Envy-free Pricing for Collaborative Consumption in Transportation Systems

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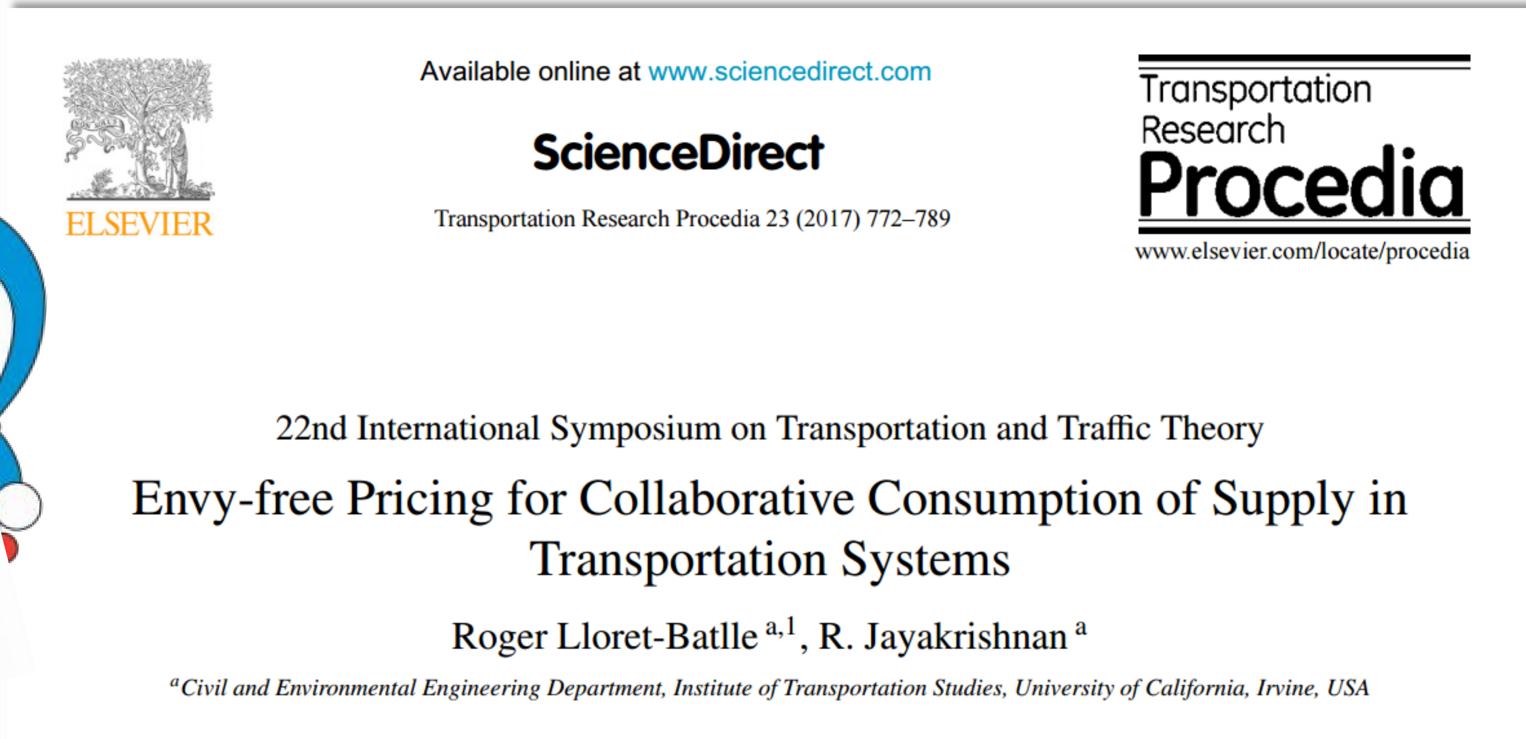
Main idea: P2P exchange and collaborative consumption of transportation supply

- Transportation systems normally operate on a FCFS basis.
- FCFS is *the fairest rule* but may not the most efficient.
- Smartphones & CV: elicit private information from drivers on real time and outperform FCFS.
- Contribution: allowing users to trade the supply they “own” to increase system efficiency and fairness.
- Two examples: Traffic signal control (Lloret-Batlle & Jayakrishnan, 2016), P2P ridesharing (Masoud & Lloret-Batlle, 2016).
- New challenges: incentives, the role of regulation, **fairness**, **public acceptance**.

