

# MEASURING FIRST-IN-FIRST-OUT VIOLATION AMONG VEHICLES

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## **ABSTRACT**

In reality, First-In-First-Out (FIFO) principle for vehicles on a link or path is often violated due to heterogeneity in drivers' behavior and traffic conditions on different lanes. It is important to understand how serious such FIFO violation can be, since the assumption of FIFO has been an important foundation for developing many dynamic traffic assignment strategies and traffic flow models. In this paper, we first present a measurement of FIFO violation among vehicles and then theoretically show that this measurement is well defined. We further apply this measurement to study scenarios generated by a microscopic traffic simulator and find that FIFO violation is linear to travel time or distance. This study can serve as a springboard for future studies on FIFO violation in real traffic, and the FIFO violation measurement can be used to calibrate and validate traffic flow models and dynamic traffic assignment models.

## 1. INTRODUCTION

The First-In-First-Out (FIFO) principle that vehicles on a link travel in the fashion of First-In-First-Out “on average” [1] has been an important assumption underlying many traffic flow models and dynamic traffic assignment models. In the dynamic traffic assignment literature, there are extensive discussions on how to enforce link FIFO for different dynamic link performance functions [1]. FIFO principle has also been incorporated when developing network-loading models, such as the dynamic network loading model [2] and the cell transmission model [3]. In addition to FIFO for vehicles on a link, one can also think that vehicles using different paths connecting the same origin-destination pair follow FIFO principle in the Wardrop’s user equilibrium [4].

In reality, it is often observed that one vehicle overtaking another on a multilane road. That is, absolute FIFO violation should exist. However, whether FIFO violation is local and tentative or system-wide and persistent is still an open problem, since people can have two types of contradictory experience. In the case when an aggressive driver keeps overtaking slow vehicles, there is system-wide and persistent FIFO violation, which increases with the distance/time traveled. Nonetheless, in the case when, usually in congested traffic, a vehicle just overtaken by another vehicle catches up the latter again, FIFO violation is local and tentative; i.e., FIFO violation should be zero in the long run. To determine the degree of FIFO violation, one has to first define a FIFO violation measurement, which is essential to understand how seriously the FIFO principle is violated. Understanding how serious FIFO violation can have important implications on a more realistic definition of dynamic user equilibrium on a road network. For example, if the difference in the arrival times of vehicles on the same path departing at the same time is about 6 minutes after 60 minutes’ travel due to FIFO violation among vehicles using the same path, then it is not realistic to require that in user equilibrium vehicles departing at the same time on different paths arrive at the same time. In addition, by measuring FIFO violation in real traffic, we can better validate and improve traffic flow models regarding overtaking behaviors.

In the literature, there exist a number of systematic studies on FIFO violation. Jin and Jayakrishnan [5] defined measurements of FIFO violation among commodities or groups of vehicles for situations where two types of vehicles, e.g. white and black cars, are initially separated but mix with each other after some time. With this measurement, it was shown that a commodity-based kinematic wave model developed in [6] violates but converges to FIFO. Jin [7] defined a FIFO violation measurement among path traffic, from which a dynamical process was derived to find user equilibrium.

In this study, we are interested in FIFO violation among vehicles on a link or a path based on vehicles’ trajectories, which can be obtained with technologies such as vehicle re-identification [8] and Automated Vehicle Identification (AVI) [9]. We first propose a measurement of FIFO violation by considering the change in relative orders of vehicles passing two locations. We then discuss the properties of the measurement and further apply the measurement to study simulated scenarios. Note that such FIFO violation is not the same as variation in travel times, which has been widely used as a measurement of the reliability of a transportation network. The reason is that travel time variation depends more on variations in traffic conditions caused by capacity constraints while FIFO violation is more related to heterogeneity in traffic conditions on different lanes and drivers’ behavior. For example, we expect to observe more overtaking and higher FIFO violation on a road due to the increasing proportion of aged drivers, who usually have longer reaction time and lower aggression.

The rest of the paper is organized as follows. In Section 2, we define a measurement of FIFO violation among vehicles. In Section 3, we derive some properties of the measurement theoretically. In Section 4, we study FIFO violation for some simulated scenarios. Finally, in Section 5, we discuss future research directions.

## 2. A MEASUREMENT OF FIFO VIOLATION AMONG VEHICLES

As shown in Figures 1(a) and 1(b), the white car overtakes the leading black car on a multilane road, and the overtaking process can be divided into three stages: (i) the two cars get closer to each other, (ii) the white car overtakes the black car, and (iii) the two cars segregate from each other. Moreover, the distances between the two cars in Figure 1(b) are both larger before and after overtaking than in Figure 1(a). Thus, it is reasonable to say that the FIFO violation is more serious in the scenario shown in Figure 1(b). In contrast, there is no FIFO violation for scenarios shown in Figures 1(c) and 1(d), although they could be considered as stages of overtaking processes. Different from scenarios in Figures 1(a) and 1(b), scenarios in Figures 1(c) and 1(d) could occur on a single-lane road with the existence of a shock or rarefaction wave respectively [10,11]. In general, FIFO violation can be caused by aggressive drivers actively switching to less congested lanes, by the difference in traffic conditions between lanes, or by control measurements such as traffic lights, but it is hard to distinguish what proportion of FIFO violation is caused by each of these reasons. Here the only criterion of the occurrence of FIFO violation is whether the order of vehicles changes within the studied section.

### 2.1 Definition of FIFO violation measurement

We define a measurement of FIFO violation by comparing vehicles' passing times at two locations,  $x_1$  and  $x_2$ . We denote  $N$  studied vehicles by  $n = 1, \Lambda, N$ , their times passing  $x_1$  and  $x_2$  by  $t(n, x_1)$  and  $t(n, x_2)$  respectively, and their passing orders by  $z(n, x_1)$  and  $z(n, x_2)$  respectively. We assume a one-to-one relationship between the identity and order of a vehicle; i.e.,  $z(n, x)$  is a one-to-one function of  $n$ . In addition, we denote the time for the  $z$ th vehicle to pass location  $x$  by  $t_z(z, x)$ , to differentiate from  $t(n, x)$ , the passing time of vehicle  $n$ .

