Experimental Study on the Sensitivity Analysis for Dynamic Traffic Assignment

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Abstract:
Dynamic Traffic Assignment (DTA) has received much attention for the last several decades. Varieties of methodologies and techniques have been developed and deployed into the modeling and solution algorithms for DTA. However, it still remains unsolved how the variations of input parameters impact the results of DTA model. This paper aims to bridge this gap by first defining the sensitivity of the solution of DTA, then proposing the measurement methods of sensitivity in DTA. Three scenarios for analyzing the sensitivity in DTA are also discussed in this paper together with some numerical experiments.

1. Introduction and Motivation

DTA remains a popular research topic in transportation study for the past two decades. A large number of methods have been proposed by many researchers to model and solve the DTA problem from the macroscopic point of view (Ran, et al. 1996) or microscopic point of view (Mahmassani, 1998). However, there are few research work that focused on the sensitivity or stability analysis, which is how the perturbations of the input parameters can impact the solution results of DTA. Fiacco (1983) defined sensitivity analysis as the impact of local perturbation over solution and stability analysis as the impact of finite perturbation over solution behavior. However, in this paper, we will not differentiate
them. Rather, we use “sensitivity analysis” to represent the impacts of perturbations of input parameters over solution results of DTA.

There are several important applications of sensitivity analysis of DTA model. First, it would indicate those input parameters that are sensitive to the assignment solution, and therefore, help us to identify the data items that deserve the most effort to obtain accurately. For example, if the assignment solution is seen to be particularly sensitive to the link travel time function, extra effort may be worth expending on the data collection in order to better calibrate this function. Secondly, if the results from the traffic assignment model are to be believed, some assessments on the robustness of the model, i.e., how vulnerable these results are to inaccuracy to the input data, are required. Thirdly, information from sensitivity analysis allows “what-if” analysis without having to re-solve the assignment problem, which generally requires expensive computational efforts. Typical question for “what-if” analysis could be how additional cost on a route or link will impact the assignment of traffic in the network. Finally, sensitivity analysis results also provide us knowledge on the network reliability and help us to identify the most critical links in the network.

Smith (1984) is among the first to study on the sensitivity of traffic assignment. He deployed dynamic system to model the traffic equilibrium problem and obtained some stability properties of the system using Lyapunov theorem. Dafermos (1984, 1988) applied Variational Inequality (VI) to model the traffic assignment problem and investigated the sensitivity properties of these problems. They obtained some insightful results of the impacts of small changes of traffic demand, supply and cost function over the equilibrium pattern, as well as the directions of the changes of traffic flow and cost function. Their results were based on certain monotonicity and continuity (or Lipschitz continuity) assumptions for both the feasible region and the cost function of the problem. Tobin (1986, 1988) and Friesz (1990) extended the work of Fiacco (1983) from nonlinear programming areas to VI fields and provided the solution behavior of DTA over a small change of some input parameters. First and second order KKT conditions are the principal tools in their study. All the aforementioned research work focused on static traffic assignment without considering the dynamic characteristics of either traffic network or driver behavior.
Recently, Nagurney (1999), Zhang (1995, 1997) and Dupuis (1993) deployed the projected dynamical system (PDS) method into the modeling of traffic assignment problem. They stated that the set of stationary points of PDS is equivalent to the optimal solution of corresponding VI problem. Further, stability and sensitivity results were also achieved under certain monotonicity and continuity assumptions. They also claimed that PDS could capture the “dynamic” property of the system. However, the “dynamics” here is more like a “convergence process” rather than the time-dependent characteristics of the DTA. Leurent (1998) studied the sensitivity property of DTA under dynamic case using similar methods with Tobin (1986, 1988) and Friese (1990). However, his work mainly focused on dual criteria traffic assignment.

The contributions of this paper are that we give the definition of sensitivity analysis in dynamic traffic assignment, together with some introductory sensitivity theorems. At the same time, we propose the measurement of the “degree” of sensitivity and investigate the three scenarios of the perturbation for DTA in transportation network modeling.

This paper is organized as following. Sensitivity analysis in DTA is defined in section 2, including both general mathematical definition and VI definition. In section 3, we apply the results from section 2 to three scenarios in DTA. Measurement of sensitivity analysis in DTA is discussed in section 4 with the introduction of Sensitivity Index (SI). Some numerical experiments are given in section 5, and conclusion remarks and some future works are listed in the final section.

2. Problem Definition

The objective of sensitivity analysis in DTA is trying to determine how the small changes of transportation parameters, like travel demand, supply or driver behavior, will impact the traffic equilibrium pattern. It can help to answer the following two questions:

Will the small changes in travel demand, and/or supply, and/or driver behavior cause small changes on optimal flow and travel cost of DTA?
If there are small changes in travel demand, and/or supply, and/or driver behavior, what are the directions of the changes of the optimal traffic flow and travel cost of DTA?

In other words, sensitivity analysis will investigate if the traffic equilibrium pattern will depend continuously upon the transportation parameters (Dafermos, 1984).