Objective

- Envy-freeness: pricing scheme with potential use in transportation systems.
- Extending Envy-freeness to dynamic situations.
- Propose a theoretical framework for **dynamic envy-free pricing**.
- Provide new applications: queue-jumping operations, traffic signal control.
- New concepts developed:
  - Dynamic envy-freeness.
  - Queue-jumping operations and its fair pricing.
  - Constant Elasticity of Substitution (CES) Envy Ranking criteria.
  - Envy measurements for online problems.

Main reference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Envy-free Pricing for Collaborative Consumption of Supply in Transportation Systems

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Envy-free pricing (I)

Envy-free pricing definition:

$$u_i(x_i, p_i) \geq u_i(x_j, p_j) \quad \forall i, j \in I, i \neq j$$

- Notion of fairness and stability: no user is willing to exchange their allocations at the current prices → Behavioral concept.
- Bidder-optimality. Prices are anonymous.
- If there is envy, $e_{ij} = \max\{u_i(x_j, p_j) - u_i(x_i, p_i), 0\}$
- No interpersonal utility comparisons.
- Honest information elicitation.
- Finally, one of the properties of Walrasian equilibria.

Easy to check: Wardrop's user equilibrium is envy-free.

Envy-free pricing (II)

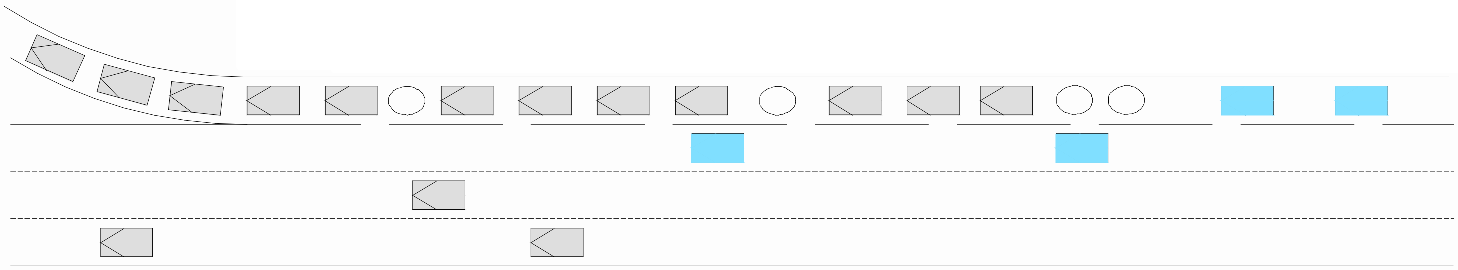
- Preliminaries. Mechanism design:
  - "Transportation environment": unit demand, linear utility: $u_i = v_i x_i - p_i$. Generally $v_i = -\theta_i$ and $x_i = d_i$. Allocation is non-transferrable.
- Existence of envy-free pricing in transportation environment.
  - Suppose WLOG $v_1 \geq v_2 \geq \cdots \geq v_n$.
  - Allocation has to be monotonous: $x_1 \geq x_2 \geq \cdots \geq x_n$.
  - Prices follow this general expression:

$$p_i = \sum_{k \leq i} \alpha_k \, , \alpha_1 \in \Re, \alpha_k \in [v_i(x_k - x_i), v_k(x_k - x_i)] \quad \forall k > 1$$

  - α_1 is found by imposing individual rationality or budget balance.
- Problem: EF allocations may not exist. In they do, they are generally not unique. → Envy relaxation methods.
- **Theorem**: every mechanism which is allocative efficient and budget balanced in quasi-linear preferences in Pareto optimal.