As we know, there is FIFO violation when  $z(n, x_1) \neq z(n, x_2)$ ; i.e., when the order of vehicle  $n$  is changed. If taking  $x_1$  as a reference point, we can switch the orders of vehicles at  $x_2$  to obtain an ideal FIFO situation, in which the ideal FIFO time for vehicle  $n$  passing  $x_2$  can be obtained as

$$\bar{t}(n, x_2) = t_z(z(n, x_1), x_2), \quad (1)$$

where  $t_z(z, x_2)$  denotes the passing time of vehicle at order  $z$ , but may not be vehicle  $z$ . Symmetrically, the ideal FIFO passing time of vehicle  $n$  at  $x_1$  is defined based on the preservation of the order of vehicles at  $x_2$

$$\bar{t}(n, x_1) = t_z(z(n, x_2), x_1). \quad (2)$$

Thus we have for two vehicles  $n$  and  $m$

$$\begin{aligned} t(n, x_1) \leq t(m, x_1) \text{ if and only if } \bar{t}(n, x_2) \leq \bar{t}(m, x_2), \text{ and symmetrically} \\ t(n, x_2) \leq t(m, x_2) \text{ if and only if } \bar{t}(n, x_1) \leq \bar{t}(m, x_1) \end{aligned} \quad (3)$$

Based on the ideal FIFO passing times of vehicles at two locations, we have the following definition of FIFO violation on time among vehicles in the section from  $x_1$  to  $x_2$ .

**Definition 1. (FIFO violation)** FIFO violation on time among vehicles in the section from  $x_1$  to  $x_2$  is given by

$$J_t(x_1, x_2) = \frac{\sum_{n=1}^N (|t(n, x_2) - \bar{t}(n, x_2)| + |t(n, x_1) - \bar{t}(n, x_1)|)}{2N}. \quad (4)$$

Let's use an example shown in Table 1 to further explain the above definition and show how to compute FIFO violation. From the actual time for ten vehicles passing two detectors, we can see that the order of vehicles is not conserved in the section, and there exists FIFO violation. As we know, if there is no FIFO violation, the order of vehicles stays the same. That is, at detector 2, the order should be 1 to 10 without FIFO violation. Then we can compute the ideal FIFO passing times for vehicles passing detectors 1 and 2. For example, the ideal passing time of vehicle 3 is 23 min at detector 2 and 13 min at detector 1. According to Equation 4, the FIFO violation in this section is calculated as the average of the differences between the actual passing time and the ideal passing time at both locations and equals 0.8 min.

## 2.2 Some basis properties

We have the following basic properties of the FIFO violation measurement defined by Equation 4. (i) The unit of FIFO violation is the unit of time. FIFO violation is an average for each vehicle, and total FIFO violation can be computed as  $NJ_t(x_1, x_2)$ . (ii) The FIFO violation is symmetric with respect to locations; i.e.,  $J_t(x_1, x_2) = J_t(x_2, x_1)$ . (iii) If there are only two vehicles as scenarios shown in Figures 1(a) and 1(b), then

$$J_t(x_1, x_2) = \frac{|t(1, x_2) - t(2, x_2)| + |t(1, x_1) - t(2, x_1)|}{2}, \quad (5)$$

which yields larger FIFO violation for the scenario in Figure 1(b), as expected. (iv) The FIFO violation is not additive with respect to location; i.e., for three locations  $x_1 < x_2 < x_3$ ,

$J_t(x_1, x_2) + J_t(x_2, x_3)$  usually is not equal to  $J_t(x_1, x_3)$ . As shown in Figure 2, the former can be bigger as in Figure 2(a) or smaller as in Figure 2(b). (v) As in [6], we can also define a measurement of FIFO violation on locations by comparing the order in locations of vehicles at two time instants. (vi) In reality, we may only be able to obtain trajectories of a subset of vehicles through AVI technologies [9] such as FastTrak in California, TranStar in Texas, E-Z Pass in New Jersey and so on. In these cases, the FIFO violation measurement can be still defined with trajectories of only a portion of vehicles, and it is possible that the FIFO violation computed from a subset of vehicles is representative for all vehicles.

Further, we can simplify the computation in Equation 4 as follows.

$$\textbf{Theorem 1. } J_t(x_1, x_2) = \frac{\sum_{n=1}^N |t(n, x_2) - \bar{t}(n, x_2) - t(n, x_1) + \bar{t}(n, x_1)|}{2N}.$$

*Proof.* Case 1. When  $z(n, x_2) > z(n, x_1)$ , vehicle  $n$  is relatively late arriving at location  $x_2$ . Since in FIFO solutions it should arrive earlier,  $\bar{t}(n, x_2) = t_z(z(n, x_1), x_2) < t(n, x_2)$ . Similarly, at location  $x_1$ , we have  $\bar{t}(n, x_1) = t_z(z(n, x_2), x_1) > t(n, x_1)$ . Thus

$$|t(n, x_2) - \bar{t}(n, x_2)| + |t(n, x_1) - \bar{t}(n, x_1)| = t(n, x_2) - \bar{t}(n, x_2) - t(n, x_1) + \bar{t}(n, x_1).$$

Case 2. When  $z(n, x_2) < z(n, x_1)$ , we have  $\bar{t}(n, x_2) = t_z(z(n, x_1), x_2) > t(n, x_2)$  and  $\bar{t}(n, x_1) = t_z(z(n, x_2), x_1) < t(n, x_1)$ . Thus

$$|t(n, x_2) - \bar{t}(n, x_2)| + |t(n, x_1) - \bar{t}(n, x_1)| = -(t(n, x_2) - \bar{t}(n, x_2) - t(n, x_1) + \bar{t}(n, x_1)).$$

Case 3. When  $z(n, x_2) = z(n, x_1)$ , we have  $\bar{t}(n, x_2) = t_z(z(n, x_1), x_2) = t(n, x_2)$  and  $\bar{t}(n, x_1) = t_z(z(n, x_2), x_1) = t(n, x_1)$ . Thus  $|t(n, x_2) - \bar{t}(n, x_2)| + |t(n, x_1) - \bar{t}(n, x_1)| = 0$ .

