Parking Fees, Congestion and Consumer Welfare

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ABSTRACT

Congestion can be caused by through-traffic and by traffic destined for the area where consumers park. It may appear that congestion should be reduced by increasing the price of parking. This paper shows that if road usage is suboptimally priced, then a lump-sum parking fee can increase welfare, but a parking fee per unit time does not. Indeed, an increase in the price of parking induces each person to park for a shorter time, allows more persons to use parking spaces each day, and can thereby increase traffic. For the same reason, consumers may prefer that parking not be free.

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Parking Fees, Congestion, and Consumer Welfare

Parking has some characteristics of a private good—only one person can use a particular space at a time, and the benefit to a person who parks is not affected by the number of others who attempt to park. But it is closely tied to road usage, and therefore to congestion. This congestion cannot be easily corrected by a parking fee. For a fee that decreases the length of time each person parks can allow more persons to park and thus increase traffic.\(^1\)

The model of Section 1 incorporates these features of parking and road congestion. We assume that all parking spaces are identical, and that demand is uniform over time. We thus do not address the role of parking fees in allocating more and less desirable parking spaces to users. For a classic treatment of this location problem see Vickrey (1954). A modern analysis is given by Arnott, de Palma, and Lindsey (1989), who consider both the timing of trips and the location of parking; they show that the parking fees charged by competitive firms are inefficient, and indeed can reduce welfare.

Section 2 examines the second-best solution: the optimal parking fees if the road-usage fee is too low. The effect of a parking fee on consumer welfare and on the number of parkers is discussed in Section 3. The analysis here can explain why many cities charge for parking on the sides of a road, but allow moving traffic (even if at a snail’s pace) to use the same road for free.\(^2\) This difference cannot be explained solely by technological factors—we should expect innovations as clever as the parking meter were governments seriously interested in congestion fees. Instead, we show that parking fees, unlike road tolls, can increase aggregate consumer welfare, even if consumers are identical and the revenue collected is not returned to them.\(^3\) Section 4 uses a queuing model to show that a parking fee can increase consumer welfare and congestion.
1. Parking and through-traffic

Consider consumers who can either park downtown, or alternatively drive through downtown on their way elsewhere.\textsuperscript{4} Heterogeneous would-be parkers are indexed by \( i \). The inverse demand function of person \( i \) is \( p(i,q) \), with \( p_q < 0 \). This function also represents his marginal valuation of the \( q \)th unit of parking time. This function represents the value to the consumer of shopping, dining, working, and so on while he parks. The continuum of consumers is arrayed in order of their decreasing willingness to pay for parking, so that we can define the partial derivative of the function \( p(i,q) \) with respect to \( i \) as \( p_i \leq 0 \). Heterogeneous through-traffic drivers are also arrayed in decreasing order of their valuations. The \( i \)th driver's valuation of a trip is \( v(i) \) with \( v' \leq 0 \).

Denote by \( E \) the number of persons who make a trip with the intention of parking. \( D \) is the number of persons who drive through the area. (To recall the notation, think of persons Driving through downtown, and of persons Ending their trip downtown.) We assume away any uncertainty and imperfect information. Trips are perfectly synchronized so that traffic flow and the demand for parking is the same throughout the day. In equilibrium a consumer can immediately find a parking space. This assumption is relaxed in Section 4.

Both parkers and through-drivers incur a fixed cost \( F \) per trip, which can include the cost of driving, the expected accident costs involved in the act of driving, and so on. This cost is independent of the length of parking time, but can depend on the level of congestion, represented by the total number of drivers; that is, \( F = F(D+E) \), where \( F' > 0 \) in the relevant range. Each parker and driver also incurs a road-usage fee, \( r \).

