Prices, capacities and service levels in a congestible Bertrand duopoly*

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Abstract

We study the duopolistic interaction between congestible facilities that supply perfect substitutes and make sequential decisions on capacities and prices, and compare the results to monopoly and first-best outcomes. At the Nash equilibrium prices and capacities, there is more congestion in the duopoly than in the social optimum. Given our assumptions, monopoly pricing and capacity choices result in the same congestion level as the social optimum. The higher congestion level under duopoly is due to strategic price responses to capacity investments. Moreover, higher marginal costs of capacity may increase duopoly profits. Lastly, when capacity is relatively cheap or demand relatively inelastic, stable asymmetric Nash-equilibria may result, where the high-capacity facility offers low time costs at a high price, and the smaller facility offers lower service levels at a lower price. In that case, there is endogenous product differentiation by ex ante identical firms.

Keywords: congestion, price-capacity games, imperfect competition
JEL code: D43, L13, L14, L86, L91, R40

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* Earlier versions of this paper were presented at the AEA-meetings in Philadelphia (January 2005), the Center for the Study of Democracy at UCI (February 2005), the ERSA meetings in Amsterdam (August 2005), and the Department of Economics at the University of Antwerp (May 2005). We thank Robin Lindsey for detailed comments. We are also grateful to Jan Bouckaert, André de Palma, Wilfried Pauwels, Ken Small, Frank Verboven and Bert Willems for additional remarks. The comments of two anonymous referees and the editor are also gratefully acknowledged. Chen Feng Ng provided valuable programming assistance. We acknowledge financial support from the Flemish Fund for Scientific Research and from the U.S. Department of Transportation and the California Department of Transportation, through the University of California Transportation Center.

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1. Introduction

Facilities like seaports, airports, internet access providers, and roads, are prone to congestion. When the volume of simultaneous users increases and capacity is constant, the time cost of using these facilities increases. More generally, the perceived quality of the service provided by a facility may decrease when it gets crowded. Facility management can respond to quality deterioration by changing prices, but also by adapting the capacity of the facility. This paper asks how capacity and price decisions are made for congestible facilities in an oligopolistic market structure, and compares the oligopoly result to the monopoly and the socially optimal outcome. More specifically, we study the duopolistic interaction between congestion-prone facilities that supply perfect substitutes in the framework of a sequential game[1]. The facilities first decide simultaneously on capacities; next, they simultaneously choose prices, given capacity decisions. Prices and capacities jointly determine consumers' time cost of accessing or using a particular facility. The level of service, defined as the inverse of time costs of using a facility, declines with crowding.

The analysis of this paper is relevant to a number of situations. Competition between airports in metropolitan areas (e.g. San Francisco Airport and Oakland Airport in the San Francisco Bay Area) is one example. The airports are congestible, so that service quality declines with the number of passengers and plane movements. If airport management maximizes profits,[2] then price decisions and capacity choices will interact with the service quality (congestion). A second example relates to competition between ports that serve the same hinterland (e.g. the ports of Long Beach and of Los Angeles in Southern California, or the ports of Antwerp and Rotterdam in Western Europe). Here too, port capacities and port charges can be chosen by the port authorities to maximize profits. Competition between internet service providers is another example, although our

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[1] The assumption of perfect substitutes is more appropriate in some applications than in others. For example, it seems quite reasonable in the case of internet access providers, but differences in location make it less appropriate for describing competition between ports or airports (see, e.g., Gillen and Morrison [11] for a model of airport competition with differentiated demand). However, our focus on perfect substitutes simplifies the analysis and has no strong implications for the qualitative results of the paper.

[2] At present, many airports may not act as profit-maximizers, especially in the U.S., as they are constrained by regulation and by long run contracts with carriers (FAA/OST [9]). In a fully deregulated environment, market power deriving from airport congestion may be more likely to accrue to airports than to airlines. Moreover, the interaction between congestion, price and capacity decisions is present when airports maximize a weighted sum of revenues and output.

maintained no entry assumption is less straightforward in this case. The quality of internet service can be measured as a weighted average of (mainly) download speed, upload speed and mail processing speed; the capacity (computing power, disk space and network capacity) that is required to keep quality constant is approximately a linear function of the number of simultaneous users.[3]

The main insights of this paper are the following. First, we find that, at the Nash equilibrium capacities and prices, the service level in a duopolistic market structure is below the socially optimal level. This is not the case for monopoly where, given our assumptions, pricing and capacity choices result in the same service level as the social optimum. Therefore, duopoly implies more congestion than either the monopoly outcome or the first-best optimum. We show that this finding is due to strategic price responses to capacity investments under duopoly. Second, strategic interaction between prices and capacities implies that lower marginal capacity costs may actually reduce duopoly profits. The intuition is that the cost reduction in the provision of capacity reduces costs and raises demand at given prices, but it also intensifies competition and reduces prices, indirectly reducing revenues. Third, the duopoly outcome may yield both symmetric and asymmetric Nash equilibria. Specifically, when capacity costs are low or demand is fairly inelastic, the only stable equilibria are asymmetric. This results in endogenous product differentiation by ex ante identical facilities. Duopolistic interaction by the congested facilities results in a large facility that provides a high service level (i.e., a low time cost) at a high price, and a small facility with a smaller market share that offers lower prices but has more congestion (i.e., a higher time cost).

Our analysis of price and capacity decisions in a homogenous goods duopoly as a sequential game in capacities and prices builds upon earlier literature. Both Braid [5] and Van Dender [21] have studied duopoly pricing decisions of congested facilities, but they do not consider capacity adjustments. De Palma and Leruth [8] do study a two-stage game in capacities and prices; however, they focus on a discrete demand representation (users either consume one or zero units of the good), which does not allow discussing the

[3] Personal communication with Francis Depuydt, Team Manager Integrated Service Platforms, Belgacom.

role of specific model parameters in much detail.[4] Baake and Mitusch [2] develop a model similar to ours, but they focus on the comparison between Cournot and Bertrand models in the pricing stage of the game and do not study the possibility of multiple equilibria. This paper provides a more detailed analysis of Bertrand pricing policies, it pays more attention to the distortion of capacity and the implications for congestion levels in the duopoly case, it contains a detailed numerical illustration of price, capacity and service levels under different market structures, and it analyzes the occurrence of multiple equilibria. Acemoglu and Ozdazgar [1] recently provide a detailed theoretical analysis of competition and efficiency on network markets. Among other things, they show that more competition among oligopolists can reduce efficiency on congested markets, and that pure strategy equilibria may not exist, especially when congestion functions are highly nonlinear. However, they exclusively focus on price competition, and do not consider capacity competition.

Our analysis of prices, capacities and service levels is also related to several strands of the literature in industrial organization. First, Shaked and Sutton [16] build upon Gabszewicz and Thisse [10] to study quality differentiation in a three-stage game of entry decisions and choice of price and quality. Assuming zero production costs, they show that in equilibrium asymmetries naturally arise, because firms offer distinct qualities in order to avoid price competition between products that are too similar. In the current paper, costly capacity endogenously determines service quality through congestion. We show that, depending on capacity costs and demand parameters, both symmetric and asymmetric equilibria may result. Second, consider the literature on sequential capacity and pricing games evolving from the seminal paper by Kreps and Scheinkman [12]. They show that, with an L-shaped marginal cost function and with an efficient capacity-sharing rule, the two-stage capacity-price game yields the same result as a one-stage Cournot game in quantities. Later papers, e.g. Maggi [13], Dastidar [6, 7] and Boccard and Wauthy [4, 5], find that this result does not hold when marginal production costs increase before capacity is reached, or when different sharing rules are used. In the current paper, marginal production costs are constant, but the introduction of

[4] In their model, the Nash equilibrium in capacities will occur where capacities are restricted up to the point of zero consumer surplus.

congestion results in an upward sloping user cost function. Moreover, we focus on interior solutions, so that the distribution of output over the facilities is endogenously determined within the model. Not surprisingly, in this context the two-stage capacity-price game does not reduce to a one-stage Cournot game.[5]

The paper is structured as follows. Section two contains the theoretical analysis. Section three uses a numerical example to clarify the properties of the model and to illustrate the role of various parameters. Section four concludes.