We can see that, in all the three possible cases,

$$|t(n, x_2) - \bar{t}(n, x_2)| + |t(n, x_1) - \bar{t}(n, x_1)| = |t(n, x_2) - \bar{t}(n, x_2) - t(n, x_1) + \bar{t}(n, x_1)|. \quad (6)$$

Therefore, from Equation 4, we obtain

$$J_t(x_1, x_2) = \frac{\sum_{n=1}^N |t(n, x_2) - \bar{t}(n, x_2)| + |t(n, x_1) - \bar{t}(n, x_1)|}{2N} = \frac{\sum_{n=1}^N |t(n, x_2) - \bar{t}(n, x_2) - t(n, x_1) + \bar{t}(n, x_1)|}{2N}. \diamond$$

### 3. REGULATION OF THE FIFO VIOLATION MEASUREMENT

For a FIFO measurement to be meaningful, we expect it to be well regulated. In the following we check its regulation properties theoretically.

#### 3.1 Theoretical properties of the FIFO violation measurement

For a reasonable FIFO violation measurement, we expect there is no FIFO violation when all vehicles follow the FIFO principle. This property is given in the following theorem for the FIFO measurement defined in Equation 4.

**Theorem 2.**  $J_t(x_1, x_2) = 0$  if and only if vehicles follow the FIFO principle. This is equivalent to saying that the order of all vehicles is preserved at two locations, or for any two vehicles  $n$  and  $m$  we have  $t(m, x_1) \leq t(n, x_1) \Leftrightarrow t(m, x_2) \leq t(n, x_2)$ .

*Proof.* The proof is straightforward and omitted here.  $\diamond$

An extreme case for FIFO violation is when all vehicles departing earlier arrive later. We call this scenario as first-in-last-out, where a traffic system works like a stack system. In the following theorem, we demonstrate that FIFO violation attains its maximum in this case.

**Theorem 3.** The most FIFO violation occurs in the case of first-in-last-out.

*Proof.* First, the FIFO violation is independent of the definition of vehicle IDs. Thus, without loss of generality, we assume vehicles arrive at  $x_1$  in the order of their IDs, like in the Table 1.

That is,  $z(n, x_1) = n$ . Second, FIFO violation can be rewritten as:

$$J_t(x_1, x_2) = J_{t,f} + J_{t,b}, \quad (7)$$

where  $J_{t,f} = \frac{\sum_{n=1}^N |t(n, x_2) - \bar{t}(n, x_2)|}{2N}$  is the forward FIFO violation at location  $x_2$  with reference

to vehicles' orders at  $x_1$ ;  $J_{t,b} = \frac{\sum_{n=1}^N |t(n, x_1) - \bar{t}(n, x_1)|}{2N}$  is the backward FIFO violation at location  $x_1$  with reference to vehicles' orders at  $x_2$ . Here we prove that, if we switch the order of any two vehicles with same relative orders at  $x_1$  and  $x_2$ ; i.e., the two vehicles observe relative FIFO, both the forward FIFO violation  $J_{t,f}$  and the backward FIFO violation  $J_{t,b}$  will never decrease.

For any two vehicles  $n_1$  and  $n_2$ ,  $n_1 < n_2$ , we assume their orders at  $x_2$  are  $z_1$  and  $z_2$  respectively, and their orders are preserved from  $x_1$  to  $x_2$ . That is,  $z_1 < z_2$  and  $t(n_1, x_2) < t(n_2, x_2)$ . We then switch the actual passing orders of the two vehicles at  $x_2$ . We can see that their ideal FIFO positions at  $x_2$  are still  $n_1$  and  $n_2$  respectively, and the actual and ideal passing orders of other vehicles are not affected. Therefore, the change in the forward FIFO violation is given by

$$\Delta J_{t,f} = |t_2 - \bar{t}_1| + |t_1 - \bar{t}_2| - |t_2 - \bar{t}_2| - |t_1 - \bar{t}_1|,$$

where  $t_1 = t(n_1, x_2)$ ,  $t_2 = t(n_2, x_2)$ ,  $\bar{t}_1 = \bar{t}_{z_1}(z_1 = n_1, x_2)$ , and  $\bar{t}_2 = \bar{t}_{z_2}(z_2 = n_2, x_2)$ .

We have that  $t_1 < t_2$  and  $\bar{t}_1 < \bar{t}_2$ , then there are the following six cases.

Case 1: When  $t_1 < t_2 \leq \bar{t}_1 < \bar{t}_2$ ,  $\Delta J_{t,f} = -t_2 + \bar{t}_1 - t_1 + \bar{t}_2 + t_2 - \bar{t}_2 + t_1 - \bar{t}_1 = 0$ .

Case 2: When  $t_1 \leq \bar{t}_1 \leq t_2 \leq \bar{t}_2$ ,  $\Delta J_{t,f} = t_2 - \bar{t}_1 - t_1 + \bar{t}_2 + t_2 - \bar{t}_2 + t_1 - \bar{t}_1 = 2(t_2 - \bar{t}_1) \geq 0$ .

Case 3: When  $t_1 \leq \bar{t}_1 < \bar{t}_2 \leq t_2$ ,  $\Delta J_{t,f} = t_2 - \bar{t}_1 - t_1 + \bar{t}_2 - t_2 + \bar{t}_2 + t_1 - \bar{t}_1 = 2(\bar{t}_2 - \bar{t}_1) > 0$ .

Case 4: When  $\bar{t}_1 \leq t_1 < t_2 \leq \bar{t}_2$ ,  $\Delta J_{t,f} = t_2 - \bar{t}_1 - t_1 + \bar{t}_2 + t_2 - \bar{t}_2 - t_1 + \bar{t}_1 = 2(t_2 - t_1) > 0$ .

