Linear Active Disturbance Rejection Control: Stabilizing Unmodeled Plants By Tuning Parameters

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Outline

INTRODUCTION

In transportation studies, there are many feedback control problems.

- Ex.1: Feedback control of traffic flow
- Ex.2: Cruise-control of a car
- Ex.3: Suspension in a car

Two ways to control

- Way 1: Based on a precise mathematical model
  - Ex.1 Capacity drop model of traffic flow
  - Ex.2 Mathematical model of cruising car

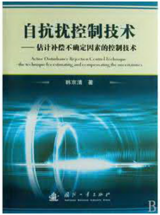
$$\ddot{x} + \frac{b}{m}\dot{x} = \frac{u}{m} \tag{1}$$

  - To identify the model parameters may be expensive and time-consuming
- Way 2: Based on knowledge and tuning parameters
  - PID controller (Proportional-Integral-Derivative)
  - "Data-driven"

Active disturbance rejection control

- Two flaws of PID
  - D may amplify the disturbance
  - I may lead to instability
- Active disturbance rejection control (ADRC) is proposed by Prof. Jingqing Han

Prof. Jingqing Han (1937-2008)



Prof. Han's book

From ADRC to LADRC

- Active disturbance rejection control (ADRC) is a practical control technology.
  - stabilizing plant by tuning several parameters
  - suitable for uncertain, nonlinear, or time-varying plants
- Linear ADRC (LADRC) is a linear version of ADRC
  - proposed by Prof. Zhiqiang Gao (Cleveland State University, USA)
  - using linear feedback and linear extended state observer (LESO)
  - less parameters to tune
  - widely discussed and applied in Chinese control community
  - see: Huang Y, Xue W. Active disturbance rejection control: methodology and theoretical analysis[J]. ISA Transac- tions, 2014, 53(4):963 – 976

In this presentation

- To introduce LADRC for second-order LTI plant
- To propose a classical control prospective and some latest stability results of LADRC

PROBLEM SETTING AND LADRC

The plant and the problem

- Consider the second-order SISO plant

$$\ddot{y} = -a_1\dot{y} - a_2 y + bu, \quad (2)$$

  - a_1, a_2 are unknown, while b is known
- The problem is to let y track a reference signal r, which is typically a step siganl

The extended state

- Re-write (2) in state space, we have

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_2 x_1 - a_1 x_2 + bu, \end{cases} \tag{3}$$

$$y = x_1, \tag{4}$$

- introduces *extended state* $x_3 = -a_2 x_1 - a_1 x_2$
- re-writes (3-4) as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 + bu, \\ \dot{x}_3 = \dot{x}_3, \end{cases} \tag{5}$$

$$y = x_1. \tag{6}$$

Tracking Differentiator (TD)

- LADRC using TD to improve transient performance, which is a signal satisfying

$$\dot{v}_1 = v_2, \tag{7}$$

$$\lim_{t \to +\infty} (r - v_1) = 0. \tag{8}$$

LESO and LADRC

- LESO

$$\begin{cases} \dot{\hat{x}}_1 = k_1(y - \hat{x}_1) + \hat{x}_2, \\ \dot{\hat{x}}_2 = k_2(y - \hat{x}_1) + \hat{x}_3 + bu, \\ \dot{\hat{x}}_3 = k_3(y - \hat{x}_1), \end{cases} \tag{9}$$

  where k_1, k_2, k_3 are the parameters to be determined.

- Controller

$$u = \frac{1}{b}(l_2(v_1 - \hat{x}_1) + l_2(v_2 - \hat{x}_2) - \hat{x}_3), \tag{10}$$

- The plant (3-4), LESO (9), and controller (10) are continuous-time LADRC.