2. Analytical model

This section provides a detailed analysis of the capacity-price game. Both duopolists are assumed to be profit-maximizers.[6] First, the structure of the model and the reduced form demand system are laid out. Then the second stage (price competition) and the first stage (capacity competition) of the duopoly game are analyzed. The duopoly solution is compared to the monopoly outcome and to the social welfare optimum. Note that we delegate technical details to appendices where appropriate.

2.1. Structure of the model and reduced form demands

There are two facilities, *A* and *B*, providing identical services. Consumers' aggregate marginal willingness to pay is described by a downward sloping linear inverse demand function

$$G = \alpha - \beta q = \alpha - \beta\left(q_A + q_B\right) \tag{1}$$

where q_i $(i \in (A, B))$ is the number of simultaneous users of facility i.[7] The generalized price for using a facility, which consists of a money price and a time cost, must be the same at both facilities. The money price is p_A at facility *A* and p_B at facility *B*. The time cost depends on consumers' marginal time cost γ and on congestion, captured by the

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[5] The focus on interior solutions is made to avoid the use of exogenous rationing rules. We return to this assumption in Section 2 below.

[6] Several authors (e.g., Starkie [19]) and Zhang and Zhang [23]) have argued that, next to profit, output may be a relevant partial objective for many airports.

[7] As observed by a referee, a simple way to introduce differentiated demands would be to make β facility-dependent. This leads to straightforward adjustments in the derivations that follow.

ratio between the number of (simultaneous) users $q_i, i \in (A,B)$ and a facility's capacity $K_i, i \in (A,B)$. Congestion can be interpreted literally, as an increase in time costs, or it can be taken to reflect quality of service; this declines as the facility gets crowded. Like de Palma and Leruth [8], we denote the inverse of capacity by R_i, so that the time cost at each facility is $\gamma q_i R_i, i \in (A,B)$. The marginal cost of capacity, $c_i, i \in (A,B)$, is assumed to be constant.

Note that the above specification implies that we assume congestion costs that are linear in the volume-capacity ratio and that, as indicated in the introduction, we focus on interior solutions throughout. The assumption of interiority is restrictive when combined with a linear congestion function, because congestion costs in themselves do not prevent full capacity utilization[8]. However, our assumptions keep the analysis tractable while guaranteeing that both firms produce positive outputs in the duopoly case, and they avoid the use of exogenous rationing rules.

Although the use of a linear congestion function is not uncommon in theoretical work (see, e.g., de Palma and Leruth [8]) and does not affect the qualitative conclusions to be derived from this paper, it does have implications for some of the more specific results. We comment on its impact on the results where appropriate.

For an interior solution, consumer equilibrium requires that generalized prices (the sum of prices and time costs) at both locations are equal to the marginal willingness to pay. The structural form of the demand system can be written as:

$$\begin{aligned} G[q_A + q_B] &= p_A + \gamma q_A R_A \\ G[q_A + q_B] &= p_B + \gamma q_B R_B \end{aligned} \tag{2}$$

where G(.) is given by (1) above. System (2) implicitly defines the reduced form demand functions that express demand at each facility as a function of prices and capacities at both facilities. Using superscript *r* for the reduced form demand functions, they can be written in general as:

[8] Strictly convex congestion functions, such that congestion costs approach infinity when demand approaches capacity, prevent full capacity usage so that interior solutions automatically result. Although for many applications (roads, airports) a convex congestion function is more realistic, we focus on linear functions to simplify the analytical work.

$$
\begin{aligned}
q_A &= q_A^r\left(p_A, p_B, R_A, R_B\right) \\
q_B &= q_B^r\left(p_A, p_B, R_A, R_B\right)
\end{aligned} \tag{3}
$$

To derive the impact of price and capacity changes on demand, we differentiate system (2), write the result in matrix notation and apply Cramer's rule. We obtain:

$$\frac{\partial q_A^r}{\partial p_A} = -\frac{\beta + \gamma R_B}{A} < 0 \tag{4}$$

$$\frac{\partial q_A^r}{\partial p_B} = \frac{\beta}{A} > 0 \tag{5}$$

$$\frac{\partial q_A^r}{\partial R_A} = -\frac{\gamma q_A\left(\beta + \gamma R_B\right)}{A} < 0 \tag{6}$$

$$\frac{\partial q_A^r}{\partial R_B} = \frac{\beta \gamma q_B}{A} > 0 \tag{7}$$

where

$$A = \gamma\left(\gamma R_A R_B + \beta\left(R_A + R_B\right)\right) > 0$$

Recalling that *R* indicates the inverse of capacity, the signs correspond to intuition: ceteris paribus, a higher price at a particular facility reduces demand at that facility and increases demand at the other; more capacity at a facility (i.e., conditional on demand being constant, a better service level) increases demand at that facility and reduces demand at the other.

2.2 Stage two: Nash equilibrium in prices

We take the point of view of facility *A*. Its objective is to maximize profits:

$$\max_{p_A} \pi_A = p_A q_A - \frac{c_A}{R_A}$$

where demand is given by (3) and operational marginal costs of production are normalized to zero. The first-order condition

$$p_A \frac{\partial q_A^r}{\partial p_A} + q_A^r = 0 \tag{8}$$

leads, using (4), to the following pricing rule (one easily verifies that the second-order conditions are satisfied as long as demand is downward sloping):

$$p_A = q_A^r(.)R_A\gamma + q_A^r(.)\gamma\frac{\beta R_B}{\beta+\gamma R_B} \tag{9}$$

A similar expression holds for facility *B*. Expression (9) is conceptually identical to the ones obtained in Braid [5], Verhoef at al. [22], and Van Dender [21].[9] The optimal price, conditional on capacities at both facilities, consists of two components. The first one implies that each facility charges the marginal congestion cost at its facility, i.e. consumers pay for the marginal reduction in quality of service that their presence at the facility imposes on other (simultaneously present) users. The second component is a positive markup; it increases when demand becomes less elastic and when the competing facility is more congestible. Note that, in the Bertrand setting, congestion costs are the only source of market power: with $\gamma=0$, prices are equal to marginal production costs (normalized to zero). Otherwise said, in the absence of congestion costs, the textbook Bertrand paradox is obtained: price equals marginal cost.

The pricing rule (9) gives an implicit representation of the price reaction function of facility *A*, conditional on capacities. By analogy we derive the price reaction function for *B*. Jointly the two reaction functions define the Nash equilibrium prices for given capacities, denoted as $p_A^{NE}\left(R_A,R_B\right)$, $p_B^{NE}\left(R_A,R_B\right)$, respectively. In appendix 1 we show that the price equilibrium is unique and stable. Moreover, we unambiguously obtain:

$$\frac{\partial p_A^{NE}}{\partial R_A} > 0 \tag{10}$$

$$\frac{\partial p_A^{NE}}{\partial R_B} > 0 \tag{11}$$

This says that a marginal capacity *de*crease at facility *A* or at facility *B* raises the Nash-equilibrium prices at *A*. In other words, a more congestible system is characterized by higher Nash-equilibrium prices.

