Development of Multi-Class, Multi-Purpose Variable Demand Network Analysis Model and Its Application to Congestion Pricing Evaluation

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Abstract

The objectives of this paper are two folders. The first objective is to develop a reasonable multi class, variable demand traffic assignment model, and the other is to apply the developed model to a network design problem (NDP). A multi-class, multi-purpose variable demand traffic assignment model is developed to analyze transportation networks with multiple traveler classes and multiple trip purposes. The model is tested for an equity problem in congestion toll pricing that has been raised as a social issue in congestion pricing implementation. In this study, we evaluate two pricing methods (link-based pricing and cordon-based pricing) by analyzing their impact on travel demand. Our analysis shows that the cordon-based pricing can more effectively lessen traffic congestion, but it may aggravate the economic discrepancy among traveler classes by depriving low-income travelers of travel opportunity. The link-based pricing, although less effective, is preferable from the traveler’s equity point of view.
1. Introduction

One of the basic assumptions in conventional traffic assignment is that all drivers' attributes are identical. However, this assumption is not realistic in urban traffic conditions. All travelers have different socio-economic backgrounds, so they would show different behavioral responses with respect to the same stimulus such as traffic management schemes to reduce network congestion. Without considering the heterogeneity of travelers, we would not be able to correctly analyze network design problems (NDP).

Let's consider the most famous example of NDP. It is a congestion toll pricing. There have been numerous studies on congestion pricing, but most studies assumed fixed-demand, a single traveler class, or single trip purpose. For transportation network analysis with heterogeneous drivers, researchers applied different values of time (VOT) for several user classes or elastic demand. It is well known that travelers would gain different economic benefits by the level of their income when congestion pricing is imposed. The optimal pricing problems for multiple users with different VOT have been investigated by Dial (1999a,b), Yang et al. (2002) and Yang and Zhang (2002). Recently Yang and Zhang (2002) applied a multi-class elastic demand model to calculate a congestion toll that satisfies the social and spatial equity constraint. They considered the social and spatial equity issues in the network problem unlike other previous studies that focused only on the efficiency of the whole network.

As known, the best solution is applying the marginal pricing. However, achieving the optimal state of network is nearly impossible since the state can be achieved only when the congestion is levies on most links. A practical solution, but not the best, is the link-based pricing in which only several major links are selected for toll collection. Another method is a cordon-based pricing in which a set of sets in a closed loop are selected as toll links.

In this paper, we develop a multi-class, multi-purpose variable demand traffic assignment model. The problem is formulated as a variational inequality (VI) problem with elastic demand. The second-best pricing problem is selected as an example. The model considers several user classes classified by VOT, and includes multiple trip purposes, such as work trip or non-work trip. In this paper, we do not attempt to find the optimal. Rather, we focus on investigating applicability and capability of the multi-class, multi-purpose variables demand traffic assignment model, leaving the bi-level optimal toll problem for next research.

2. The overview of variable demand models and congestion pricing

2.1. The variable inequality formulation of variable demand assignment

The most well known formulation for a variable demand assignment was suggested by
Sheffi (1985).

\[
\min Z(\vec{x}, \vec{q}) = \sum_r \int_0^L t_r(w) \, dw - \sum_r \int_0^L D_{nj}^r(w) \, dw
\]  
(1)

subject to

\[
\sum_r q_{rs} = q^r \quad \forall r, s
\]  
(2)

\[
f_{rs}^k \geq 0 \quad \forall k, r, s
\]  
(3)

\[
q^r \geq 0 \quad \forall r, s
\]  
(4)

The basic idea for the optimal solution of Eq (1) is using the inverse demand function (in cost or time unit) that is regarded as a pseudo path between an origin and a destination (OD). Thanks to the concept, the above problem can also be reconstructed by adding a dummy path connecting an OD pair directly in order to adopt an excessive demand. (See Sheffi (1985) Chapter 6.5)

Eq (1–4) is proposed to solve a single class trip demand. For multi-class or multi-purpose trip demand, the link travel cost should be different depending on the value of time of the travelers’ class or trip purpose. For developing more general equivalent formulation for a multi-class trip demand or multi-purpose trip demand, a variational inequality formulation was introduced. Nagurney (1993) suggested a variational inequality formulation for a variable demand assignment.

\[
\sum_r \sum_i \left( c_i^r (f_{rs}^i) (f_{rs}^i - f_{rs}^i) \right) - \sum_r D_{nj}^r(q^r) (q^r - q^r) \geq 0
\]  
(5)

This formulation shows a concept of equilibrium in a variable demand assignment explicitly. The second term represents a dummy path between rs for an excess demand. As we can represent the dummy path as a single link, the Eq(1) can be transformed to a general VI formulation for a link-based user equilibrium problem. Based on the Nagurney’s formulation, Chen(1999) developed a dynamic traffic assignment model with variable demand using a variational inequality formation.