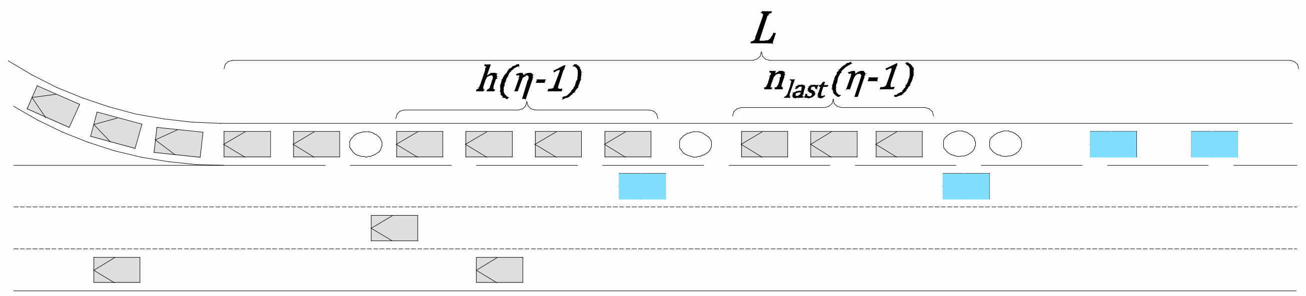
Sequential case: queue-jumping (I)

- Highway exit. Long queue builds (FCFS or even worse if anxious human drivers)



- VOT/VODS heterogeneity in users → FCFS is inefficient.
- We propose a mechanism that allows high VOT vehicles to jump positions in queue by paying the rest of arriving vehicles that join the queue at the back.
- Why "sequential"? We group vehicle arrivals in independent sets.
- Proposed mechanism: envy-free, Pareto optimal (since allocative efficient and budget balanced)
- Inspired by Position Auctions (GoogleAds) but we have externalities!
- Connected and Automated Vehicle (CAV) environment required.

Sequential case: queue-jumping (II)



- Vehicles are known at a distance from the front of the queue closer than L.
- Vehicles' arrival speed v_a. Queue moves at $v_q \ll v_a$ with spacing s_q.
  Queue headway: $h = s_q/v_q$.
- Design parameters: $\eta - 1$: number of vehicles between gaps. r reserve price.
- Algorithm:
  - Policy activates once queue is at least μ gaps.
  - Detected incoming vehicles (arrival time < time exit first gap) form participant set N.
  - Envy-free position auction: solve Program (P).
  - Vehicles who's $v_i > r$ may skip positions and pay the vehicles which join at the back.
  - If $\#\text{gaps} \geq \mu$ holds, next set N are the vehicles which could jump to any of the gaps such that first new vehicle could still join the back of the queue.

Sequential case: queue-jumping (III)

$$\text{(P)} \qquad \max \sum_{i\in I, j<m} (b_i - r)\delta_{ij}$$

$$s.t.$$

$$\delta_{ij} = \max\{h((m-1-j)(\eta-1)+n_{last}),0\}x_{ij} \quad \forall i \in I, \forall j < m$$

$$(b_i - r)\sum_{k\in J} \delta_{ik} - p_i \geq (b_i - r)\sum_{k\in J} \delta_{jk} - p_j \quad \forall i \in I, j \neq i \quad [\text{EF}]$$