Case 5: When  $\bar{t}_1 \leq t_1 \leq \bar{t}_2 \leq t_2$ ,  $\Delta J_{t,f} = t_2 - \bar{t}_1 - t_1 + \bar{t}_2 - t_2 + \bar{t}_2 - t_1 + \bar{t}_1 = 2(\bar{t}_2 - t_1) \geq 0$ .

Case 6: When  $\bar{t}_1 < \bar{t}_2 \leq t_1 < t_2$ ,  $\Delta J_{t,f} = t_2 - \bar{t}_1 + t_1 - \bar{t}_2 - t_2 + \bar{t}_2 - t_1 + \bar{t}_1 = 0$ .

We can see that, in all these cases,  $\Delta J_{t,f} \geq 0$ . Therefore, by switching the two order-preserved vehicles at  $x_2$ , the forward FIFO violation  $J_{t,f}$  will not decrease. Similarly, the backward FIFO violation  $J_{t,b}$  will not decrease after the orders of two vehicles are reversed. Thus, when the relative orders of vehicles are not reverse at two locations, we can switch their orders at  $x_2$  until we finally arrive at the state of first-in-last-out. Since FIFO violation does not decrease in these processes, it reaches its maximum at first-in-last-out.  $\diamond$

Note that, however, first-in-last-out may not be the only case when FIFO violation attains its maximum. For example, for three vehicles departing location 1 at minute 1, 2, and 3 respectively, they arrive at location 2 at minute 6, 5, 4 in case 1, and at minute 6, 4, 5 in case 2. We can see that case 1 is first-in-last-out, but case 2 is not since the orders of vehicles 2 and 3 are preserved. However, in both case, the FIFO violation for each vehicle is the same at  $\frac{4}{3}$  minutes.

The average travel time from  $x_1$  to  $x_2$  is  $ATT(x_1, x_2) = \frac{\sum_{n=1}^N (t(n, x_2) - t(n, x_1))}{N}$ . In the

following, we show that the FIFO violation is bounded by the average travel time.

**Theorem 4.**  $J_t(x_1, x_2) \leq ATT(x_1, x_2)$ .

*Proof.* Theorem 3 says that the most FIFO violation occurs in the case of first-in-last-out, where the positions of the pair of vehicles  $m$  and  $n$  ( $m + n = N + 1$ ) are switched. That is,  $z(m, x_1) = z(n, x_2)$  and  $z(m, x_2) = z(n, x_1)$ . We then have

$$\bar{t}(n, x_2) = t_z(z(n, x_1), x_2) = t_z(z(m, x_2), x_2) = t(m, x_2)$$

and

$$\bar{t}(n, x_1) = t_z(z(n, x_2), x_1) = t_z(z(m, x_1), x_1) = t(m, x_1).$$

Similarly,  $\bar{t}(m, x_2) = t(n, x_2)$  and  $\bar{t}(m, x_1) = t(n, x_1)$ . Without loss of generality, we can assume that  $z(m, x_1) > z(m, x_2)$ . That is, vehicle  $m$  overtakes vehicle  $n$ . Thus  $t(n, x_2) \geq t(m, x_2)$  and  $t(n, x_1) \leq t(m, x_1)$ , which leads to

$$\begin{aligned} & |t(n, x_2) - \bar{t}(n, x_2) - t(n, x_1) + \bar{t}(n, x_1)| + |t(m, x_2) - \bar{t}(m, x_2) - t(m, x_1) + \bar{t}(m, x_1)| \\ &= |t(n, x_2) - t(m, x_2) - t(n, x_1) + t(m, x_1)| + |t(m, x_2) - t(n, x_2) - t(m, x_1) + t(n, x_1)| \\ &= 2t(n, x_2) - 2t(m, x_2) - 2t(n, x_1) + 2t(m, x_1) \leq 2t(n, x_2) - 2t(n, x_1) + 2t(m, x_2) - 2t(m, x_1) \end{aligned}$$

Hence we obtain

$$J_t(x_1, x_2) \leq \frac{\sum_{n=1}^N (t(n, x_2) - t(n, x_1))}{N} = ATT(x_1, x_2).$$

That is, the FIFO violation is never larger than the average travel time.

Further, we show that theoretically FIFO violation can be as large as average travel time. For example, for two vehicles,  $t(1, x_1) > t(2, x_1)$  and  $t(1, x_2) < t(2, x_2)$ , the FIFO violation is:

$$J_t(x_1, x_2) = \frac{|t(2, x_2) - t(1, x_2)| + |t(1, x_1) - t(2, x_1)|}{2} = \frac{t(2, x_2) - t(1, x_2) + t(1, x_1) - t(2, x_1)}{2},$$

and average travel time as

$$ATT(x_1, x_2) = \frac{t(1, x_2) - t(1, x_1) + t(2, x_2) - t(2, x_1)}{2}$$

Assuming that vehicle 1 runs at a constant speed, but the speed of vehicle 2 is near 0, i.e.,  $t(2, x_2) \rightarrow \infty$ , then we have

$$\frac{J_t(x_1, x_2)}{ATT(x_1, x_2)} = \frac{t(2, x_2) - t(1, x_2) + t(1, x_1) - t(2, x_1)}{t(1, x_2) - t(1, x_1) + t(2, x_2) - t(2, x_1)} \rightarrow 1.$$

Thus, the average travel time is actually the supreme of the FIFO violation.  $\diamond$

### 3.2 Normalized FIFO violation

From the theorems in the preceding subsection, we can see that the FIFO violation measurement has expected properties and is well regulated. According to Theorem 4, we can normalize the FIFO violation with respect to the average travel time as follows.

**Definition 2. (Normalized FIFO violation)** The normalized FIFO violation can be defined as:

$$\mathcal{G}(x_1, x_2) = \frac{J_t(x_1, x_2)}{ATT(x_1, x_2)}. \quad (8)$$

From Theorem 4, we have  $0 \leq \mathcal{G}(x_1, x_2) \leq 1$ . Since the average travel time is highly related to the average traffic conditions on all lanes, the normalized FIFO violation can emphasize the heterogeneity in lane conditions and drivers' behavior. Note that, since the FIFO violation in the case of first-in-last-out is not straightforward to compute, we do not normalize the FIFO violation with respect to it.