[9] None of these papers study the role of capacity and capacity competition. Verhoef et al. [22] focus on the monopoly case but do allow for nonlinear demands and costs. Similarity of (9) to their result suggests that the expression also holds for more general specifications of demands and costs.

2.3 Stage one: Nash equilibrium in capacities

The first order condition for profit maximization in stage 1 is:

$$\frac{\partial p_A^{NE}}{\partial R_A} q_A^r(.) + p_A \frac{dq_A^r}{dR_A} + \frac{c_A}{R_A^2} = 0 \qquad (12)$$

where

$$\frac{dq_A^r}{dR_A} = \underbrace{\frac{\partial q_A^r}{\partial R_A}}_{<0} + \underbrace{\frac{\partial q_A^r}{\partial p_A}}_{<0} \underbrace{\frac{\partial p_A^{NE}}{\partial R_A}}_{>0} + \underbrace{\frac{\partial q_A^r}{\partial p_B}}_{>0} \underbrace{\frac{\partial p_B^{NE}}{\partial R_A}}_{>0} < 0 \qquad (13)$$

is the total effect of a capacity change in A on demand. It consists of the direct effect, holding prices constant, and indirect effects through Nash equilibrium price adjustments at the pricing stage of the game. The signs of the partial derivatives of the reduced form demand system and of the Nash-equilibrium prices – indicated beneath the expressions – were defined in (4), (5), (6), and in (10) and (11). According to (10), the partial derivative in the first term of (12) is positive. Since quantities, capacities, prices and costs are non-negative, and the left-hand side of (12) equals zero, $\frac{dq_A^r}{dR_A}$, as developed in (13), is negative. This says that the direct effect of capacity on reduced-form demand dominates the indirect effects through price reactions of capacity changes. Hence, marginally increasing R_A – marginally decreasing capacity at *A* – reduces demand at *A*.

Expression (12) basically equates marginal cost and benefit of a capacity change. Note that, combining (12) and (13) and using the first order condition for optimal pricing behavior in *A* (see (9)), condition (12) for optimal capacity choice can be formulated equivalently as follows:

$$p_A \frac{\partial q_A^r}{\partial R_A} + p_A \frac{\partial q_A^r}{\partial p_B} \frac{\partial p_B^{NE}}{\partial R_A} + \frac{c_A}{R_A^2} = 0 \qquad (14)$$

This shows that the decision to supply higher capacity depends on capacity costs per unit (third term), on the extent to which capacity directly raises demand (first term), and on the extent to which it reduces demand via price adjustments by the competitor (second term): higher capacity in *A* reduces the Nash equilibrium price of the competitor *B*, which in turn reduces demand in *A*.

Equation (14) implicitly defines the reaction function in capacity for facility *A*. It explicitly depends on the competitor's capacity, R_B, and on the capacity cost c_A:

$$R_A \equiv R_A^R(R_B, c_A)$$

where the reaction function is denoted by superscript 'R'. Rewriting (14) in implicit form:

$$\psi(R_A, R_B, c_A) = p_A \frac{\partial q_A^r}{\partial R_A} + p_A \frac{\partial q_A^r}{\partial p_B} \frac{\partial p_B^{NE}}{\partial R_A} + \frac{c_A}{R_A^2} = 0$$

and applying the implicit function theorem, we immediately find that a higher capacity costs shifts the reaction function upwards:

$$\frac{\partial R_A^R}{\partial c_A} = -\frac{\psi_{c_A}}{\psi_{R_A}} = -\frac{1}{\psi_{R_A}} \left[\frac{1}{R_A^2} \right] > 0 \qquad (15)$$

Note that ψ_{R_A} is negative by the second order condition for profit maximization in capacity.

The slope of the capacity reaction function is given by:

$$\frac{\partial R_A^R}{\partial R_B} = -\frac{\psi_{R_B}}{\psi_{R_A}} \qquad (16)$$

In general, one expects the sign of this slope to be ambiguous because two opposite forces are at play. More capacity in *B* provides *A* an incentive to defend its market share by responding with a capacity increase as well. The size of this effect will depend on capacity costs. However, higher capacity in *B* reduces Nash equilibrium prices at both facilities. Firm *A* then has an incentive to reduce capacity in order to increase prices (and at the same time deliberately creating extra congestion). However, despite the ambiguity in general, we show in Appendix 2 that, given the linear specifications of demand and cost functions, reaction functions in capacity are highly plausibly downward sloping. For example, we show that the slope will necessarily be negative at a symmetric equilibrium of the two-stage game.

What is the impact of capacity costs on Nash equilibrium capacities and prices? To derive these effects, first note that at a Nash equilibrium of the first stage of the game we have:

$$\begin{aligned} R_A^{NE} &\equiv R_A^R(R_B^{NE}, c_A) \\ R_B^{NE} &\equiv R_B^R(R_A^{NE}, c_B) \end{aligned} \tag{17}$$

Differentiating system (17) then immediately yields:

$$\frac{\partial R_A^{NE}}{\partial c_A} = m \frac{\partial R_A^R}{\partial c_A} \tag{18}$$

$$\frac{\partial R_A^{NE}}{\partial c_B} = m \frac{\partial R_B^R}{\partial c_B} \frac{\partial R_A^R}{\partial R_B} \tag{19}$$

$$\frac{\partial R_B^{NE}}{\partial c_B} = m \frac{\partial R_B^R}{\partial c_B} \tag{20}$$

$$\frac{\partial R_B^{NE}}{\partial c_A} = m \frac{\partial R_A^R}{\partial c_A} \frac{\partial R_B^R}{\partial R_A} \tag{21}$$

where

$$m = \frac{1}{1 - \dfrac{\partial R_A^R}{\partial R_B} \dfrac{\partial R_B^R}{\partial R_A}} .$$

These expressions imply that, if reaction functions are downward sloping and $m > 0$ so as to guarantee stability, an increase in the capacity cost in *A* reduces the Nash equilibrium capacity in *A* and raises it in *B*. Moreover, under these assumptions, (18)-(19) and simple algebra show that a simultaneous increase in capacity costs in both *A* and *B* raises equilibrium values of R_A^{NE} and R_B^{NE}.

Finally, the effect of capacity costs on the Nash equilibrium price at facility *A* is given by:

$$\frac{dp_A^{NE}}{dc_A} = \frac{\partial p_A^{NE}}{\partial R_A} \frac{\partial R_A^{NE}}{\partial c_A} + \frac{\partial p_A^{NE}}{\partial R_B} \frac{\partial R_B^{NE}}{\partial c_A} \tag{22}$$

The overall impact is the sum of two terms; the first one is positive, the second one is negative, because capacity costs in *A* raise Nash equilibrium capacities in *B*. If the direct effects dominate the indirect effects due to capacity adjustments at the other facility, a capacity cost increase induces a facility to raise prices.

We conclude this section with an important remark. Unlike the price reaction functions at stage two, the capacity reaction functions are nonlinear, so that multiple equilibria may result. This issue will be illustrated in the numerical application.

Moreover, stability of equilibria is not guaranteed. Not surprisingly, capacity costs are likely to be crucial in determining stability of equilibria, because they directly affect slopes of the capacity reaction functions. To see this, use (16) to get:

$$\frac{\partial^2 R_A^R}{\partial R_B \partial c_A} = -\frac{\psi_{R_A} \frac{\partial \psi_{R_B}}{\partial c_A} - \psi_{R_B} \frac{\partial \psi_{R_A}}{\partial c_A}}{(\psi_{R_A})^2} \tag{23}$$

Simple algebra shows that $\frac{\partial \psi_{R_B}}{\partial c_A} = 0$ and $\frac{\partial \psi_{R_A}}{\partial c_A} = -\frac{2}{R_A^3} < 0$, so that we have

$$\frac{\partial^2 R_A^R}{\partial R_B \partial c_A} = -\frac{2\psi_{R_B}}{(R_A^3)(\psi_{R_A})^2} \tag{24}$$

If reaction functions are downward sloping then ψ_{R_B}<0 so that higher capacity costs in A raise the slope (i.e., make it less negative); it becomes smaller in absolute value. These findings suggest that, starting from a symmetric stable equilibrium, sufficiently low capacity costs may generate unstable symmetric equilibria. This useful insight will also be illustrated numerically in section 3.2.