$$\sum_{j\in J} x_{ij} = 1 \;\; \forall i \in I \quad [FEAS1]$$

$$\sum_{i\in I} x_{ij} \leq 1 \quad \forall j < m \quad [FEAS2]$$

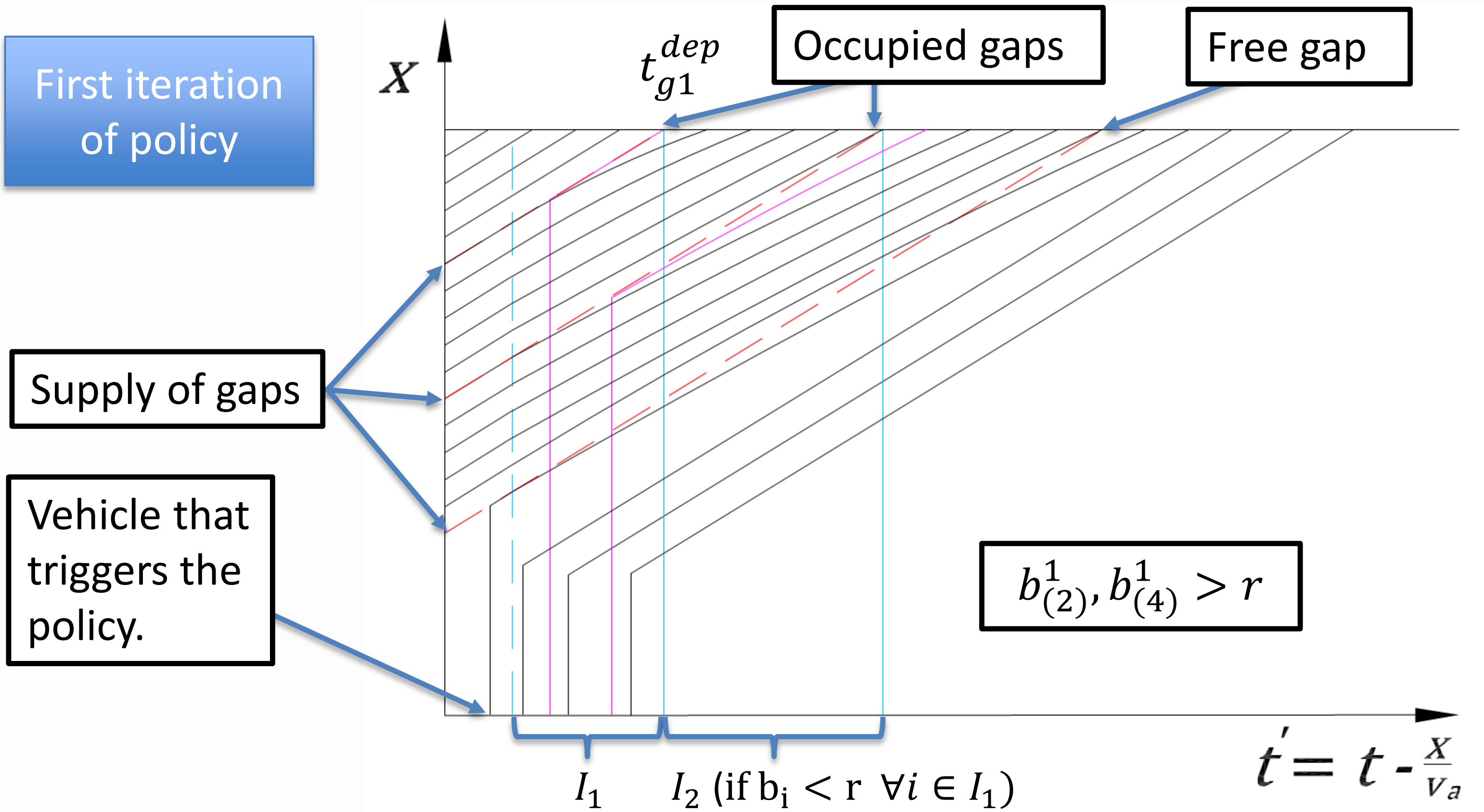
$$(b_i - r)\sum_{k<m} \delta_{ik} - p_i \geq 0 \quad \forall i \in I \quad [IR]$$

$$\sum_{i\in I} p_i = 0 \quad \forall i \in I \quad [BB]$$

$$x_{ij} \in \{0,1\} \;\; \forall i \in I, j \in Q$$

Sequential case: queue-jumping (IV)