2.2. A travel demand model based on OD travel time

Two types of demand model can be applied to a variable demand traffic assignment model: one is linear type model and another is non-linear type model. The linear type model assumes that the elasticity of demand is constant throughout the whole range of demand. Chen (1999) used a linear demand model as follows.
The linear model is simple to use, but it cannot represent the change of sensitivity with respect to the level of costs. To overcome this shortcoming, a non-linear demand model has been used. Yang and Bell (1997) suggested an exponential type model and later Yang et al. (2004b) this model in their study.

\[ q'' = q''_{\text{max}} \cdot \exp[p : t'' - t''_{\text{max}}] \]  

Above two models generate maximum travel demand when the travel time between rs is zero. It is, however, impossible because travel time cannot be zero. Zhang et al. (2004) suggested an improved exponential type model as follows.

\[ q'' = q''_{\text{max}} \cdot \exp[p : 1.0 - \frac{t''}{t''_{\text{max}}}] \]  

As Kanafani (1983, pp12-13) pointed out, however, in general travel demand is less sensitive at very low and high cost range, and relatively more sensitive at the middle range of cost. This shape of demand curve cannot be represented both of above models. In this study, for simplicity, we use a linearity assumption on the shape of demand curve. However, the travel cost point at which maximum demand is generated is set as non-zero cost.

2.3. Congestion pricing problem in the network design problem (NDP)

The principle of congestion pricing has come from the fundamental economic theory of marginal cost pricing. That is, road users using congested roads should pay a toll equal to the difference between the marginal cost and average cost in order to maximize social net benefit or eliminating social external cost. In Fig. 1, the Average cost (AC) curve represents the average cost (or average) cost of congestion at each level of demand (number of trips), accounting for the amount that the road users are willing to pay. While the Marginal cost (MC) curve represents the marginal cost, which is additional cost of adding one extra vehicle to the congested road. MC may not be aware by road users and may be interpreted as a social cost in that it is the cost to the society of road users. Therefore the difference between the AC and MC curves at any level of travel demand represents the economic costs of congestion at that demand. In Fig. 1 the optimal flow is \( T_x \) corresponding to point G where marginal cost and demand are intersected. While the actual demand without toll tends to be \( T_x \) because road
users ignore the additional cost that they impose on other users. From a social point of view, the actual demand, $T_a$, is excessive because the $T_a$-th user is only enjoying a benefit of $\overline{T_a}A'$, but imposing costs of $\overline{T_a}M$. The additional traffic beyond the optimal level $T_o$ can be seen to be generating costs equal to the area $T_o M G T_o$, but only enjoying a benefit equal to the area $T_o A G T_o$, a deadweight welfare loss of the area $A M G$ is apparent. A demand level lower than $T_o$ is also sub-optimal because the potential consumer surplus gained from trip-making is not being fully exploited. Therefore, the optimal toll to be charged is equal to $\overline{BG}$. Under this toll charge, the net economic benefit (NEB), as given by the area $BGECG \times (total$ $user$ $benefit$ $representing$ $area$ $OT_oGE$ $minus$ $total$ $social$ $cost$ $representing$ $area$ $OT_oBCG)$, will be maximized. The net economic benefit (NEB) consists of consumer's surplus, area $C_oGE$, and total government revenue, area $C_oBGC_o$, gained from levying congestion pricing.

In a practical side, there is a serious problem. We should find optimal tolls for all links in a network to optimize travel demand because a network consists of large link set. However, because of undeveloped tolling technique and the resistance of drivers, a traffic administration cannot apply the marginal pricing method. Therefore, only several link sets are selected as toll links. This kind of pricing scheme is called the second-best pricing scheme. The link-based pricing and the cordon-based scheme are classified into this scheme.