Parameters and the bandwidth method

- let $l_1, l_2 > 0$. For example(Gao 2003), suppose $\omega_c > 0$ is a positive parameter called "controller bandwidth" and let

$$l_1 = 2\omega_c, \quad l_2 = \omega_c^2. \tag{11}$$

- chose a tuning parameter $\omega_o > \omega_c$ called "observer bandwidth" and let

$$k_1 = 3\omega_o, \quad k_2 = 3\omega_o^2, \quad k_3 = \omega_o^3. \tag{12}$$

STABILITY OF LADRC

The subsystem of LESO error

- Let the error of LESO be $\tilde{x}_i = x_i - \hat{x}_i,\ i = 1, 2, 3$, we have

$$\begin{cases} \dot{\tilde{x}}_1 = -k_1 \tilde{x}_1 + \tilde{x}_2, \\ \dot{\tilde{x}}_2 = -k_2 \tilde{x}_1 + \tilde{x}_3, \\ \dot{\tilde{x}}_3 = -k_3 \tilde{x}_1 + \dot{x}_3. \end{cases} \tag{13}$$

- let

$$\tilde{r} = l_2 v_1 + l_1 v_2, \tag{14}$$

$$\nu = l_2 \tilde{x}_1 + l_1 \tilde{x}_2 + \tilde{x}_3, \tag{15}$$

$$e = \tilde{r} + \nu. \tag{16}$$

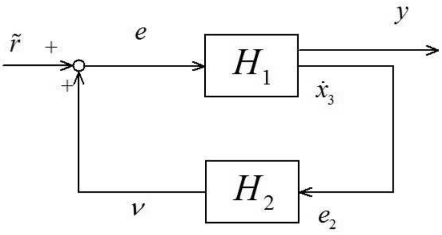
- Look (13) and (15) as a subsystem H_2 with input $\dot{x}_3$ and output ν

- Substituting controller (10) into (5), and look e and $[y, \dot{x}_3]^T$ as the input and output, we have a single-input-double-output subsystem

$$H_1 : \qquad \begin{cases} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -l_2 x_1 - l_1 x_2 +, e \end{cases} \tag{17}$$

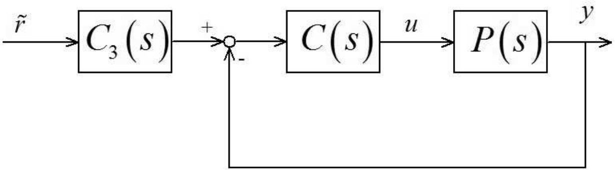
$$\begin{bmatrix} y \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ a_1 l_2 x_1 + (a_1 l_1 - a_2) x_2 - a_1 e \end{bmatrix} \tag{18}$$

- H_1 and H_2 has interconnection



- Stability results: If $l_1, l_2 > 0$, k_1, k_2, k_3 are tuned with (12), then there exist a suitable large ω_o, which guarantee LADRC stable.
- Proof details: See Huiyu Jin, Lili Liu, Jianping Zeng. Stability of Sampled-Data Linear Active Disturbance Rejection Control: A Classical Control Approach[C]. Proceedings of 13th IEEE International Conference on Control and Automation (ICCA), Ohrid, Macedonia, July 3-6, 2017.

Another block diagram of LADRC



$$P(s) = \frac{b}{s^2 + a_1 s + a_2},$$

$$C(s) = \frac{1}{s}\frac{(k_1 l_2 + k_2 l_1 + k_3)s^2 + (k_2 l_2 + k_3 l_1)s + k_3 l_2}{b[s^2 + (k_1 + l_1)s + k_1 l_1 + k_2 + l_2]},$$

$$C_3(s) = \frac{s^3 + k_1 s^2 + k_2 s + k_3}{(k_1 l_2 + k_2 l_1 + k_3)s^2 + (k_2 l_2 + k_3 l_1)s + k_3 l_2}.$$

- $C(s)$ is a PI-like controller

CONCLUSIONS

- For a two-order LTI plant, LADRC is a PI-like controller which guarantee stability.
- It can be used to eliminate the steady-state error for step reference, while the parameters a_1, a_2 are unknown.
- These results can be extended to high-order plants

Thank You!

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