleration constraint: $a^*(x) \leq A(v^*(x))$

- Theorem. The following statements are equivalent:
  - New entropy condition
  - max $q^*$ s.t. $q^* \leq d_1, q^* \leq s_2, H(d_1 - s_2) \cdot (q^* - C^-) \leq 0$
  - $q^* = \min\{d_1, s_2, l_2 C_1 (1 - \epsilon H(d_1 - s_2))\}$

- $C^-$: dropped capacity
  - $\epsilon$: capacity drop ratio
Phenomenological model

• Three characteristics
  • the discharge flow-rate can reach the downstream capacity when the upstream link is uncongested
  • capacity drop occurs with the formation of an upstream queue
  • the downstream link cannot be stationary at all densities, and the observed flow-density relation is discontinuous.

• Model: \( q = \min\{d_1, s_2, l_2 C \left(1 - \epsilon H(d_1 - s_2)\right)\} \)

• References:
Application to sag/tunnel bottleneck
Conclusion
Summary

• Macroscopic models are extended in various ways
  • Fundamental diagram of lane-changing traffic
  • New entropy condition for capacity drop
  • Constraints on stationary states

• Lane-changing=lane reduction⇒ capacity reduction
  • but lane drops and sag/tunnels lead to more significant capacity reduction

• Bounded acceleration+Capacity reduction⇒capacity drop
  • lane changes are neither sufficient nor necessary conditions of capacity drop
  • there’s correlation at some locations
Applications

• Simple car-following models for lane-changing, bounded acceleration, and capacity drop
• Variable speed limit
• Ramp metering
• Control through connected and autonomous vehicles
• ...
Thank you!

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