2.4 Duopoly, monopoly and the social optimum

The comparison of different market structures provides further insight into the effects of the oligopolistic interaction on which this paper focuses. In this subsection we first derive price and capacity rules for a monopolist and for a social welfare-maximizer, and then provide a detailed comparison of the implications of these rules with those under duopoly.

Assume first that both facilities are operated by a single profit-maximizer. Profits are given by: $\sum_{i=A,B} p_i q_i^r (p_A, p_B, R_A, R_B) - \sum_{i=A,B} \frac{c_i}{R_i}$

and maximized with respect to the two prices and capacity levels. In Appendix 3 we show that the first-order conditions yield, after simple manipulation:

$$p_i = q_i \gamma R_i + \left(q_A + q_B\right)\beta, i \in \{A, B\} \tag{25}$$

$$\frac{1}{R_i} = \left(\frac{\gamma}{c_i}\right)^{1/2} q_i, \quad i \in \{A, B\} \tag{26}$$

According to (25), the price at each facility is the sum of the marginal congestion cost at that location and a term relating to the elasticity of demand. Comparing (9) and (25) it follows that, for given demand and capacity, the elasticity-related markup is higher under monopoly than in the duopoly case. According to (26), capacity – the inverse of R_i – is inversely related to the marginal cost of capacity, it is increasing in the marginal value of time, and it is proportional to demand at the facility. Because the monopolist fully controls all instruments, his choice of capacity does not directly take account of effects on the equilibrium price. This contrasts to the duopoly case, where capacity choices do affect the Nash equilibrium price through strategic interactions (see (14)).

Next, assume the facilities are operated by a welfare-maximizing government. It maximizes the difference between total net surplus and total social costs:

$$\sum_{i=A,B}\left(\int_0^{q_i} G[u]du\right) - \sum_{i=A,B}\left((G-p_i)q_i + \frac{c_i}{R_i}\right)$$

In Appendix 3, we derive the following price and capacity rules.

$$p_i = q_i \gamma R_i, i \in \{A, B\} \tag{27}$$

$$\frac{1}{R_i} = \left(\frac{\gamma}{c_i}\right)^{1/2} q_i, \quad i \in \{A, B\} \tag{28}$$

Social welfare maximization internalizes the externality: the price equals the marginal external congestion cost. The capacity rule is identical to that of the monopoly case (but as it holds at a different price and a different level of demand, the optimal capacity level will be different). This confirms recent findings of Oum et al. [15], who show that this result also holds for nonlinear congestion functions. Moreover, note that our assumptions of constant returns to scale in the provision of capacity and a linear congestion function together imply that optimal pricing and optimal provision of capacity lead to exact cost recovery (see, e.g., Small [17])[10]. This follows because (27) implies total revenues equal

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[10] As shown in Morrison [14] and Zhang and Zhang [23], exact cost recovery is not generally obtained with nonlinear congestion functions.

to $\sum_i p_i q_i = \sum_i q_i^2 \gamma R_i$, and (28) allows us to write total expenditures as $\sum_i \frac{c_i}{R_i} = \sum_i q_i^2 \gamma R_i$.[11] Self-financing facilities imply that the social welfare maximum can be implemented without distortionary taxes, viz. by a combination of congestion tolls and competitive pricing at each facility. The competitive price equals the marginal private production cost at each facility (here normalized to zero).

A more detailed comparison of prices, capacities and service levels under the three market structures leads to a number of observations. First, compare monopoly with the social optimum. One immediately shows that the levels of service (as measured by time costs of using a facility, $\gamma R_i q_i$) are identical under both market structures. To see this, note that the optimal capacity rules (26) and (28) imply that, both under monopoly and at the social welfare optimum, the time cost equals

$$\gamma R_i q_i = (\gamma c_i)^{1/2} \tag{29}$$

Moreover, using (25) and (27), equal service levels then immediately imply that monopoly prices necessarily exceed prices at the social optimum. Finally, consistent with earlier results of Spence [18] it follows that, at his observed output level, the monopolist supplies less capacity than the welfare optimal level.[12] In other words, although the monopoly capacity level is distorted, pricing behavior of the monopolist implies that users face identical congestion levels as at the welfare optimum. Note that this strong result hinges on the linearity of the congestion function: it is not guaranteed for general nonlinear congestion functions.

Second and more importantly, compare the outcomes under duopoly with the other market structures. We obtain that the duopolist unambiguously provides lower service levels: duopoly implies more congestion compared to either monopoly or the

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[11] Total expenditures are $\sum_i c_i / R_i$. Rewriting (28) as $c_i / R_i = \gamma R_i q_i^2$, and substituting in the expression for total expenditures produces the statement in the text.

[12] Spence [18] shows that quality at the monopolists' output level is below (above) the socially optimal level when the second partial derivative of willingness to pay with respect to output and to quality is negative (positive). As noted by a referee, if we treat capacity as the facilities' quality choice variable and given our specifications of demand and congestion, under-provision of capacity by a monopolist immediately follows from this, because the second derivative with respect to demand and capacity is easily shown to equal - γ .

first-best. To show this statement formally, combine the first-order conditions for the optimal price and the optimal capacity level (expressions (8) and (14), respectively) for facility *A*, and use (4), (5) and (6). This leads to:

$$q_A\left[\gamma q_A-\left(\frac{\beta}{\beta+\gamma R_B}\right)\frac{\partial p_B^{NE}}{\partial R_A}\right]=\frac{c_A}{R_A^2} \tag{30}$$

Multiplying both sides by (γR_A^2) and slightly manipulating the result yields:

$$\gamma R_A q_A=\left(\gamma c_A+[Z]\right)^{1/2} \tag{31}$$

where $$[Z]=\frac{\gamma\beta q_A R_A^2}{\beta+\gamma R_B}\frac{\partial p_B^{NE}}{\partial R_A}>0\,.$$

Comparing (29) and (31) shows that the time cost under duopoly will exceed the socially optimal one. Hence users face higher congestion under duopoly.

This finding can be explained by the strategic price responses to capacity changes under duopoly. A capacity increase reduces the generalized cost and, therefore, boosts demand at both facilities, implying that the benefits of a capacity increase at one location partially accrue to the other. This externality is fully internalized in both the social optimum and the monopoly case, but it is not under duopoly. To see the effect of price responses to capacity changes, note that a capacity increase at facility *A* affects not only capacity at the competing facility *B* but also reduces the price there. The price reduction at the competing facility negatively affects demand at *A*. This strategic price response is clearly visible in the term Z appearing in (31). If there were no price response to a capacity increase by the competitor, Z = 0 and the socially optimal service level would result. The price response, however, implies a 'leakage of benefits' of a capacity investment to the competitor. The joint implication of price and capacity choice is lower service levels and higher congestion, as shown by (31). Note that this type of behavior is similar, but not identical, to a "puppy dog" strategy where firms under-invest to soften price competition (e.g. Tirole [20]). Here firms under-provide service levels, i.e., they allow more congestion, in order to be able to raise prices.