Parking fees consist of two parts: a fixed lump-sum fee of \( l \), and a charge of \( m \) per unit time. Consumer \( i \)'s surplus from parking for \( q \) units of time is

\[
S(i,q) = \int_0^q p(i,x) dx - mq - l - r - F.
\]  \hspace{1cm} (1)

Consumer \( i \) maximizes utility by choosing \( q \) to satisfy the first-order condition
\[
\frac{dS(i,q)}{dq} = p(i,q) - m = 0. \tag{2}
\]

This determines \(q(i,m)\), the length of time person \(i\) will park as a function of the fee \(m\).

This section describes the socially optimal parking fees, road-usage fee, and number of parking spaces. The cost \(F\) is a social cost. The parking fees \((m\text{ and } l)\) and the road-usage fee \((r)\) are income transfers. The cost of supplying \(N\) units of parking hours per day (the number of parking spaces times the length of the day) is \(c(N)\), with \(c' > 0\).

To restrict attention to the interesting case, we assume that both types of trips are made in equilibrium. Social welfare is the sum of utilities of the \(E\) persons who park each day and of the \(D\) persons who drive through, minus the costs of providing the parking spaces:

\[
W = \sum_0^E \int p(i,x)dx \, di + \sum_0^D \int v(i)di - (D+E)F(D+E) - c(N). \tag{3}
\]

Since at the optimum there should be no excess capacity, the total demand for parking (as a function of \(m\)) must equal the supply. That is,

\[
\sum_0^E q(i,m)di = N. \tag{4}
\]

The social optimization problem is then

\[
\text{Max}_{D,m,E} \quad W = \sum_0^E \int p(i,x)dx \, di + \sum_0^D \int v(i)di - (D+E)F(D+E) - c\left(\sum_0^E q(i,m)di\right). \tag{5}
\]

The first-order conditions are

\[
W_D = v - F - (D+E)F' = 0 \tag{6}
\]
\[ W_m = \int_{0}^{E} p(l, q(i, m))q_m(i, m)di - c' \int_{0}^{E} q_m(i, m)di = 0 \] (7)

\[ W_E = \int_{0}^{q(E, m)} p(E, x)dx - (D+E)F' - c'q(E, m) = 0. \] (8)

By definition the marginal through-driver has zero consumer surplus: \( \nu(D) - rF(D+E) = 0 \). This condition and equation (6) yield:

\[ r = (D+E)F'. \] (9)

Equations (7) and (2) yield

\[ m = c'. \] (10)

By definition, the marginal parker has zero consumer surplus:

\[ \int_{0}^{q(E, m)} p(E, x)dx - mq(E, m) - l - r - F(D+E) = 0. \]

This condition, with (8), (9), and (10), yield

\[ l = \varnothing. \] (11)

Equation (9) states that the conventional first-best rule for determining a road-usage fee holds: it equals the congestion externality generated by an additional vehicle on the road. Equation (10) shows that an optimum parking fee per unit time equals the marginal cost of providing a unit of parking service. Equation (11) shows that no lump-sum fee for parking is required.

Equation (7), which describes the optimal parking fee per unit time, requires further explanation. The first term on the right hand side is the social cost of a marginal increase in \( m \); this is the decline in consumer welfare following a decrease in the length of time the \( E \) consumers park. The second term shows the cost reduction from providing fewer parking spaces.
An increase in congestion may call for a decrease rather than an increase in the parking fee. To see this, consider the effects of changes in the capacity of a road, defined by its tendency to congest. Suppose for simplicity that the congestion function is linear: \( F = (D+E)k \), where \( k > 0 \) is a constant. Solve the first order condition (6)-(8) for the optimal \( D, m, \) and \( E \) as functions of \( k \). Define \( |J| \) as the determinant of the Jacobian, where the Jacobian is the matrix of the partial derivatives of the left-hand side of the three equations with respect to the variables \( D, m, \) and \( E \). The value of \( |J| \) is unambiguously negative if \( c'' = 0 \). It is also negative for sufficiently small values of \( c'' \), which we henceforth assume.