The average travel time function is for a section of a road and approaches 0 when the two locations get closer. It can be written as  $ATT(x_1, x_2) = \int_{x_1}^{x_2} \Lambda(x) dx$ , where

$$\Lambda(x) = \frac{\sum_{n=1}^N 1}{N v(n, x)},$$
 and  $v(n, x)$  is the speed of vehicle  $n$  at location  $x$ . The function  $\Lambda(x)$  can

be considered as the average tardity or latency [12]. In this sense, the average travel time is differentiable with respect to location. In contrast, the FIFO violation function is not additive and therefore not differentiable with respect to location. Thus, we have to indicate two locations whenever computing FIFO violation.

## 4. APPLICATION TO SIMULATED SCENARIOS

In this section, we apply the measurement of FIFO violation to study several scenarios generated by Paramics simulator.

### 4.1. Simulation set-up

Paramics is a suite of microscopic simulation tools used to model the movement and behavior of individual vehicles on urban and highway road networks [13]. The car-following and lane-changing models are principally based on the model in [14]. Each simulated vehicle is regarded as a Driver Vehicle Unit (DVU), which has four major driving behavior parameters: target headway, driver's reaction time, aggression, and awareness.

- (1) The target headway describes the target time distance that a DVU needs to maintain when following a leading car. If the DVU wants to make a lane-change, a gap must exist both in front of and behind the position it would occupy and be at least as large as the target headway for the DVU [15];
- (2) The reaction time determines how fast a DVU reacts to changes in environment traffic conditions;
- (3) A high aggression value will cause drivers to accept a smaller headway when following a car or switching to another lane. Usually, vehicles with higher aggression values have a higher traveling speed than those with lower ones;
- (4) A high awareness value will cause a DVU to be aware of a downstream hazardous situation (such as lane drop) earlier and then apply a longer headway in order to give way to merging vehicles.

By default, both the target headway and reaction time of a DVU follow a continuous normal distribution with the mean of 1 second, and the aggression and awareness values are uniformly distributed from 1 to 8.

Although vehicles' overtaking behavior is not explicitly modeled in Paramics, overtaking is allowed, and we can observe the following overtaking phenomena:

- (1) A faster vehicle catches up a slower one on the same lane, switches to another lane, and bypasses the slower one;
- (2) A slower vehicle on the fast lane switches to the right lane to let a faster, following vehicle to bypass itself;
- (3) In some occasions, a slow vehicle followed by several fast vehicles will not change its lane if it could not find an acceptable gap on its adjacent lanes.

The base scenario in this study is a 2-lane highway link of 60 miles with the speed limit of 50 mph. The network has only one OD pair with the demand of 2000 veh/hr, and traffic is in a free flow state. In order to compute FIFO violation, we place 120 detectors along the highway, one every half a mile. The position of detector  $i$  ( $i = 1, \Lambda, 120$ ) is denoted by  $x_i$ . Using Paramics's strong API programming capability, a plug-in is developed to capture the passing times of all vehicles at each detector. We first carefully study FIFO violation for the base scenario and then investigate the FIFO violation for (i) different market penetration rates, (ii) different number of lanes, (iii) different aggression values of vehicles, and (iv) different traffic demand levels. Here the unit of FIFO violation and average travel time is minute, and that of distance is mile.

### 4.3 Result and analysis

For the base scenario, Figure 3 shows a bimodal distribution of travel times, from which we can clearly see the existence of two types of vehicles: one with average travel time of 60 min and the other 80 min. Figure 4(a) shows the relationships between the location and FIFO violation at any location relative to detector 1 at  $x_1$ . Here the dash line represents the sum of FIFO violation among two consecutive detectors, which is smaller than the FIFO violation relative to  $x_1$ , because the FIFO violation is mostly in the form shown in Figure 2(b), and  $J_t(x_1, x_2) + J_t(x_2, x_3) < J_t(x_1, x_3)$ . Figures 4(b-d) show the relationships between the location and the average travel time of all vehicles, between the average travel time of all vehicles and the FIFO violation, and between the standard deviation of travel times (i.e.  $stdTT(x_1, x)$ ) and the FIFO violation. All these four figures follow linear relationships as follows:

(a) Figure 4(a):  $J_t(x_1, x) = 0.13(x - x_1) - 0.17$

(b) Figure 4(b):  $ATT(x_1, x) = 1.22(x - x_1) - 0.66$

(c) Figure 4(c):  $J_t(x_1, x) = 0.1ATT(x_1, x) - 0.1$

(d) Figure 4(d):  $J_t(x_1, x) = 0.83stdTT(x_1, x) + 0.01$

These linear relationships are statistically significant with R-squared values higher than 0.99. From relationship (c), we can have the normalized FIFO violation  $\mathcal{A}(x_1, x) \approx 0.1$ . From these figures, we can see that FIFO violation increases with the time/distance traveled.

In reality, we might only be able to obtain the trajectory data of a certain portion of vehicles through an AVI system. Here we would like to check the influence of the market penetration rate of such devices on FIFO violation by randomly picking vehicles based on a certain penetration rate. For each penetration rate, five Monte Carlo simulation runs are conducted [13]. The means and standard deviations of speed (mile/min) and normalized FIFO violation at different penetration rates are then computed with respect to 119 detectors. Note that

the mean of speed is computed as  $\frac{\sum_{i=2}^{120} (x_i - x_1) / ATT(x_1, x_i)}{119}$ , which is not vehicle-based average

speed and equals the latter when all vehicles have the same speed. As shown Table 2, there exist no significant differences in the means and standard deviations of speed, and the mean of normalized FIFO violation for 5% market penetration is almost the same as that for 100% market penetration. Although lower penetration rate leads to higher variation in the normalized FIFO violations, these results suggest that FIFO violation computed from a very small portion of vehicles is sufficiently representative. If this is also true in reality, FIFO violation can be obtained with rather low cost. We note that, however, the proper market penetration rate, which yields representative FIFO violation for total traffic, should be obtained from observations in real traffic, and both the mean and standard deviation should be considered in this process.