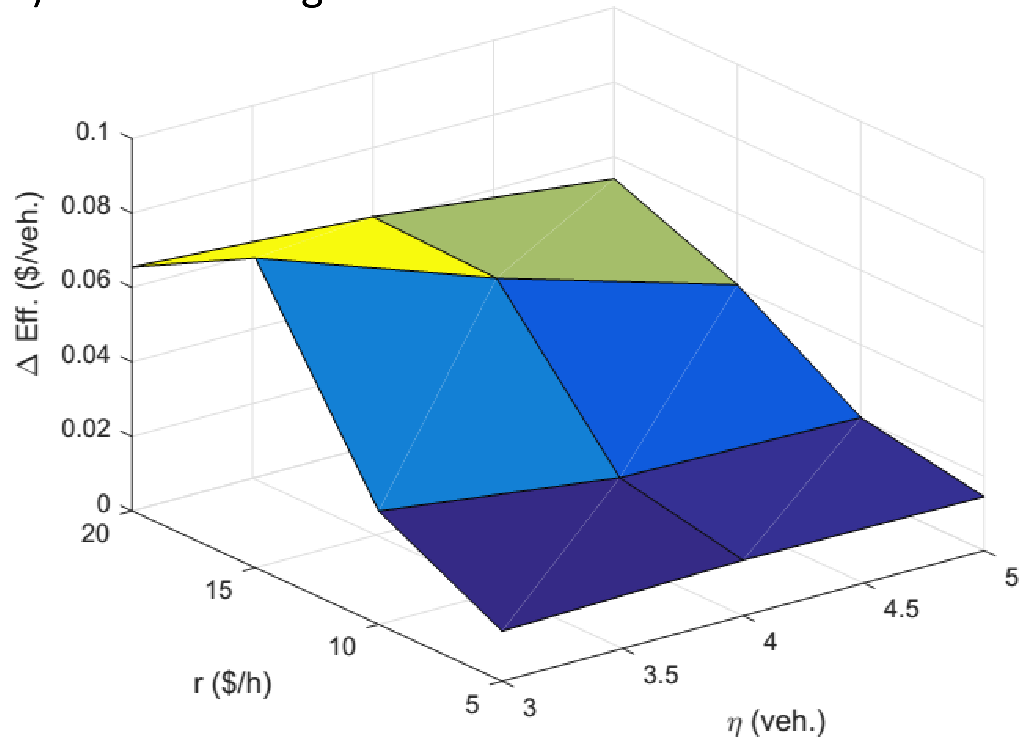
- Gap Allocation example with $\mu = 3, \eta = 5$
- Note: affine transformation time axis.



- Note 2: gaps are physically created only once a vehicle is assigned to them.

Sequential case: queue-jumping (V)

- MATLAB Simulation sensitivity analysis ($L = 300$ m, $s_q = 6$ m, $v_q = 5$km/h, Poisson arrivals $\lambda = 900$ veh/h, $v \sim N(2.16, 0.7)$ {Abou-Zeid et al. 2011}, $Q_0 = 0$ veh). Car following model: Newell.



- We expect concavity as well on queued batch size η: more queue interactions, policy more sensitive to adjacent main lane congestion.

Dynamic Envy-freeness (I)

- Envy is well defined for a static or sequential problems.
- Not so clear for dynamic problems. Questions:
  1. How is envy instantaneously *felt* by agents? → Behavior.
  2. How should it be modelled? → Analysis
  3. How should it be measured? → Policy evaluation.
- Need for new concepts…
- Let's define the environment for dynamic transportation problems.
- States $s_t \in S_t, \forall t \in \Re^+$.
- **Axiom**: Constant envy from state invariance.

$$\forall t \in \Re^+, if\ s_t = s_{t+1} \Longrightarrow e_{it} = e_{i,t+1} \quad \forall i \in I_t \cap I_{t+1}$$

- $s_t = (\boldsymbol{d}_t, \boldsymbol{p}_t, \boldsymbol{\pi}_t) \in D_t \times P_t \times \Pi_t$.
  - Predicted cumulated delays, prices charged at t, cumulated prices at t.
  - Suppose that at instant t, each agent $i \in I_t$ experiences excess envy e_{it}.
  - e_{it} should change only when s_t changes.
- **Definition**: dynamic envy-freeness.