We obtain

\[
\frac{dm}{dk} = \frac{1}{|J|} 2\nu'(D+E)c''m \int q_m(i,m)di.
\] (12)

The value of \( dm/dk \) is zero if \( c'' = 0 \) or \( \nu' = 0 \). It is negative if \( c'' > 0 \) and \( \nu' < 0 \). That is, a decrease in the capacity of the road calls for a decrease in the optimal parking fee if (1) through-traffic drivers are heterogeneous and (2) the marginal cost of providing parking spaces increases as the number of spaces increases.

We also find that

\[
\frac{dE}{dk} = \frac{1}{|J|} 2\nu'(D+E) \left[ \int q_m(i,m)di \left( 1 - c'' \int q_m(i,m)di \right) \right].
\] (13)

The sign of \( dE/dk \) is ambiguous if \( c'' > 0 \); negative if \( c'' = 0 \) and \( \nu' < 0 \); zero if \( c'' = 0 \) and \( \nu' = 0 \). Thus, a decrease in road capacity has an ambiguous effect on the optimal number of parkers.

2. Second-best solutions

So far we have considered a first-best optimum. Assume now that the road-usage fee is suboptimal (perhaps zero), so that \( r < (D+E)F' \). The social objective is then to
\[
L = \max_{D,m,E} \int_{D}^{E} p(i,x)dx \, dl + \int_{0}^{D} \psi(t)dt - (D+E)F(D+E)
- c\left[\int_{0}^{E} q(i,m)di\right] - \tau(v(D) - F(D+E) - r). \tag{14}
\]

The first order conditions are

\[
L_{D} = v - F - (D+E)F' - \tau(v'-F') = 0 \tag{15}
\]

\[
L_{m} = \int_{0}^{E} p(i,q(i,m))q_{m}(i,m)di - c'\int_{0}^{E} q_{m}(i,m)di = 0 \tag{16}
\]

\[
L_{E} = \int_{0}^{q(E,m)} p(E,x)dx - F - (D+E)F' - c'q(E,m) + \tau F' = 0. \tag{17}
\]

Equation (16) is identical to (7) above. The second-best optimum therefore satisfies (10): \(m = c'\), so the optimal parking fee per unit time equals the marginal cost of providing a unit of parking space. The road congestion externality generated by a parker is corrected by the lump-sum parking fee, \(l\). To determine its value, use equations (15) and the zero consumer-surplus condition to get

\[
r = (D+E)F' + \tau(v'-F').
\]

Solve this equation for \(\tau\):

\[
\tau = \frac{r - (D+E)F'}{v' - F'}.
\]

Substitute into (17), use the zero consumer surplus condition and solve for \(l\):

\[
l = -\frac{v'}{v' - F'}(r - (D+E)F'). \tag{18}
\]
We conclude that an optimal lump-sum parking fee is positive if through-drivers are heterogeneous. For a parking fee will then reduce the number of parkers and thus reduce the number of people on the road. The lump-sum parking fee, however, should be zero if \( v' = 0 \) in the relevant range, that is if through-drivers are homogeneous. For then the marginal公园er does not create an unpriced congestion externality. Indeed, the zero consumer surplus condition for through-drivers shows that an increase in the number of parkers causes an equal decrease in the number of through-traffic trips.

3. Consumer welfare

Thus far we have considered socially efficient allocations. We examine next the effect of a parking fee on consumer welfare if the revenue is not returned to consumers. Since an increase in the lump-sum fee, \( l \), clearly decreases consumer welfare if the revenue is not returned to them, we shall consider only changes in \( m \), the parking fee per unit time. We shall assume that the number of parking spaces, \( N \), is fixed.

An excess supply of parking spaces implies that in equilibrium the marginal parker’s consumer surplus is zero. An increase in the parking fee will then reduce the consumer surplus of inframarginal parkers, decrease the number of parkers, and decrease the aggregate welfare of parkers. Considering also the welfare of through-drivers makes the effect on aggregate consumer welfare ambiguous: through-drivers may benefit so much from the reduced congestion that aggregate consumer surplus increases.