As we explained in introduction, links on a corridor or important path are selected as toll links in link-based pricing scheme. On the other hand, links on a closed loop around CBD are selected as toll links in cordon-based pricing scheme. These two schemes have different properties and give different effect to network drivers.
3. Multi-class/purpose travel demand model with variable demand

3.1. The derivation of demand model based on linear relationship assumption

3.1.1 A linear relationship assumption

Generally the travel time-demand relationship has a non-linear, non-monotonic shape. However, for simplicity, let’s assume a linear relationship between travel time and trip demand for a specific OD pair rs.

\[ q^* = a \cdot t^* + b \]  \hspace{1cm} (9)

Then

\[ t^* = D^{-1}(q^*) = \frac{q^* - b}{a} = \frac{1}{a} q^* - \frac{b}{a} \]  \hspace{1cm} (10)

If we assume that \( q_{\text{max}}^* = 20000 \) trip/hr and \( t_{\text{min}}^* = 3 \) hr, then we can have boundary conditions as follows. If \( t^* = 0 \), then \( q_{\text{max}}^* = 20000 \), and if \( t_{\text{max}}^* = 3 \), then \( q_{\text{max}}^* = 0 \). Therefore, \( a = -6666.67 \), and \( b = 3 \).

\[ D^{-1}(q^*) = 3 - 0.00015 \cdot q^* \]
Fig. 2. A linear relationship between travel time and travel demand

3.1.2 A consideration for a minimum travel time between an OD pair

In the previous section, we presented the way of deriving the linear inverse demand model. In the formulation, the minimum value of travel time between an OD pair is zero. It is very unrealistic because travel time could not be zero unless the travel distance is zero.

To consider the minimum OD travel time \( U^\alpha \), we assume the travel time and demand relationship as follows.

\[
q^\alpha = a \cdot (t^\alpha - U^\alpha) + b \tag{11}
\]

\[
t^\alpha = t^\alpha = U^\alpha + \frac{a}{b} \cdot q^\alpha + U^\alpha - \frac{b}{a} \tag{12}
\]
The boundary conditions is that if \( t'' = u'' \), then \( q''_{\text{max}} = 20000 \), and if \( t'' = 3 \), then 

\[ q''_{\text{max}} = 0. \]

\[ D'(q'') = 3 + u'' - 0.00015 \cdot q'' \]

### 3.1.3 The consideration of the ratio of travel time increase and minimum travel time

The inverse travel demand function suggested above is based on the sensitivity of travel demand to the travel time increase. In the formulation, the amount of travel demand decrease is directly related to the amount of travel time increase.

![Travel demand model](image)

**Fig. 4. A travel demand model based on the increasing ratio of travel time**

However, in real world, the same decrease of travel time can cause the different demand increase depending on the total length of travel time. 5 min decrease of travel time is large when the minimum travel time is 10 minute, but 5 min is trivial when the minimum travel time is 2 hr. To consider this, we suggest the ratio of travel time increase and the minimum travel time as follows.

\[
q'' \sim a \cdot \frac{(t'' - u'')}{u''} + b
\]

\[
= a \cdot \frac{t''}{u''} - a + b
\]

(13)

Therefore

\[
\frac{a}{u''} \cdot t'' = q'' + a - b
\]
\[ r'' = \frac{u''}{a} \left( q'' + a - b \right) = \frac{u''}{a} q'' + \frac{u''}{a} + \frac{b}{a} u'' \]

When \( r'' = u'' \), \( q'' = q_{\text{max}}'' \).

Therefore

\[ u'' = \frac{u''}{a} q_{\text{max}}'' + \frac{u''}{a} + \frac{b}{a} u'' \quad (14) \]

\[ \frac{u''}{a} q_{\text{max}}'' = \frac{b}{a} u'' \]

Therefore, \( b = q_{\text{max}}'' \),

and

\[ r'' = D^{-1}(q'') = \frac{u''}{a} q'' + \frac{u''}{a} q_{\text{max}}'' \quad (15) \]

To find the value of \( a \), we assume that the maximum point of \( r'' \) as \( a \cdot u'' \). It means that if \( r'' = a \cdot u'' \), then \( q'' = 0 \).

\[ a \cdot u'' = u'' - \frac{u''}{a} q_{\text{max}}'' \quad (16) \]

Therefore

\[ a = 1 - \frac{q_{\text{max}}''}{u''} \text{, and then } a = q_{\text{max}}'' / \left( 1 - a \right) \]

The final formulation is

\[ r'' = D^{-1}(q'') = u'' \cdot \frac{1 - a}{q_{\text{max}}''} + u'' - u'' \cdot \frac{1 - a}{q_{\text{max}}''} q_{\text{max}}'' \]

\[ = u'' \cdot (1 - a) + \frac{q_{\text{max}}''}{u''} - u'' \cdot (1 - a) \]

\[ = u'' + u'' \cdot (1 - a) \cdot \left( \frac{q_{\text{max}}''}{u''} - 1 \right) \quad (17) \]