$$v_i x_i^t - p_i^t - \pi_i^{t-1} \geq v_i x_j^t - p_j^t - \pi_j^{t-1} \quad \forall i,j \in I, i \neq j, \forall t \in T$$

Dynamic Envy-freeness (II)

- What if envy-free allocations do not exist? → Relax envy-free conditions.

$$\boldsymbol{\epsilon_{ij}} + u_i(x_i, p_i) \geq u_i(x_j, p_j)$$
$$\boldsymbol{\epsilon_{ijt}} + v_i x_i^t - p_i^t - \pi_i^{t-1} \geq v_i x_j^t - p_j^t - \pi_j^{t-1}$$

- New issue: how to rank the envy terms → Need for criteria (obj. functions)
- Existing criteria Minimax: (Diamantaras and Thomson (1990), Envious/envied criteria (Fleurbaey, 2008).
  - $\min_i \sum_j \max\{\epsilon_{ij}\}$ AND $\min_j \sum_i \max\{\epsilon_{ij}\}$
- These criteria overlook all envy relations that are not binding.
- We propose a new criteria: **Constant Elasticity of Substitution (CES) envy intensity criterion**

$$\boldsymbol{p} \in f(e) \Leftrightarrow \forall \boldsymbol{p}' \in P, \sum_{i \in I_h} \gamma_i \left(\sum_{j \neq i} \left(\beta_j \epsilon_{ij}(\boldsymbol{p}) \right)^{\rho} \right)^{\frac{1}{\rho}} \leq \sum_{i \in I_h} \gamma_i \left(\sum_{j \neq i} \left(\beta_j \epsilon_{ij}(\boldsymbol{p}) \right)^{\rho} \right)^{\frac{1}{\rho}}, \epsilon_{ij} \geq 0 \ \forall i, j \in I_h$$

- All the envy relations compute in this ranking → Better and more redistribution across all the valuation range for the traffic signal case.

Dynamic Envy-freeness (III)

- Which are the properties of the **CES envy intensity criterion**?
- **Proposition**: this criteria selects the set of EF allocations, when such set is non-empty.

- **Proposition**: CES envy intensity criterion satisfies the axiom Equal Payment for Uniform Allocation (EPUA)

$$\forall e \in E, \forall \boldsymbol{\pi}_t \in \Pi_t, \forall i,j \in I \mid d_i^t = d_j^t\text{:}\;\; \pi_i^t = \pi_j^t$$

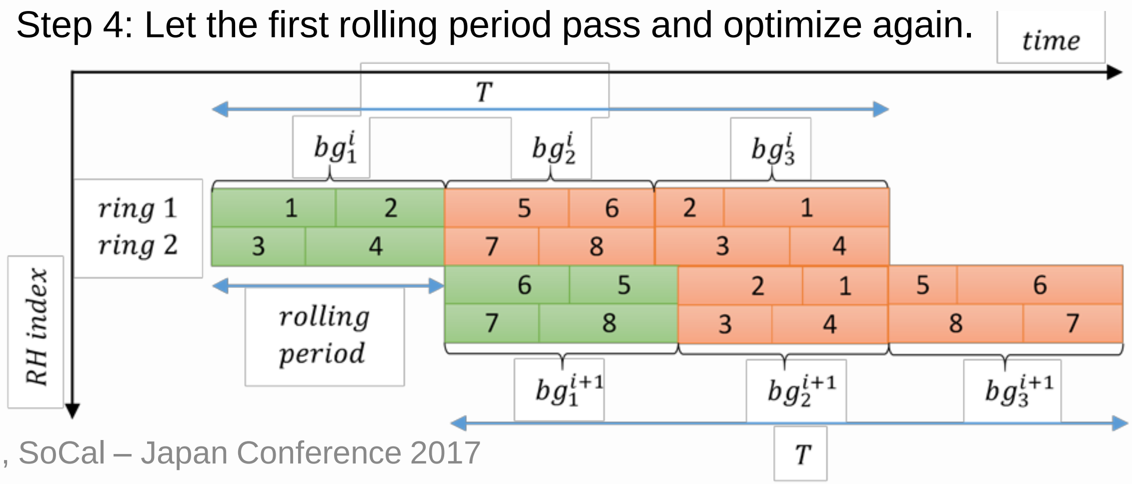
- **Proposition**: CES envy intensity criterion satisfies equal utility for Uniform Type (EUUT)