Consider next excess demand for parking. That is, suppose some potential parkers are prevented from parking. We can imagine, for example, a municipality that limits the number of parking permits and that imposes a sufficiently low parking fee per unit time to make parking spaces used to capacity. Alternatively, suppose the number of persons who drive downtown solely to park is sufficiently small so that they alone do not fill the parking spaces. Some through-drivers, however, may want to park if they immediately find an empty parking space; otherwise they will continue their journey and avoid the costs of searching for a space. The consumer surplus from parking of a through-driver who parks could then be positive.
An increase in the parking fee per unit time necessarily decreases the consumer surplus of the original parkers. But because each of the original persons parks for a shorter time, more persons can park. The new parkers, by assumption, have positive consumer surplus. If each parker's marginal utility is a diminishing function of the length of time he parks, then the gain of the new parkers can exceed the losses of the original parkers who now park a shorter time. Though this effect increases aggregate consumer welfare, the increased congestion caused by the additional parkers reduces welfare. The net effect of an increase in the parking fee is therefore ambiguous.

A numerical example illustrates the effect of a parking fee on the welfare of parkers. Assume that all consumers are identical, and that there are no through-traffic drivers. The inverse demand function of each parker is \( p(q) = 1/(q+1) \). For a given parking fee, \( m \), a person who parks maximizes utility by parking for that length of time, \( q \), where marginal benefit equals marginal cost, so that \( q = q(m) = 1/m - 1 \). To concentrate on parking rather than on congestion, let the fixed cost of a trip, \( F \), be a constant. Maximizing consumer welfare then requires maximizing the utility, \( S \), of each parker, times the number of consumers who park each day, \( N/q \), or maximizing

\[
\frac{N}{q(m)} S = \frac{N}{q(m)} \left[ \int_0^{q(m)} \frac{1}{x+1} dx - mq(m) - F \right].
\]

The first order condition is

\[-\ln(m) - F - (1-m)(2-m) = 0.\]

Numerical solution shows that if, for example, \( F = 0.2 \), aggregate consumer welfare is maximized at \( m = 0.19 \); the corresponding value of \( q \) is 4.26. Each consumer's net benefit at this solution is \( S = 0.65 \); aggregate consumer welfare is \((N/q)S = 0.15\). When \( F = 0 \), the optimal solution is \( m = 0.32 \), \( q = 2.13 \), and \((N/q)S = 0.22\). These calculations show that aggregate consumer welfare is at a maximum when \( m > 0 \); consumers should not prefer free parking. We also see that though an increase in the fixed cost \( F \) increases the consumer surplus of any one consumer, the reduction in the number of persons who park causes aggregate consumer welfare to decline.
4. Searching for parking

We now return to a model with no excess demand. The zero consumer surplus condition for the marginal parker implies that an increase in the parking fee \( m \) increases the number of parkers \( E \), thereby increasing road congestion.\(^7\) To consider further the relation between the parking fee and congestion, we relax the assumption that search costs are prohibitively high. We describe a queuing model that allows the length of time each person searches for a parking space to vary with the number of other persons who search for parking and with the length of time each person parks. This makes the fixed cost of parking, \( F \), an endogenous variable. For simplicity let there be one parking space. Similar results apply for multiple parking spaces.

The arrival rate of (identical) consumers is \( \alpha \) per unit time. A person who on arrival finds the space empty parks immediately. Otherwise he waits until all persons who arrived before him parked and left. There is no balking and no reneging. Each person parks for as long as he wishes to, given the parking fee. These assumptions lead to the M/D/1 queuing model.\(^8\)

Let the marginal valuation function be \( p(q) = 1/(q+1) \), so that \( q(m) = 1/m-1 \). Standard results in queuing theory show that the wait per person is

\[
\text{Wait} = \frac{\alpha(m-1)^2}{2m[\alpha(m-1)+m]}
\]

Let \( c \) be a consumer's cost per unit time of waiting, so that \( F = c \text{ Wait} \). In equilibrium the number of consumer arrivals per unit time, \( \alpha \), must make consumer surplus equal zero, so that

\[
\ln(1+q) - c \text{ Wait} - mq = 0.
\]