Third, with linear demands, duopoly prices will not only structurally but also numerically be between those under monopoly and at the social optimum. Indeed, (8) implies that a duopolist will operate where the price elasticity of reduced-form demand equals minus one, whereas the monopolist operates at an elasticity exceeding one in absolute value. Finally, the higher generalized prices (both higher time costs and higher prices) under duopoly as compared with the social optimum yield lower demand under duopoly. Noting the definition of the time cost (i.e., $\gamma R_i q_i$), lower demand and higher time costs under duopoly jointly imply that duopolists will provide less capacity than in the first-best. The numerical analysis in the next section confirms this finding.

3. Numerical analysis

This section explores the properties of the capacity-price game using parameterized versions of the model analyzed in the previous section. All the scenarios use ex ante symmetric parameterizations. Parameters were chosen to produce reasonable elasticities of demand, but they do not reflect a particular real world empirical example.

In Section 3.1 we discuss a parameterization for which the model produces a single, stable and symmetric Nash equilibrium (the 'central scenario'). This scenario mainly illustrates the results of theoretical analysis, and is accordingly brief. Section 3.2 considers alternative values of marginal capacity costs, and illustrates that ex ante symmetric parameterizations may lead to multiple Nash equilibria.

3.1 Central scenario: duopoly, monopoly and surplus maximization

Table 1 presents the parameters selected for the central scenario (rows 1 through 4) and the results (rows 5 through 21) for the unique and stable Nash-equilibrium of the duopoly game, the monopoly outcome, and the surplus-maximizing solution.[13] Figure 1 depicts the capacity reaction functions of the duopoly game;[14] they are are negatively sloped, and their intersection produces a single, stable and symmetric Nash equilibrium for this particular set of parameters.

[13] In all cases reported, second-order conditions were satisfied.
[14] The price reaction functions are not shown; they are linear and upward-sloping, producing a single and stable Nash equilibrium in prices, as was shown to be a general property in section 2.

Figure 1 Inverse capacity reaction functions for the Nash equilibrium of the central scenario

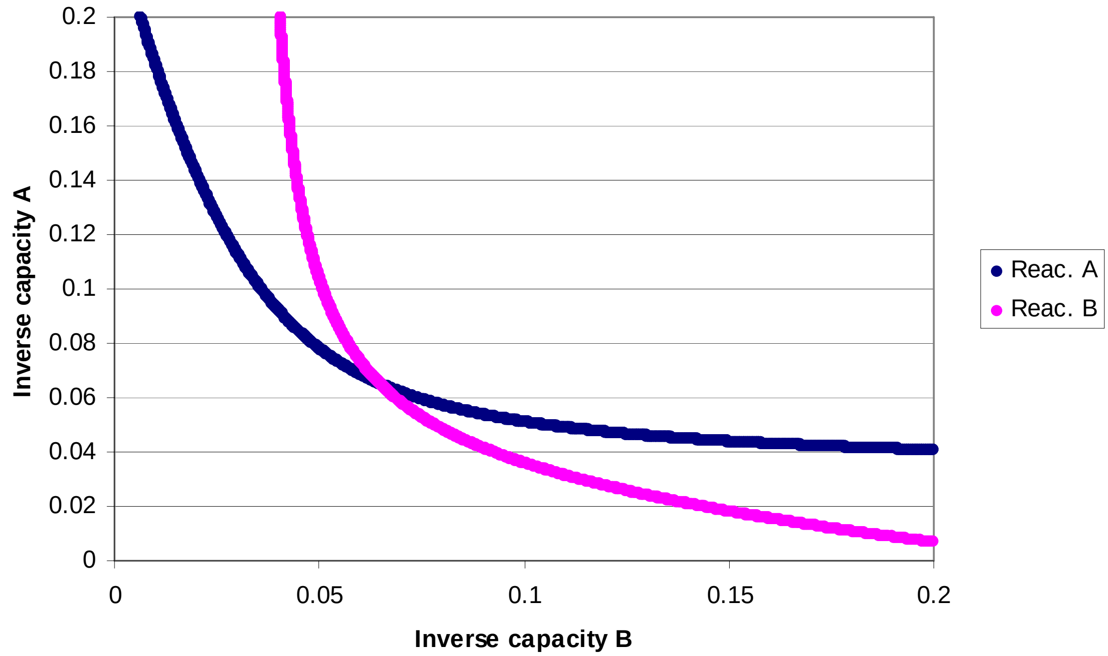


Table 1 shows that prices and profits in the monopoly outcome are higher than in the Nash equilibrium. While monopoly capacities are below duopoly capacities, the service level (the inverse of time costs) is higher in the monopoly case than under duopoly. As shown in section 2, time costs are the same in the monopoly and the social welfare maximum. In contrast, duopolists cannot capture as much surplus generated by high quality as a monopolist can: since an expansion of a facility's capacity implies an expansion of the overall network, the benefits of capacity expansion partly accrue to the competitor. The consequence is lower service levels (higher time costs) in the duopolistic equilibrium as compared to the monopoly or the social welfare maximum. However, despite the higher congestion levels under duopoly, consumer surplus and welfare are lower in the monopoly case than under duopoly due to the output distortion of monopolistic pricing.

Table 1 Parameters and solutions of the central scenario under alternative assumptions on market structure

Parameter or variable	Symbol	Duopoly: Nash equilibrium	Monopoly	Surplus maximization
1. Intercept inverse demand function	α	13.8		
2. Slope inverse demand function	β	0.2		
3. Marginal value of time	γ	1		
4. Marginal cost of capacity	c	1		
5. Quantity demanded	q	47.671	29.500	59.000
6. Quantity demanded at A	q_A	23.836	14.750	29.500
7. Quantity demanded at B	q_B	23.836	14.750	29.500
8. Generalized price	g	4.266	7.900	2.000
9. Price at A	p_A	2.717	6.900	1.000
10. Price at B	p_B	2.717	6.900	1.000
11. Time cost to A	a_A	1.548	1.000	1.000
12. Time cost to B	a_B	1.548	1.000	1.000
13. Inverse capacity at A	R_A	0.065	0.068	0.0339
14. Inverse capacity at B	R_B	0.065	0.068	0.0339
15. Capacity at A	K_A	15.393	14.749	29.500
16. Capacity at B	K_B	15.393	14.749	29.500
17. Profits at A	π_A	49.375	87.025	0
18. Profits at B	π_B	49.375	87.025	0
19. Consumer surplus	CS	227.3	87.0	348.1
20. Welfare (consumer surplus plus profits)	W	326.0	261.1	348.1
21. Generalized price elasticity	ε_{QG}	-0.45	-1.34	-0.17
22. Money price elasticity at A	ε_{QPA}	-0.29	-1.17	-0.08
23. Money price elasticity at B	ε_{QPB}	-0.29	-1.17	-0.08

We also considered the implications of various parameter changes for the final results. In particular, we looked at variations in the marginal value of time, in the unit cost of capacity provision, and in the slope of the demand function. With one exception, the results were as expected.[15] Intuitively, reducing the value of time directly reduces the

[15] More detailed results are included in the working paper version of this article. The linear structure of the model implies that the effects of an equal percentage reduction in the marginal value of time or in the marginal cost of capacity on all relevant properties of the duopoly and monopoly outcomes are identical. A reduced value of time implies that physical congestion levels are less costly, while a reduced marginal cost of capacity implies that alleviating congestion is cheaper. The effect on service levels is identical in our model. The only difference is that lower capacity costs result in higher capacity levels in the social optimum.