$$\forall e \in E, \forall \boldsymbol{\pi}_t \in \Pi_t, if\ \forall i,j \in I, v_i = v_j\text{:} \forall i,j \in I, u(\pi_i, d_i, v_i) = u(\pi_j, d_j, v_j)$$

- Note: for the static case, it holds also. Just consider p_i instead of π_i^t.
- Our new criteria fits into an existing axiomatic framework (Fleurbaey-Maniquet) of social welfare literature, by satisfying the aforementioned two (strong) axioms.

Application: PEXIC traffic signal control (I)

- PEXIC: **P**riced **EX**changes in **I**ntersection **C**ontrol (Lloret-Batlle & Jayakrishnan, 2016)
  - Heterogeneity in drivers Value of Delay Savings (VDS).
  - Controls traffic intersection as a computational exchange economy.
  - Multi-agent adaptive traffic signal control.
  - Generally, high VDS drivers pay compensate VDS drivers to wait less.
  - PEXIC outline:
    - Step 1: At time t, find best phasing for next T seconds. (Aprox. dynamic programming algorithm)
    - Step 2: Solve program PEXIC-EF.
    - Step 3: Charge the prices to vehicles.
    - Step 4: Let the first rolling period pass and optimize again.



Application: PEXIC traffic signal control (II)

Program (PEXIC-EF) for every horizon h.

- CES criterion with $\rho = 1$ and $\beta_{ij} = 1\ \forall i, j \in I_t$.

$$\min \sum_{i,j \in I_h} \gamma_i \epsilon_{ij}$$

$$s.t.$$

$$\epsilon_{ij} - \theta_i d_i - p_i^t - \pi_i^{t-1} \geq -\theta_i d_j - p_j^t - \pi_j^{t-1} \quad \forall i, j \in I_h, i \neq j \quad [REF]$$

$$\sum_i p_i^t = 0 \quad [BB]$$

$$\epsilon_{ij} \geq 0 \ \ \forall i, j \in I_h, i \neq j \quad [ELB]$$

$\gamma_i > 0$ are decreasing with type θ_i.

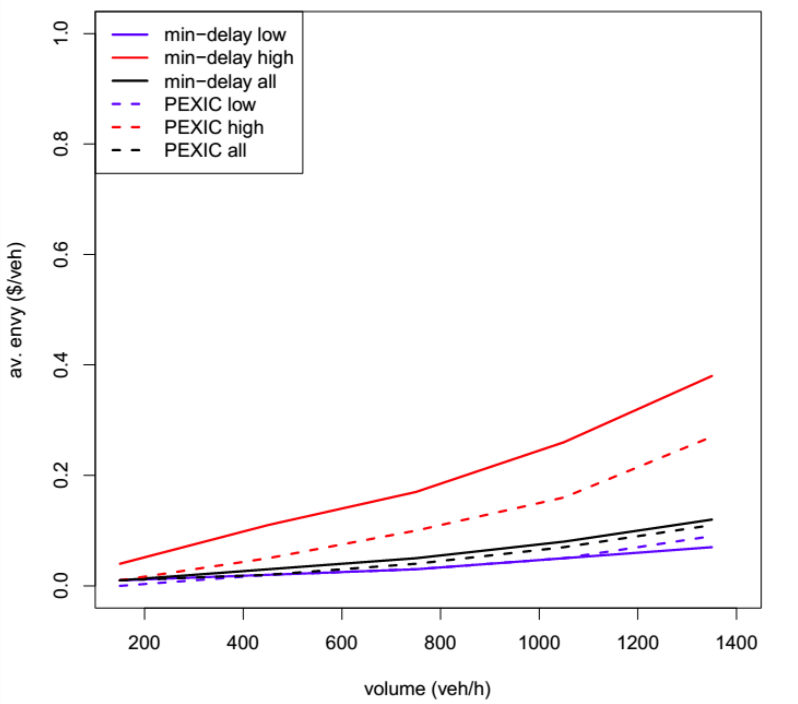
Application: PEXIC traffic signal control (III)