For different number of lanes and traffic demand proportional to the number of lanes, the normalized FIFO violation and average speed are shown in Table 3. In all scenarios, the density on each lane is almost identical. Thus the number of lanes is the major factor for determining FIFO violation. It is asserted that no overtaking can occur when there is only one lane. However, according to Table 3, the maximum average FIFO violation occurs on the two-lane road. This is contradictory to the intuition that roads with more lanes cause more lane-changes and therefore

overtakings. In the future, we would be interested in a detailed study on this phenomenon with real traffic data.

Table 4 shows eight cases with different aggression values. In each case, we randomly assign aggression value 1 to 50% of vehicles and aggression value  $j$ ,  $j=1,2,\dots,8$  to the other half. We observe that the larger difference of aggression between two group vehicles, the higher FIFO violation, as expected. However, as shown in Table 4, when all vehicles have the same aggression value of 1, the average speed and normalized FIFO violation are higher than those in some cases with mixed aggression values. We suspect that, when all vehicles have the same aggression value, heterogeneity in other behaviors such as vehicles' reaction times dominates the difference in the average speed and FIFO violation.

Table 5 shows simulation results for different demand levels. We observe that higher demands cause lower speeds, and normalized FIFO violation is the largest at the demand of 1500 vehicles/hr. It is consistent to our expectation that both sparse traffic and congested traffic conditions allow smaller FIFO violation, since it is harder to catch up a vehicle at a lower traffic density, and it is also harder to overtake by switching to another lane at higher density.

Note that, the number of simulation runs is one for all the scenarios studied in this subsection. Here we do not perform more simulation runs for the following two reasons. First, when comparing the effect of different factors such as number of lanes on FIFO violation, we would like to fix all other parameters, including the seed number of the random number generator used by Paramics. Second, the purpose of studying FIFO violation for different scenarios is to check whether the measurement of FIFO violation is well defined, and we do not intend to draw any conclusion about real traffic characteristics regarding FIFO violation. Therefore, we limit ourselves with only one simulation run without checking the effect of randomness induced in Paramics.

## 5. CONCLUSION

In this paper, we first defined a measurement of FIFO violation among vehicles and theoretically showed it is well defined. Then with a microscopic traffic simulator, we studied FIFO violation for different road networks, traffic conditions, and drivers' characteristics. In all these uniform traffic flows, FIFO violation is proportional to average travel time. However, since some results for simulated scenarios are consistent with our expectation and some are not, we do not intend to draw any conclusion about real traffic characteristics regarding FIFO violation. Rather, with these applications, we demonstrate that the measurement of FIFO violation is practically well defined and can be effectively calculated from vehicles' trajectories.

The FIFO violation measurement could be an important characteristic of traffic streams at the aggregate level, e.g., as a measurement of level of service [17]. Since it is highly related to heterogeneity in drivers' characteristics and lane distribution of traffic, it could be an indicator of the happening of road rages or even accidents caused by lane-changes or overtakings. In practice, it could be applied to guide the development of policies on overtaking and lane-changing.

In the future, we will be interested in detecting FIFO violation among vehicles in real traffic based on observed trajectories of all or a portion of vehicles. Such data could be obtained from video data by Berkeley Highway Lab [18], which video-tapes all vehicles' trajectories on a half-mile section of Interstate 80 between Ashby Avenue and Powell Street in Emeryville, California, by an FHWA's research project in 1980s [19], or by the FHWA NGSIM project [20]. With the new measurement of FIFO violation proposed in this study, we will be able to compute FIFO violation for different number of lanes, speed limits, traffic conditions, and market penetration.

The current study, together with future studies, on FIFO violation could have fundamental impacts on developments of both traffic flow models and dynamic traffic assignment models. First, the FIFO violation measurement can be used to validate and calibrate lane-changing and overtaking behavior models, so that they can yield FIFO violation consistent with observations. Second, the FIFO violation measurement can also be used to detect the degree of FIFO violation among vehicles of the same origin-destination (O-D) pair. Since the ideal DUO solutions allow no FIFO violation among vehicles of the same O-D pair, better understanding of FIFO violation among vehicles on the same link or of the same O-D pair would allow us to obtain a practical definition of dynamic user equilibrium, in which a certain level of FIFO violation is allowed. Thus, these studies would be helpful for developing dynamic traffic assignment algorithms based on traffic simulators such as Paramics which allows FIFO violation.

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Table 1. An example of FIFO violation

Detector 1			Detector 2		
<b>Veh. ID</b>	Actual time	<i>FIFO time</i>	<b>Veh. ID</b>	Actual time	<i>FIFO time</i>
<b>1</b>	10	<i>10</i>	<b>1</b>	20	<i>20</i>
<b>2</b>	11	<i>11</i>	<b>2</b>	21	<i>21</i>
<b>3</b>	12	<i>13</i>	<b>4</b>	23	<i>24</i>
<b>4</b>	13	<i>12</i>	<b>3</b>	24	<i>23</i>
<b>5</b>	14.5	<i>14.5</i>	<b>5</b>	25.5	<i>25.5</i>
<b>6</b>	15	<i>16</i>	<b>9</b>	27.5	<i>29.5</i>
<b>7</b>	16	<i>17.5</i>	<b>6</b>	27.5	<i>27.5</i>
<b>8</b>	17.5	<i>19</i>	<b>7</b>	29	<i>27.5</i>
<b>9</b>	19	<i>15</i>	<b>8</b>	29.5	<i>29</i>
<b>10</b>	21	<i>21</i>	<b>10</b>	32.5	<i>32.5</i>

Table 2. The means and standard deviations of speed (mile/min) and normalized FIFO violation with respect to 119 detectors v.s. the market penetration rate

