Linear Active Disturbance Rejection Control: Stabilizing Unmodeled Plants By Tuning Parameters

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Outline

1. INTRODUCTION
2. PROBLEM SETTING AND LADRC
3. STABILITY OF LADRC
4. CONCLUSIONS
In transportation studies, there are many feedback control problems.

- Ex.1: Feedback control of traffic flow
- Ex.2: Cruise-control of a car
- Ex.3: Suspension in a car
Two ways to control

- **Way 1:** Based on a precise mathematical model
  - Ex.1 Capacity drop model of traffic flow
  - Ex.2 Mathematical model of cruising car
    
    \[ \ddot{x} + \frac{b}{m} \dot{x} = \frac{u}{m} \]
    
    - To identify the model parameters may be expensive and time-consuming

- **Way 2:** Based on knowledge and tuning parameters
  - PID controller
    (Proportional-Integral-Derivative)
  - "Data-driven"
Active disturbance rejection control

- Two flaws of PID
  - D may amplify the disturbance
  - I may lead to instability

- Active disturbance rejection control (ADRC) is proposed by Prof. Jingqing Han

Prof. Jingqing Han (1937-2008) Prof. Han’s book
From ADRC to LADRC

- Active disturbance rejection control (ADRC) is a practical control technology.
  - stabilizing plant by tuning several parameters
  - suitable for uncertain, nonlinear, or time-varying plants

- Linear ADRC (LADRC) is a linear version of ADRC
  - proposed by Prof. Zhiqiang Gao (Cleveland State University, USA)
  - using linear feedback and linear extended state observer (LESO)
  - less parameters to tune
  - widely discussed and applied in Chinese control community
In this presentation

- To introduce LADRC for second-order LTI plant
- To propose a classical control prospective and some latest stability results of LADRC
PROBLEM SETTING AND LADRC
The plant and the problem

Consider the second-order SISO plant

\[ \dot{y} = -a_1 \dot{y} - a_2 y + bu, \]  

- \( a_1, a_2 \) are unknown, while \( b \) is known
- The problem is to let \( y \) track a reference signal \( r \), which is typically a step signal
The extended state

- Re-write (2) in state space, we have

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -a_2 x_1 - a_1 x_2 + bu, \\
y &= x_1,
\end{align*}
\]  

(3)

- introduces extended state \( x_3 = -a_2 x_1 - a_1 x_2 \)

- re-writes (3-4) as

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3 + bu, \\
\dot{x}_3 &= \dot{x}_3, \\
y &= x_1.
\end{align*}
\]  

(5)

(6)
Tracking Differentiator (TD)

LADRC using TD to improve transient performance, which is a signal satisfying

\[ \dot{v}_1 = v_2, \quad \text{(7)} \]

\[ \lim_{t \to +\infty} (r - v_1) = 0. \quad \text{(8)} \]
LESO and LADRC

**LESO**

\[
\begin{align*}
\dot{x}_1 &= k_1 (y - \hat{x}_1) + \hat{x}_2, \\
\dot{x}_2 &= k_2 (y - \hat{x}_1) + \hat{x}_3 + bu, \\
\dot{x}_3 &= k_3 (y - \hat{x}_1),
\end{align*}
\]

where \( k_1, k_2, k_3 \) are the parameters to be determined.

**Controller**

\[
u = \frac{1}{b} (l_2 (v_1 - \hat{x}_1) + l_2 (v_2 - \hat{x}_2) - \hat{x}_3),
\]

The plant (3-4), LESO (9), and controller (10) are continuous-time LADRC.
let $l_1, l_2 > 0$. For example (Gao 2003), suppose $\omega_c > 0$ is a positive parameter called "controller bandwidth" and let

$$l_1 = 2\omega_c, \quad l_2 = \omega_c^2. \quad (11)$$

chose a tuning parameter $\omega_o > \omega_c$ called "observer bandwidth" and let

$$k_1 = 3\omega_o, \quad k_2 = 3\omega_o^2, \quad k_3 = \omega_o^3. \quad (12)$$
STABILITY OF LADRC
The subsystem of LESO error

Let the error of LESO be \( \tilde{x}_i = x_i - \hat{x}_i \), for \( i = 1, 2, 3 \), we have

\[
\begin{align*}
\dot{\tilde{x}}_1 &= -k_1 \tilde{x}_1 + \tilde{x}_2, \\
\dot{\tilde{x}}_2 &= -k_2 \tilde{x}_1 + \tilde{x}_3, \\
\dot{\tilde{x}}_3 &= -k_3 \tilde{x}_1 + \dot{x}_3.
\end{align*}
\] (13)

let

\[
\tilde{r} = l_2 \nu_1 + l_1 \nu_2, \\
\nu = l_2 \tilde{x}_1 + l_1 \tilde{x}_2 + \tilde{x}_3, \\
e = \tilde{r} + \nu.
\] (14), (15), (16)

Look (13) and (15) as a subsystem \( H_2 \) with input \( \dot{x}_3 \) and output \( \nu \).
Substituting controller (10) into (5), and look \( e \) and \([y, \dot{x}_3]^T\) as the input and output, we have a single-input-double-output subsystem

\[
H_1 : \quad \begin{cases} 
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -l_2 x_1 - l_1 x_2 + e 
\end{cases}
\tag{17}
\]

\[
\begin{bmatrix}
y \\
\dot{x}_3 
\end{bmatrix} = \begin{bmatrix} x_1 \\
a_1 l_2 x_1 + (a_1 l_1 - a_2) x_2 - a_1 e 
\end{bmatrix}
\tag{18}
\]
- $H_1$ and $H_2$ has interconnection

![Block Diagram](image)

- Stability results: If $l_1, l_2 > 0$, $k_1, k_2, k_3$ are tuned with (12), then there exist a suitable large $\omega_o$, which guarantee LADRC stable.

Another block diagram of LADRC

\[
P(s) = \frac{b}{s^2 + a_1 s + a_2},
\]
\[
C(s) = \frac{1}{s} \frac{a_1 (k_1 l_2 + k_2 l_1 + k_3) s^2 + (k_2 l_2 + k_3 l_1) s + k_3 l_2}{b(s^2 + (k_1 + l_1) s + k_1 l_1 + k_2 + l_2)},
\]
\[
C_3(s) = \frac{s^3 + k_1 s^2 + k_2 s + k_3}{(k_1 l_2 + k_2 l_1 + k_3) s^2 + (k_2 l_2 + k_3 l_1) s + k_3 l_2}.
\]

- \( C(s) \) is a PI-like controller
For a two-order LTI plant, LADRC is a PI-like controller which guarantee stability. It can be used to eliminate the steady-state error for step reference, while the parameters $a_1$, $a_2$ are unknown. These results can be extended to high-order plants.
Thank You!

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