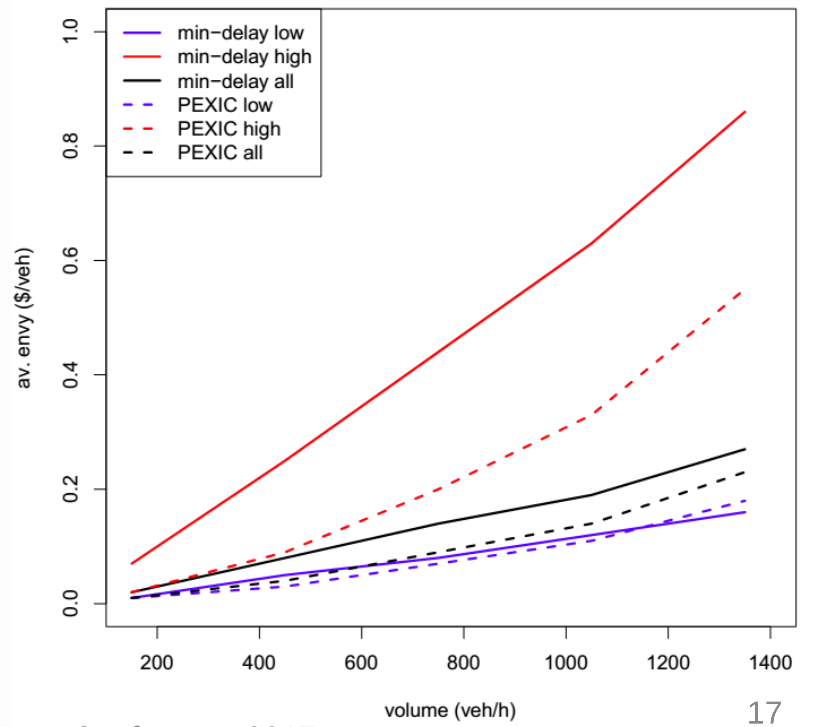
- New measures of envy in dynamic problems:
  - (average) Maximal dynamic envy measure:

$$\epsilon^{max} = \frac{1}{N}\sum_i \max_{h:I_h \ni i}\{\max_j\{\epsilon_{ijh}\}\}$$

  - (average) Cumulative dynamic envy measure:

$$\epsilon^{sum} = \frac{1}{N}\sum_i \sum_{h:I_h \ni i} \max_j\{\epsilon_{ijh}\}$$





Conclusion

- FCFS consumption of transportation supply is inefficient and unfair.
- We presented envy-free pricing as a flexible technique to violate FCFS in many transportation applications.
  - Traffic signal control.
  - Queues.
  - (Also P2P ridesharing in previous work from author).
- New problem introduced: queue-jumping operations.
  - We proposed: Envy-free Pareto Optimal mechanism for this problem.
- New concept: dynamic envy-freeness.
- We proposed a new envy ranking criterion: Constant Elasticity of Substitution (CES) envy ranking criterion.
  - Useful for dynamic problems in transportation.
- We presented two new indicators for dynamic envy.

Thank you!

This presentation corresponds to:

Lloret-Batlle, R. and Jayakrishnan, R. (2017) “Envy-free Pricing for Collaborative Consumption of Supply in Transportation Systems” **ISTTT2017**. Transportation Research Procedia, 23, 2017, 772–789.

PEXIC system reference:

Lloret-Batlle, R., & Jayakrishnan, R. (2016), Envy-minimizing Pareto efficient intersection control with brokered utility exchanges under user heterogeneity. ***Transportation Research Part B: Methodological***, 94, 22-42.