time cost of congestion; reducing capacity costs indirectly reduces the cost of congestion by raising capacity. Not surprisingly, we find that reducing the marginal value of time or lowering the marginal capacity cost raises capacity and increases the service level. Output increases in all market structures. One unexpected finding is related to the effects of lower marginal capacity costs on firm profits. Under monopoly, the resulting capacity increase does not lead to pricing adjustments, so that profits rise. However, in the Nash equilibrium under duopoly, a reduction in the unit cost of capacity yields less congestion and higher capacity. This induces the duopolist to lower prices, so that profits may fall despite the increase of output.[16]

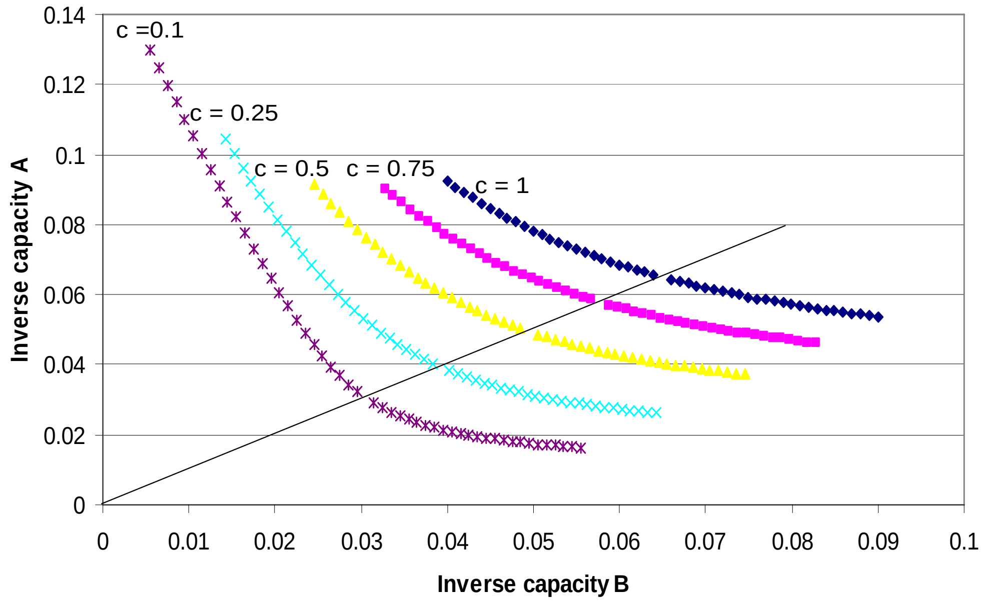
3.2 Marginal costs of capacity and asymmetric equilibria

We now focus on the duopoly model, and look in more detail at the effect of changes in marginal capacity costs, while retaining ex ante symmetry: both facilities face identical demand and cost conditions before the price-capacity game is played. The main insight from this exercise is that capacity costs strongly affect the slope of reaction functions and, as a consequence, the nature of the resulting equilibria.[17] At relatively high values of capacity costs we find a stable, symmetric Nash equilibrium. However, for relatively low values of capacity costs, the only stable equilibria of the two-stage game are asymmetric.

When marginal costs of capacity decline, the capacity reaction functions become more convex and steeper in the neighborhood of the symmetric Nash-equilibrium, as is illustrated in Figure 2 (see also (24)). The symmetric intersections of these reaction functions are on a ray through the origin. For relatively high capacity costs the reaction functions intersect once and produce a single symmetric Nash equilibrium. Increased convexity at low marginal capacity costs implies that, below a threshold value for marginal capacity costs, the symmetric equilibrium becomes unstable. The model then yields multiple intersections, and stable asymmetric equilibria result.

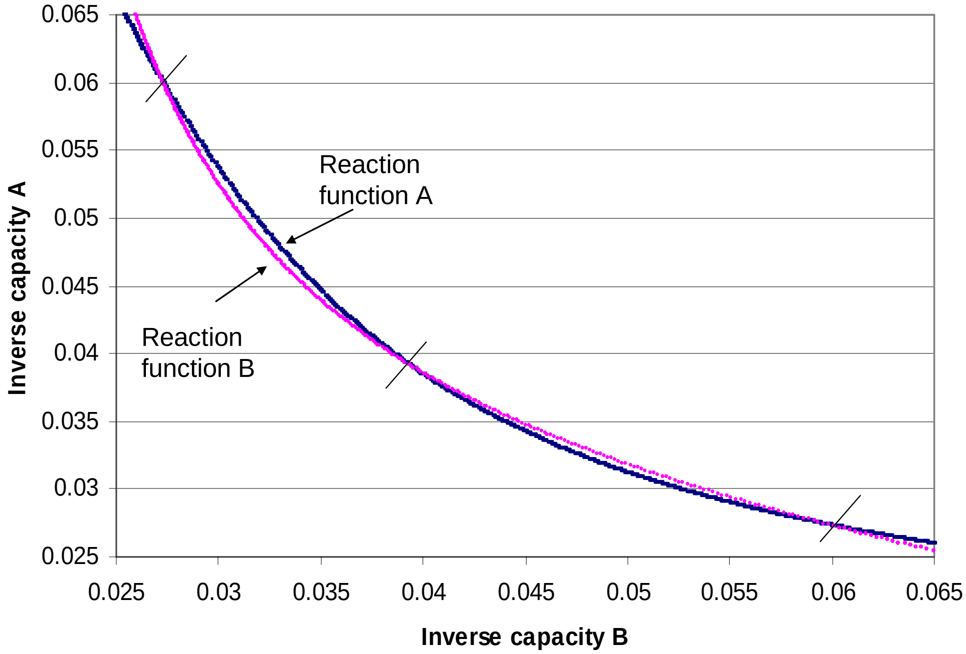
[16] This was the case in the numerical simulations we did with our model. Note that the effect on profits can go both ways. One easily shows that the effect of a capacity cost increase may raise or reduce profit of a facility depending on the size of two different effects, viz., the effect of extra capacity on demand at constant prices and the impact through price adjustments.
[17] Higher capacity costs and more inelastic structural demand have similar effects on the shape of reaction functions. Results are available from the authors.

Figure 2 Inverse capacity reaction functions for various marginal capacity cost levels



To illustrate the existence of multiple asymmetric equilibria at low capacity costs, Figure 3 shows the reaction functions for both facilities for the case where marginal capacity costs have been reduced to 0.25, keeping all other parameters at the level of the central scenario. It shows that there are three Nash equilibria, of which the asymmetric ones are stable. Table 2 shows the (unstable) symmetric and the stable asymmetric equilibria. Total output in the latter is slightly lower than in the symmetric unstable equilibrium, while the generalized price is slightly higher; this indicates that the asymmetry reduces consumer surplus. Total welfare is only slightly lower in the asymmetric equilibrium than in the symmetrical one.

Figure 3 Inverse capacity reaction functions and Nash equilibria for marginal capacity costs of 0.25



The emergence of asymmetric equilibria can be interpreted as endogenous product differentiation at low capacity costs and/or relatively inelastic structural demand. The interpretation is that asymmetric outcomes are more likely when competition is more intense. Low capacity costs affect the slope of capacity reaction functions and make capacity competition more intense. Relatively inelastic demand implies intense price competition. Note that in asymmetric equilibria the large facility (here labeled facility *B*) caters to a larger share of the market than the small facility. It charges a higher price, but time costs are lower because the capacity investment is larger. So the picture emerges of a market served by a large facility that provides high quality at a high price, and by a smaller facility that provides lower quality at a lower price. While the large facility grosses a larger profit, profit per unit of capacity investment is larger at the small facility (2.17 instead of 1.71 at the large facility).