Since \( q \) and \( \text{Wait} \) are both functions of the parking fee per unit time, \( m \), this equation can be solved for the equilibrium value of \( \alpha \) as a function of \( m \):

\[
\alpha = \frac{2m^2\ln(1/m)+m-1}{(m-1)(2m\ln(1/m) + (1-m)(c-2m))}.
\quad (19)
\]

The effect of a parking fee on demand is surprising. Let \( c = 1 \). Solving equation (22) shows that when \( m = 0.001 \), the equilibrium arrival rate is \( \alpha = 0.000012 \); when \( m = 0.01 \), the equilibrium
arrival rate is \( \alpha = 0.00069 \); when \( m = 0.1 \), the equilibrium arrival rate is \( \alpha = 0.026 \). Indeed, throughout the region \((0,1)\) \( \alpha \) is an increasing function of \( m \). This means that an increase in the parking fee increases the arrival rate of parkers. To put it in conventional terms, an increase in parking fees increases congestion.

To summarize, a parking fee has two effects. First, it reduces the length of time each person parks, and for a given arrival rate reduces the expected waiting time. Second, for a given waiting time, the parking fee reduces consumer surplus. These two effects have opposing implications on demand. The first effect increases the arrival rate and the second reduces it. Under some conditions the first effect can dominate the second, and therefore parking fees can increase congestion.
References


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<td>Number of through-traffic drivers</td>
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<td>$E$</td>
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1. Policymakers seem to think that parking fees can substitute for road usage fees. Mayor Tom Bradley of Los Angeles, for example, proposed in 1990 to increase taxes on parking in downtown Los Angeles in order to reduce congestion. In New Zealand, the Parliamentary Commissioner for the Environment recommended that employees pay a tax on the free parking granted by employers, so as to encourage people not to drive to work, and thereby reduce congestion and pollution. As will be seen, these analyses can be greatly flawed. For alternative methods of restraining traffic see Heggie (1973), Gomez-Ibanez and Fauth (1980), and May (1986). In Tokyo the shortage of parking spaces, and the externalities generated by on-street parking, are so severe that to register an automobile the owner must have written proof that he has a parking space.

2. For a review of pricing policies see Howitt (1980) and Higgins (1986). Some of the political opposition to road fees may arise from cognitive effects. A congestion toll increases social welfare by inducing fewer people to use the road. No driver, however, sees a person who is deterred from driving—the effect is an abstraction to any voter or consumer. In contrast, a potential parker may see someone else rushing to an expiring meter and moving his car. The effects of parking fees can be quite obvious.

3. If consumers differ in their aversion to congestion, then road tolls can increase consumer welfare; see Glazer (1981) and Niskanen (1987). Also, if the road is so heavily congested that an increase in the number of cars reduces the total flow of traffic, then tolls increase consumer welfare even if no compensation is paid (DeMeza and Gould 1987, p. 1324).

4. Higgins (1986, p. 148) states that "there is a great variation in the number of vehicles that can be affected by parking pricing, with as little as 25% and as much as 50% of traffic in congested downtowns bound for parking destinations there." See also Heggie (1973, p. 108). The actual percentage of course depends on the geography of the city. The ability to control traffic by parking fees is also limited by the fact that there are usually alternative opportunities for off-street parking, which are unpriced, or whose prices are not under the control of a planner. In this respect, however, parking fees do not in principle differ from road tolls, which can be escaped when alternative roads are available.
5. We continue to assume that some potential parkers with positive consumer surplus cannot find a parking space.

6. Notice that the lump-sum fee for parking, \( l \), can also be interpreted as a fixed cost of parking.

7. On the other hand, we have also seen from equation (12) that an increase in the road's tendency to congest may call for a decrease in the parking fee.

8. The model here formalizes the relation Vickrey (1954) noted between the length of time a person parks and the average wait for a parking space.