The above inverse demand relationship implies the below demand model.

\[ q'' = \frac{q_{\text{max}}''}{1 - a} \left( \frac{u'' - u''}{u''} + q'' \right) \quad (18) \]

where, \((u'' - u'')u''\) is named “the increase ratio of travel time” and then \(q''\) becomes a function of “the increase ratio of OD travel time”
3.2. Travel demand models for different trip purposes

In this paper, we consider multiple trip purposes: work trip and non-work trip. Each trip purpose has a different demand function. In general, commuter’s trip is less sensitive to the travel cost than other trips. Consequently, the $\alpha$ value of the work trip is larger than that of non-work trip. In Fig. 5 (a), two shapes of demand function is postulated: one for work trip, the other for non-work trip. We assume that the minimum travel time is 0.5 hr, maximum trip demand is 5000 trip/hr, the $\alpha$ value of work trip is 10 and the $\alpha$ value of non-work trip is 5.

(a) Basic demand function  
(b) Modified demand function

Fig. 5. Demand functions for for work trip and non-work trip

This paper focuses on the peak time travel demand, so the maximum demand of work trip will be bigger than the maximum demand of non-work trip. When assuming that the maximum non-work demand is a half of the maximum work demand, the maximum demand for non-work trip becomes 2500 trip/hr and the corresponding $\alpha$ value is value is 2.5.

3.3. The value of time (VOT) consideration for heterogeneous traveler classes

To consider heterogeneous travelers, we introduce two traveler classes by the level of income: high-income class, and a low-income class. As Yang et al. (2004a) explained, the value of time could be considered in two ways. One way is to construct demand models based on a monetary cost, and the other way is to construct demand models based on travel time. When applying the monetary cost, the path travel time is transformed to monetary value by multiplying the VOT. On the contrary, when the travel time unit is applied, the monetary cost for trip, such as toll, is transformed to travel time unit by divided by VOT.

Generally, the two approaches give same result in NDP problem as Yang et al. (1999) showed with a fixed demand model. We consider this problem in a variable demand case. Let us assume two traveler classes whose VOTs are $40/hr and $20/hr, respectively. When
travel time between an OD pair is one hour, their travel costs are $40 and $20, respectively. Presumably, the demand elasticity of low-income class (class 2) is larger than that of high-income class (class 1).

Fig. 6. Demand models

Fig. 6(a) shows two demand functions at a monetary unit assumed to be independent each other. When the same amount of congestion toll δ (in monetary unit) is imposed, more trips are decreased for the low-income class due to larger elasticity with respect to travel costs. Fig. 6(b) explains with the travel time unit. In this case, we can apply a common trip demand function for two classes assuming that the demand elasticity with respect to travel time is same for both classes. In Fig 6(b), the amount of shift for each class is different because of the discrepancy in VOT between classes. Therefore, the decrease of demand for each class will be different in a same demand model for both classes. That is, in Fig. 6(b), we will have
same results as in Figure 6(a) by applying VOT. However, considering the UE principle, an unrealistic result could be brought out when using a common demand model for multi-class travelers. Under the UE route choice principle, all used paths should have the same travel cost. However, in Fig. 6(b), there are two different generalized costs for two classes traveling the same OD pair. To make this happen, the two classes should use different routes. In a general congested network, it is impossible, and two classes will generate the same amount of demand based on a common demand model because they have the same generalized minimum cost.

![Diagram](image)

**Fig. 7. Modified demand model based on OD travel time**

To prevent the unreasonable result, we use independent demand function for each class. In Fig 7, demand models give different volumes of travel demand for the same increase ratio of travel time. In the figure, A means the increase of generalized travel time resulted in levying of congestion toll.

To help readers understand the above model issues, a simple example is provided in Table 1. Assume a network with two routes and two traveler classes with different income level. The case 1 shows that the rout A is not used by the low-income (LI) class due to the toll of $10 that is equivalent to 2 hours for the LI class. At equilibrium, the general equilibrium OD costs for both classes are same. In the case 1, with different amount of toll, the demands for both classes are equally affected despite the difference in their income level, which is not realistic. In the second case, each class’ general equilibrium OD costs are mutually independent since they are taking different route each other. In such circumstances, each group’s demand is affected independently by their costs. This shows that a different demand
function should be applied to each group.