Finally note that further reductions in the marginal cost of capacity result in more asymmetric equilibria. When capacity costs are very low and/or demand is highly inelastic, the asymmetric equilibria converge to a corner solution. In that case there is no stable duopoly equilibrium, and the outcome is effectively the monopoly solution. This produces an extreme form of asymmetry, where only one facility makes positive investments in capacity. The relevant solution in a one shot game then is the monopoly solution, in the sense that once capacity investments are made and one facility has decided not to enter the market, the other facility is in a position to charge monopoly prices.

Table 2 Parameters and solutions for reduced marginal capacity costs

Parameter or variable	Symbol	Symmetric equilibrium - unstable	Asymmetric equilibrium - stable
1. Intercept inverse demand function	α	13.8	
2. Slope inverse demand function	β	0.2	
3. Marginal value of time	γ	1	
4. Marginal cost of capacity	c	**0.25**	
5. Quantity demanded	q	53.978	53.230
6. Quantity demanded at A	q_A	26.989	21.931
7. Quantity demanded at B	q_B	26.989	31.299
8. Generalized price	g	3.004	3.154
9. Price at A	p_A	1.945	1.841
10. Price at B	p_B	1.945	2.298
11. Time cost to A	a_A	1.059	1.313
12. Time cost to B	a_B	1.059	0.856
13. Inverse capacity at A	R_A	0.039	0.060
14. Inverse capacity at B	R_B	0.039	0.027
15. Capacity at A	K_A	25.476	16.670
16. Capacity at B	K_B	25.476	36.753
17. Profits at A	π_A	46.124	36.194
18. Profits at B	π_B	46.124	62.788
19. Consumer surplus	CS	291.4	283.3
20. Welfare (consumer surplus plus profits)	W	383.6	382.3
19. Generalized price elast.	ε_{QG}	-0.28	-0.30
20. Money price elast. at A	ε_{QPA}	-0.18	-0.17
21. Money price elast. at B	ε_{QPB}	-0.18	-0.22

4. Concluding remarks

This paper has studied the duopolistic interaction between congestible facilities that supply perfect substitutes and that make sequential decisions on capacities and prices. Congestion increases consumers' time costs of using a facility – alternatively, it reduces the service level – and is determined by the ratio of the number of users and capacity. We compared the duopoly outcome to the monopoly and the surplus maximizing results using both theoretical analysis and numerical simulation. This leads to a number of insights. First, theory and numerical examples suggest that under duopoly capacity provision is lower, and congestion levels are higher, than in the social optimum. This contrasts with the monopoly outcome, where pricing and capacity provision are such that the monopolist provides the same service level as in the social optimum. Higher congestion under duopoly can be explained by strategic price responses to capacity investments. A capacity increase reduces generalized costs and equilibrium prices at both facilities, implying that the benefits of a capacity increase at one location partially accrue to the other. This externality is fully internalized in both the social optimum and the monopoly case, but it is not under duopoly. Second, price responses to capacity changes also imply that under duopoly higher marginal capacity costs may raise profits. Third, asymmetric Nash-equilibria may result even when firms are ex ante identical. More specifically, when capacity is cheap or demand is relatively inelastic, the only stable equilibria are asymmetric. In such an asymmetric equilibrium, there is one large facility that provides high service levels (low time cost) at a high price, and a small facility with a smaller market share and less service quality (high time cost) at lower prices. This implies endogenous product differentiation by ex ante identical facilities.

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Appendix 1. Properties of the price reaction functions and the Nash equilibrium prices

The optimal pricing rules for *A* (see (9)) and its equivalent for *B* are implicit representations of the price reaction functions (superscript *R)* $p_A^R = p_A^R\left(p_B, R_A, R_B\right)$ and $p_B^R = p_B^R\left(p_A, R_A, R_B\right)$, conditional on capacities. To find the slope of the price reaction function for *A*, write the price rule in implicit form as follows:

$$\omega(p_A, p_B, R_A, R_B) = p_A - q_A^r(p_A, p_B, R_A, R_B)\left[R_A\gamma + \frac{\gamma\beta R_B}{\beta + \gamma R_B}\right] = 0,$$

where the dependence of demand on capacities and prices, see (3), has been made explicit. Then use the implicit function theorem to find:

$$\frac{\partial p_A^R}{\partial p_B} = -\frac{\dfrac{\partial\omega}{\partial p_B}}{\dfrac{\partial\omega}{\partial p_A}} = \frac{\beta}{2(\beta + \gamma R_B)} > 0 \tag{A1.1}$$

$$\frac{\partial p_A^R}{\partial R_A} = -\frac{\dfrac{\partial\omega}{\partial R_A}}{\dfrac{\partial\omega}{\partial p_A}} = 0 \tag{A1.2}$$

$$\frac{\partial p_A^R}{\partial R_B} = -\frac{\dfrac{\partial\omega}{\partial R_B}}{\dfrac{\partial\omega}{\partial p_A}} = \frac{\gamma\beta(\alpha - p_B)}{2(\beta + \gamma R_B)^2} > 0 \tag{A1.3}$$

Analogous results hold for *B*. As shown by (A1.1), the price reaction functions, conditional on capacity, are linear in the price of the competing facility and upward sloping. The slope is between zero and one, guaranteeing (given positive intercept, which is easily shown to be the case) a unique interior Nash equilibrium in prices, for given capacities. As is clear from(A1.3), the reaction of prices to capacities at the competitor's facility is not linear. As could be expected, the expression implies that a marginal capacity decrease at *B* (i.e. a marginal increase in R_B) leads to a higher price at *A*.

Remarkably, equation (A1.2) shows that along the reaction function, a facility's price does not respond to a change in its capacity determined at the previous stage of the game. Intuitively, there are two opposing effects from a marginal capacity increase. The first one is that, holding demand in *A* constant, an increase in capacity in *A* reduces the time cost in *A*, so reducing the optimal price. The second effect is that more capacity at *A* increases demand in *A*, and this increases both the time cost and the markup, raising the price. Given the specific model structure used (linear demands and congestion cost functions), one easily shows that these two effects cancel out. Of course, in more general models (e.g. with nonlinear congestion functions), the two effects will have opposite signs but their absolute size need not be identical.

The Nash-equilibrium prices, for given capacities, are denoted $p_A^{NE}\left(R_A,R_B\right)$, $p_B^{NE}\left(R_A,R_B\right)$, respectively. Formally, they are determined by the intersection of the reaction functions:

$$\begin{aligned} p_A^{NE}\left(R_A,R_B\right) &\equiv p_A^R\left(p_B^{NE},R_A,R_B\right) \\ p_B^{NE}\left(R_A,R_B\right) &\equiv p_B^R\left(p_A^{NE},R_A,R_B\right) \end{aligned} \tag{A1.4}$$

The sign of the effect of a marginal capacity increase at *A* and at *B* on these prices is determined by differentiating system (A1.4). We find, using (A1.1)-(A1.3) and the analogous effects for the reaction function in B:

$$\frac{\partial p_A^{NE}}{\partial R_A}=\frac{\dfrac{\partial p_A^R}{\partial p_B}\dfrac{\partial p_B^R}{\partial R_A}}{1-\dfrac{\partial p_A^R}{\partial p_B}\dfrac{\partial p_B^R}{\partial p_A}}>0 \tag{A1.5}$$

$$\frac{\partial p_A^{NE}}{\partial R_B}=\frac{\dfrac{\partial p_A^R}{\partial R_B}}{1-\dfrac{\partial p_A^R}{\partial p_B}\dfrac{\partial p_B^R}{\partial p_A}}>0 \tag{A1.6}$$

By (A1.1) and its equivalent for *B*, the denominator of these expressions is positive and smaller than one. By (A1.1) and (A1.3), the numerator is positive.