$\mu$	1	0.75	0.5	0.25	0.1	0.05	0.025	0.01
Mean of speed	0.8415	0.8413	0.8408	0.8424	0.8434	0.8379	0.8433	0.8564
Std of speed	0.0187	0.0188	0.0187	0.0184	0.0186	0.0198	0.0196	0.0156
Mean of $\mathcal{G}(x_1, x_i)$	0.1010	0.1006	0.1011	0.0994	0.0940	0.0955	0.0820	0.0576
Std of $\mathcal{G}(x_1, x_i)$	0.0091	0.0096	0.0108	0.0132	0.0173	0.0224	0.0249	0.0214

Table 3. Simulation results for different number of lanes

Number of lanes	1	2	3	4
Mean of speed	0.7243	0.8415	0.8511	0.8439
Std of speed	0.0128	0.0187	0.0161	0.0193
Mean of $\mathcal{G}(x_1, x_i)$	0	0.101	0.0952	0.0712
Std of $\mathcal{G}(x_1, x_i)$	0	0.0091	0.0069	0.0037

Table 4. Simulation results for different aggression values

Aggression	1&1	1&2	1&3	1&4	1&5	1&6	1&7	1&8
Mean of speed	0.7518	0.7469	0.7503	0.7965	0.7971	0.7978	0.7982	0.7987
Std of speed	0.0131	0.0149	0.0127	0.0094	0.0101	0.0105	0.0108	0.0112
Mean of $\mathcal{G}(x_1, x_i)$	0.0619	0.0527	0.0569	0.0847	0.0856	0.0862	0.0868	0.0873
Std of $\mathcal{G}(x_1, x_i)$	0.0099	0.0109	0.0092	0.0007	0.0013	0.0017	0.002	0.0024

Table 5. Simulation results for different demand levels

Demand	500	1000	1500	2000	2500	3000	3500	4000
Mean of speed	0.8819	0.8635	0.8501	0.8415	0.8274	0.8207	0.8116	0.8012
Std of speed	0.0159	0.0178	0.02	0.0187	0.0202	0.0184	0.0164	0.012
Mean of $\mathcal{G}(x_1, x_i)$	0.0926	0.1021	0.1057	0.101	0.0975	0.0915	0.0938	0.0884
Std of $\mathcal{G}(x_1, x_i)$	0.0093	0.0081	0.0084	0.0091	0.0075	0.0065	0.0049	0.0043