Table 1. Comparison of Toll Effects

<table>
<thead>
<tr>
<th></th>
<th>Route A (with a $10 toll)</th>
<th>Route B</th>
<th>General Equilibrium OD cost (Demand)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HI class</td>
<td>Cost: 2 hr + $10 = 3 hr</td>
<td>3.0 hr</td>
<td>3.0 hr (100)</td>
</tr>
<tr>
<td></td>
<td>Traffic: 70</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>LI class</td>
<td>Cost: 2 hr + $10 = 4 hr</td>
<td>3.6 hr</td>
<td>3.0 hr (100)</td>
</tr>
<tr>
<td></td>
<td>Traffic: 0</td>
<td>10c</td>
<td></td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HI class</td>
<td>Cost: 2 hr + $10 = 3 hr</td>
<td>3.5 hr</td>
<td>3.0 hr (100)</td>
</tr>
<tr>
<td></td>
<td>Traffic: 100</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>LI class</td>
<td>Cost: 2 hr + $10 = 4 hr</td>
<td>3.5 hr</td>
<td>3.5 hr (100)</td>
</tr>
<tr>
<td></td>
<td>Traffic: 0</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Note) VOT of HI-class = $10 and VOT of LI-class = $5.

As a result, the trip assignment problem for multi-purpose, multi-class with variable demand is formulated as a VI form as follows:

\[
\sum_{\alpha} \sum_{\rho} \sum_{\epsilon} c_{\epsilon, \rho, \alpha} (f_{\rho, \epsilon}^+ - f_{\rho, \epsilon}^-) + \sum_{\alpha} \sum_{\rho} D_{\alpha, \rho} (q_{\rho, \alpha}^+ - q_{\rho, \alpha}^-) = 0 \quad (19)
\]

where, subscript \( \rho \) means trip purpose, and subscript \( \epsilon \) means the travelers' classes classified by their income.

4. Numerical Experiment

4.1. Test Network

In Fig. 8 a test network is presented. The network consists of 10 nodes and 21 links. There are three OD pairs in the network. Table 2. shows the maximum OD volume and the parameter \( a \) for each purpose and each class between OD pairs in detail.

The VOT value is determined based on the result of previous researches on congestion pricing. Brownstone et al. (2003) found that the median willingness of pay for San Diego I-15 freeway was about $40 per hour, and the range was roughly $20–$40. Yang et al. (2004a) found the VOT of 1.0 (HKD/min, $7.8/hr) for low-income travelers and 2.0 (HKD/min, $15.6/hr) for high-income travelers.
$15.6/hr) for high-income travelers in their Hong Kong study. Generally, the willingness to pay might be higher than the VOT. Therefore, we assume that the VOT is $20 per hour for high-income travelers and $10 per hour for low-income travelers.

Fig. 8. Sample network

<table>
<thead>
<tr>
<th>O→D</th>
<th>Purpose</th>
<th>Class</th>
<th>( \alpha )</th>
<th>Max OD vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1→10</td>
<td>Work</td>
<td>High-Income</td>
<td>10.0</td>
<td>4,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low-Income</td>
<td>8.0</td>
<td>4,000</td>
</tr>
<tr>
<td></td>
<td>Non-Work</td>
<td>High-Income</td>
<td>6.0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low-Income</td>
<td>4.0</td>
<td>2,000</td>
</tr>
<tr>
<td>2→10</td>
<td>Work</td>
<td>High-Income</td>
<td>10.0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low-Income</td>
<td>8.0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>Non-Work</td>
<td>High-Income</td>
<td>6.0</td>
<td>1,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low-Income</td>
<td>4.0</td>
<td>1,500</td>
</tr>
<tr>
<td>4→10</td>
<td>Work</td>
<td>High-Income</td>
<td>10.0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low-Income</td>
<td>8.0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>Non-Work</td>
<td>High-Income</td>
<td>6.0</td>
<td>1,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low-Income</td>
<td>4.0</td>
<td>1,500</td>
</tr>
</tbody>
</table>

4.2. Evaluation of Congestion Pricing Schemes

In this paper, we test two second-best pricing schemes. In a link-based scheme, a congestion toll is levied on link 8 only, and in a cordon-based scheme, congestion tolls are levied on links 5, 8, and 10. In both cases, various congestion tolls are applied from 0 (no toll case) to $7.
In our case, the amount of toll is an exogenous variable. As Zhang et al. (2004) showed, the optimal solution for heterogeneous traveler’s equity or maximizing social welfare (Consumer surplus plus supplier surplus) can be studied by applying a bi-level optimization approach, but we left this for future research.