Appendix 2 The slope of the capacity reaction functions

In this appendix we study the slope of the capacity reaction functions; in particular, we show that at a symmetric Nash equilibrium of the two-stage game the reaction functions of the capacity game are downward sloping.

The slope can be written in general as:

$$\frac{\partial R_A^R}{\partial R_B} = -\frac{\psi_{R_B}}{\psi_{R_A}}$$

where $\psi(.)$ is the reaction function in implicit form defined in section 2.3, and ψ_{R_A} is negative by the second order condition for profit maximizing capacity choice. The numerator can be written as:

$$\psi_{R_B} = p_A^{NE}\left[\frac{\partial^2 q_A^r}{\partial R_A \partial R_B} + \frac{\partial q_A^r}{\partial p_B}\frac{\partial^2 p_B^{NE}}{\partial R_A \partial R_B} + \frac{\partial p_B^{NE}}{\partial R_A}\frac{\partial^2 q_A^r}{\partial p_B \partial R_B}\right] + \left[\frac{\partial q_A^r}{\partial R_A} + \frac{\partial q_A^r}{\partial p_B}\frac{\partial p_B^{NE}}{\partial R_A}\right]\frac{\partial p_A^{NE}}{\partial R_B} \qquad \text{(A2.1)}$$

Using results derived earlier in the paper we obtain expressions for the individual terms appearing in this equation.

First, differentiating (6) with respect to inverse capacity in B yields:

$$\frac{\partial^2 q_A^r}{\partial R_A \partial R_B} = \frac{1}{A}\left\{\beta^2\gamma^2(q_A^r - q_B^r) - \beta\gamma^3 q_B^r R_B\right\} \qquad \text{(A2.2)}$$

where A>0 was defined in section 2.1. Note that the above expression is negative at an ex post symmetric equilibrium ($q_A^r = q_B^r$). At a sufficiently asymmetric equilibrium it may be positive.

Second, differentiating the equivalent expression of (A1.6) for the price at B yields:

$$\frac{\partial^2 p_B^{NE}}{\partial R_A \partial R_B} = \frac{\dfrac{\partial p_A^R}{\partial R_B}\dfrac{\partial^2 p_B^R}{\partial p_A \partial R_A}}{\left(1 - \dfrac{\partial p_A^R}{\partial p_B}\dfrac{\partial p_B^R}{\partial p_A}\right)^2} < 0 \qquad \text{(A2.3)}$$

where the superscript 'R' refers to the reaction functions in prices at the second stage of the game. Note that the expression is necessarily negative for our specification, because $\frac{\partial p_A^R}{\partial R_B} > 0$ (see (A1.3)) and, using the equivalent of (A1.3) for the price at B, we easily show $\frac{\partial^2 p_B^R}{\partial p_A \partial R_A}$<0.

Third, similar procedures as before easily show that:

$$\frac{\partial^2 q_A^r}{\partial p_B \partial R_B} = \frac{-\beta\gamma\left(\beta + \gamma R_A\right)}{\left(A\right)^2} < 0 \tag{A2.4}$$

Again, this is negative. Moreover, from the first order condition of the capacity choice problem (see (14)) we have that:

$$\frac{\partial q_A^r}{\partial R_A} + \frac{\partial q_A^r}{\partial p_B}\frac{\partial p_B^{NE}}{\partial R_A} = -\frac{c_A}{p_A^{NE} R_A^2} < 0 \tag{A2.5}$$

Finally, earlier results reported in the paper imply:

$$\frac{\partial q_A^r}{\partial p_B} > 0, \frac{\partial p_B^{NE}}{\partial R_A} > 0, \frac{\partial p_A^{NE}}{\partial R_B} > 0$$

We have now determined the signs of all terms appearing in ψ_{R_B} as given in (A2.1). Using these results implies that the slope of the reaction function in capacities is highly plausibly downward sloping. Unless (A2.2) is very largely positive (which requires an extreme form of asymmetry) we have $\psi_{R_B} < 0$, implying the slope of the capacity reaction function is negative. At a symmetric equilibrium (so that (A2.2) is necessarily negative), it follows that $\psi_{R_B} < 0$. As a consequence, we have shown that, for our specifications and at a symmetric equilibrium (we have used the first order conditions of both the price and capacity game as well as the symmetry assumption to show the result), the slope of the reaction function must be negative.

Appendix 3 The monopoly case and the social optimum

Assume first that both facilities are operated by a single profit-maximizer. Profits are given by:

$$\sum_{i=A,B} p_i q_i^r(p_A, p_B, R_A, R_B) - \sum_{i=A,B} \frac{c_i}{R_i}$$

and maximized with respect to the two prices and capacity levels. The first-order conditions can be written as:

$$p_A \frac{\partial q_A^r}{\partial p_A} + q_A^r(.) + p_B \frac{\partial q_B^r}{\partial p_A} = 0; \quad p_A \frac{\partial q_A^r}{\partial p_B} + q_B^r(.) + p_B \frac{\partial q_B^r}{\partial p_B} = 0$$

$$p_A \frac{\partial q_A^r}{\partial R_A} + p_B \frac{\partial q_B^r}{\partial R_A} + \frac{c_A}{(R_A)^2} = 0; \quad p_A \frac{\partial q_A^r}{\partial R_B} + p_B \frac{\partial q_B^r}{\partial R_B} + \frac{c_B}{(R_B)^2} = 0$$

These equations can be manipulated, using the reduced-form derivatives derived before (see (4)-(7) in the main body of the paper), to yield:

$$p_i = \left(q_A + q_B\right)\beta + q_i \gamma R_i, i \in \{A, B\}$$

$$\frac{1}{R_i} = \left(\frac{\gamma}{c_i}\right)^{1/2} q_i, \quad i \in \{A, B\}$$

Next, assume the facilities are operated by a welfare-maximizing government. It maximizes the difference between total net surplus and total social costs:

$$\sum_{i=A,B}\left(\int_0^{q_i} G[u]du\right) - \sum_{i=A,B}\left((G - p_i)q_i + \frac{c_i}{R_i}\right)$$

where, as before, demands are given by (3) and G is defined in (2). This last expression implies

$$G - p_i = \gamma R_i q_i$$

Using this information, the first order conditions can be written as:

$$(G - 2\gamma R_A q_A)\frac{\partial q_A^r}{\partial p_A} + (G - 2\gamma R_B q_B)\frac{\partial q_B^r}{\partial p_A} = 0; \quad (G - 2\gamma R_A q_A)\frac{\partial q_A^r}{\partial p_B} + (G - 2\gamma R_B q_B)\frac{\partial q_B^r}{\partial p_B} = 0$$

$$(G - 2\gamma R_A q_A)\frac{\partial q_A^r}{\partial R_A} + (G - 2\gamma R_B q_B)\frac{\partial q_B^r}{\partial R_A} - \gamma q_A^2 + \frac{c_A}{(R_A)^2} = 0$$

$$(G - 2\gamma R_A q_A)\frac{\partial q_A^r}{\partial R_B} + (G - 2\gamma R_B q_B)\frac{\partial q_B^r}{\partial R_B} - \gamma q_B^2 + \frac{c_B}{(R_B)^2} = 0$$

Again using (2), we have $G - 2\gamma R_i q_i = p_i - \gamma R_i q_i, \quad i = A, B$. Substitution then immediately implies the following price and capacity rules.

$$p_i = q_i \gamma R_i, i \in \{A, B\}$$

$$\frac{1}{R_i} = \left(\frac{\gamma}{c_i}\right)^{1/2} q_i, \quad i \in \{A, B\}$$