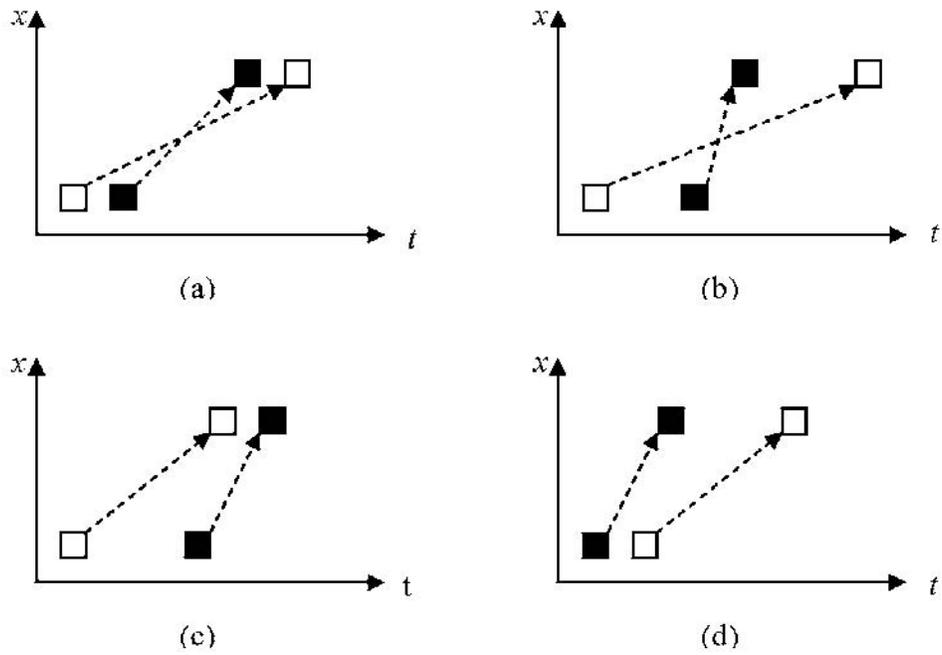


Figure 1. An illustration of First-In-First-Out violation for two cars

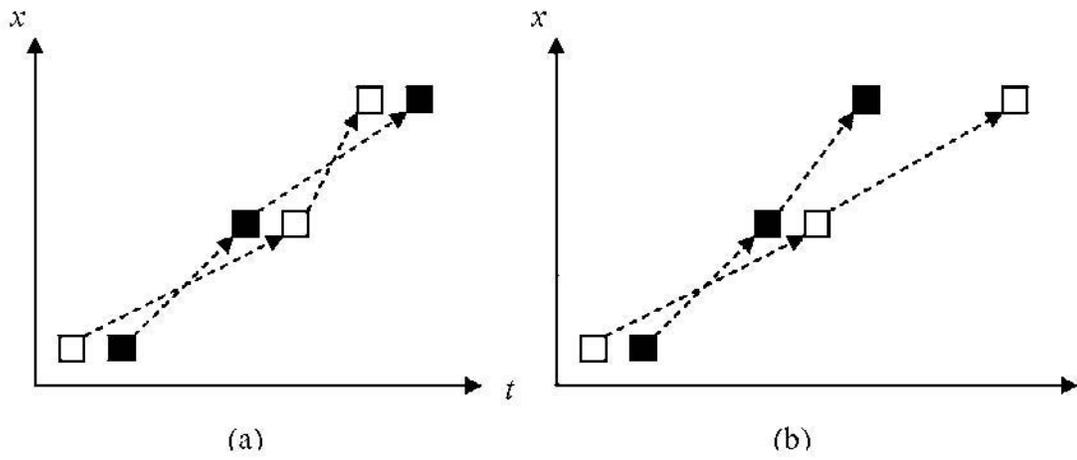


Figure 2. FIFO violation is not additive

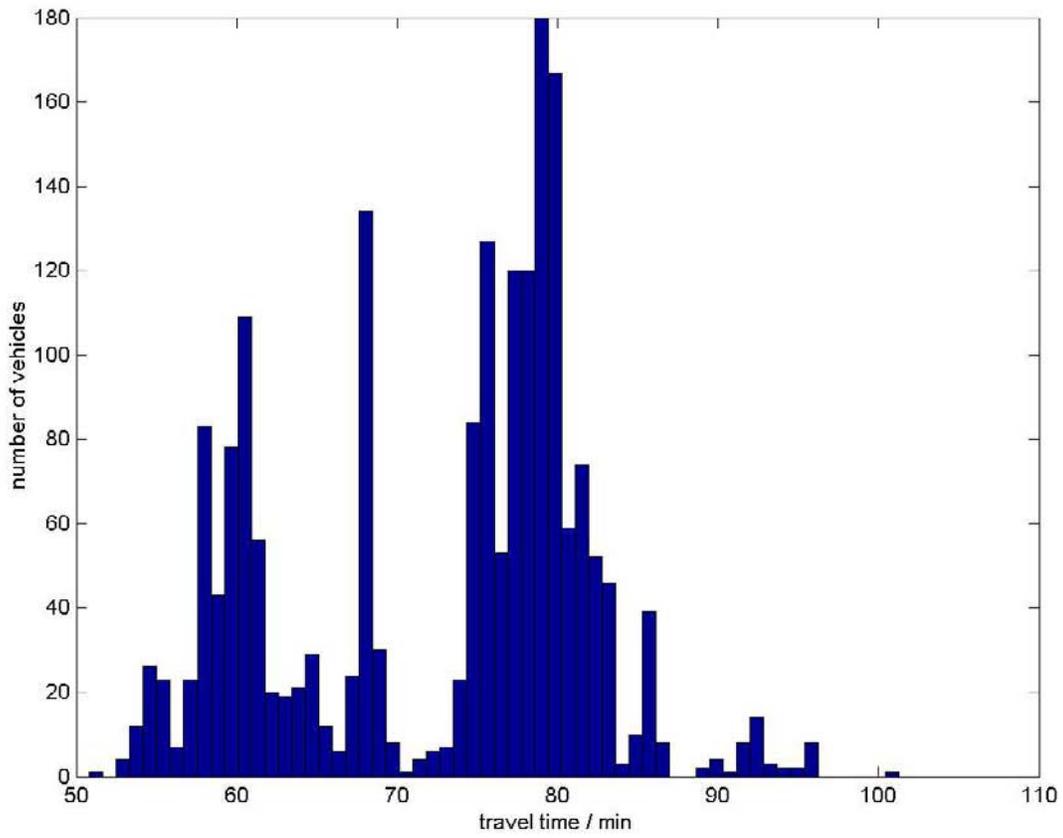


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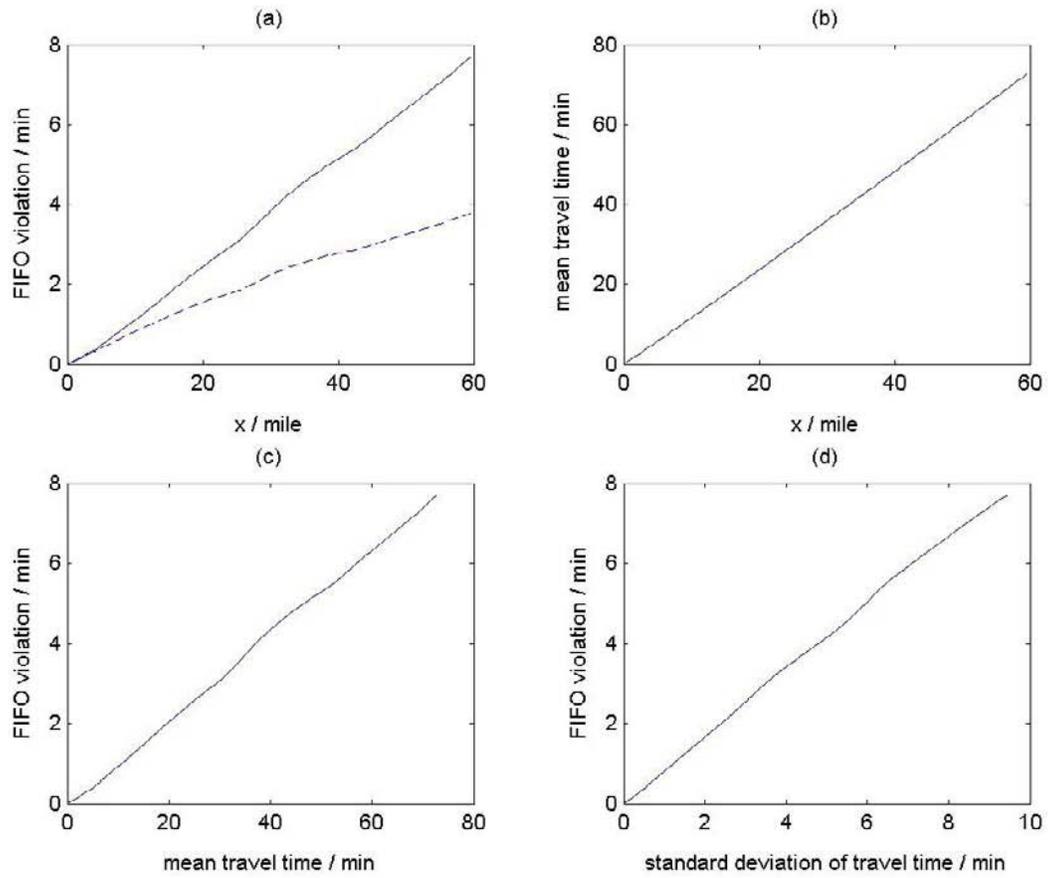


Figure 4. Relationships between the location and FIFO violation or mean travel time, and between FIFO violation and mean travel time or its standard deviation