Fig. 9 shows changes in travel speed in the network. In the figure, CBD consists of link 12, 13, 16, 18, 19, 20, and 21 (a sub-network of right side of cordon line). A cordon-based pricing is more effective in improving the network condition. The cordon-based pricing improved the average speed of network from 27.54 km/h (with no toll) to 40.57 km/h (with a toll of $7) while the link-based pricing improved only about 2 km/h. The cordon-based toll increases the average speed of the CBD area from 14.78 km/hr to 27.38 km/hr while the link-based method improves the speed from 14.78 km/hr to 18.15 km/hr; the total network travel time (TNTE, veh-hr) also shows similar tendency. The link-based scheme reduced TNTE by 6.51% while the cordon-based scheme reduced TNTE by 49.89%.

Fig. 9. Average speed improvement by congestion pricing

In terms of effectiveness, the cordon-based congestion pricing method is superior to the link-based method. It is because while the benefit of the cordon-based toll comes mostly from drivers’ route switching, the cordon-based toll effectively control the travel demand by imposing tolls to all traffic to CBD.

In the rest of this section, we investigate equity issues in congestion pricing. From Table 3 shows that shows results by trip purpose and driver class, several interesting results are found. Firstly, the link-based method decreases travel demand in all classes with similar proportion, but the cordon-based method decreases mainly low-income drivers’ trips. Fig. 10 shows such tendency in detail. As observed in the figure, the proportion of each demand does
not change much in the case of the link-based toll scheme. However, in the corordon-based case, the proportion of high-income class increases as the amount of toll increases. That is, the corordon-based toll leads to increase of high-income travelers including non-work trip and decrease of low-income drivers.

Comparing demand by purpose, the corordon-based toll tends to increase more work trips. While the link-based toll of $7 increases the proportion of work trip from 73.1% to 75.5%, the corordon-based toll of $7 increases the proportion to 78.5%. It is also found that the corordon-based toll also increases the proportion of work-trip more than the link-based toll.

<table>
<thead>
<tr>
<th>Link-based</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-HI</td>
<td>6124</td>
<td>6133</td>
<td>6109</td>
<td>6074</td>
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[W : work trip, NW : Non-work trip, HI : High-income class, LI : Low-income class]
Fig. 10. The proportion change of travel demand by congestion pricing

Another finding is that the small amount of toll may ease the equity problem by just increasing the high-income travelers. Especially when the cordon-based pricing is applied, the high-income traveler demand increases up to the toll amount of $4. It happens from the interaction between the demand sensitivities with respect to the amount of toll. In this case, the amount of toll was enough to reduce the low-income trips, but not enough to reduce high-income trips. Therefore, we may consider the minimum amount of toll that reduces demand in all classes to guarantee the minimum equity.

In summary, the cordon-based pricing scheme is a more effective way to improve the overall network condition than the link-based pricing scheme. However, from the equity point of view, the cordon-based scheme may be the better way since it tends to reduce the low-income traveler’s trips. It means that the cordon-based scheme may deprive low-income travelers of the opportunity for economic activity. To be fair to low-income travelers, we should develop an advanced approach to determine the optimal location of toll and the optimal amount of toll. We may prefer the cordon-based pricing, but some links should be free to guarantee the activity opportunity for the low-income class.

5. Discussions

To solve NDP, a single traveler class-fixed demand assignment model has been mainly used. However, to analyze the interaction among multiple traveler classes and trip purposes, we need a model for multi-class traffic assignment with variable demand model. This paper introduced a multi-class, multi-purpose trip assignment model with variable demand. We
Kim, Lim, and Oh

attempted to develop a realistic travel demand model based on a linearity assumption, and
applied it to multi-class multi-purpose variable traffic assignment model.

There is increasing interest in social equity problems. Considering that the most travel
demand is derived from economic activities, the equity problem has also been an issue in
transportation network management. The model developed in this paper is capable of
investigating the interaction among multiple traveler classes as well as among trip demand
by purposes by incorporating the elastic demand. The model was applied to the congestion
pricing problem in this study, and we found a couple of interesting points by comparing two
pricing schemes. We found that the cordon-based toll was more effective, but might cause
more equity problem by discouraging more low-income travel. A suggestion was to leave a
free corridor for low-income travel.

Although not attempted in this study, the model in this study can further improved by
treating consumer surplus, social welfare, and social net benefit endogenously. By doing this,
the models will be able to find optimal and/or second-best solution for NDP problem. As a
concluding remark, we like to emphasize the importance of demand models that realistically
reflect traveler behavior.

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