2.1 Mathematical Definition

2.1.1 Static case

The small changes of input parameters mentioned above are denoted as “perturbation”. We are to find the impacts of perturbation over DTA solution. In static traffic assignment, perturbation is denoted as a vector $\lambda$ mathematically with $n$ elements, where $n$ is the number of parameters. In another word, we are interested in:

$$S = \nabla_{\lambda} F$$  \hspace{1cm} (1)

where $F$ is the solution of static traffic assignment, like route flow vector or link flow vector. $\lambda$ is the vector of the perturbation parameter. $\nabla$ denotes partial derivative. Therefore, equation (1) is actually a Jacobian matrix of vector $F$ over vector $\lambda$.

If we model static traffic assignment as a non-linear programming problem, then some results of sensitivity analysis of equation (1) can be found in Fiacco’s book (1983).

2.1.2 Dynamic case

Under dynamic case of traffic assignment, sensitivity analysis becomes much more complicated due to the introduction of the time dimension. Since perturbation of one time interval is very likely to impact the solution of DTA at another time, we have to study the impacts of a perturbation at time $t$ to the solution vector at any time $t'$. Denotes it as:

$$S(t,t') = \nabla_{\lambda} F(t,t')$$  \hspace{1cm} (2)

The right side of equation (2) is read as "the partial derivative of solution at time $t'$, $F(t')$, over the perturbation at time t, $\lambda(t)$".
2.4.3 Types of Perturbation

The complication of equation (2) also lies in the property of time-dependent perturbation \( \lambda(t) \). Perturbations can be divided into two groups: Unpredictable Perturbation (UPP) and Predictable Perturbation (PP).

**Unpredictable Perturbation (UPP)**

Intuitively, the perturbation of some time should have no effect over the solution of the time before this perturbation ever happened. We call this kind of perturbation as unpredictable perturbation (UPP). Most of the perturbations in DTA analysis are UPP, like OD perturbation, incident, congestion and many others. UPP type of perturbation has a very important property regarding its impacts over the DTA solution:

\[
S(t, t') = 0 \text{ for } t > t' \tag{3}
\]

**Predictable Perturbation (PP)**

Nevertheless, other perturbations, like road constructions or special events, can be predicted and transmitted to the public, thus they have impacts over the traffic equilibrium even before they happen. We denote this kind of perturbations as Predictable Perturbation (PP). For PP perturbation, equation (3) is not valid any more.

Equation (2) is a general formulation for sensitivity analysis in dynamic traffic assignment. More specific equations can be obtained for a particular problem according to the properties of the feasible region and the nature of the problem.
2.2 VI Definition

Static traffic assignment problem was modeled by Dafermos and Nagurney by using VI methods. Later on, Ran and Boyce (1996) extended it to the dynamic counterpart as following (an Ideal/Instantaneous DTA problem using VI model):

\[
\begin{bmatrix}
\sum_{n} \sum_{p} g(t)[f_p^*(t) - f_p^*(t)] \\
f_p^*(t) \geq 0
\end{bmatrix} \geq 0
\]  \hspace{1cm} (4)

where \( g(t) \) is the generalized cost function and \( f_p^*(t) \) is path flow.

Normally, we can model DTA using the following VI formulation:

\[
\begin{bmatrix}
< G(F(t), t), F(t) - F^*(t) > \\
F(t) \in K(t)
\end{bmatrix} \geq 0 \hspace{1cm} \text{for each } t \in [0, T] \hspace{1cm} (5)
\]

where \( G \) is the time-dependent generalized cost function defined on \( R^m \times R \), \( m \) is the number of paths in the network. \( F \) is the time-dependent path flow vector and \( F \in R^m \times R \). \( K \) is the time-dependent constraints of path flow vector and \( K \in R^m \times R \). If \( t \) is fixed, then \( G(F(t), t) \) is defined on \( R^m \), \( F(t) \) is defined on \( R^m \), and \( K(t) \in R^m \). \( <x,y> \) denotes the inner product of two vectors \( x \) and \( y \).

If we denote perturbation as \( \delta(t) \in \Lambda \subset R^4 \times R \), equation (5) can be rewritten as:

\[
\begin{bmatrix}
< G(F(t), t), F(t) - F^*(t) > \\
F(t) \in K(\delta(t), t)
\end{bmatrix} \geq 0 \hspace{1cm} \text{for each } t \in [0, T] \hspace{1cm} (6)
\]

By extending Nagurney’s static definition for sensitivity analysis in traffic assignment (1999), we can have the following sensitivity definition for DTA.

Definition 1. Sensitivity of VI Model in DTA

Suppose \( F^*(t) \) is the solution of equation (6) for \( \delta(t) = \Lambda(t) \). If \( \forall \delta(t) \in N_\delta(\Lambda(t)) \), the solution of (6) is \( F^*(t) \) and \( F^*(t) \in N_\delta(F^*(t)) \), we then define VI (6) is not sensitive
with parameter $\lambda(t)$; otherwise, it is sensitive. Here, $N_2(\lambda(t))$ denotes the neighbor of $\lambda(t)$ at $\lambda(t)$ and $N_2(F^*(t))$ is the neighbor of $F(t)$ at $F^*(t)$.

Dynamic parametric sensitivity analysis can also be generalized from the static parametric sensitivity analysis proposed by Nagurney (1999).

Theorem 1. Dynamic Parametric Sensitivity Analysis

Suppose $\overline{F}^*(t)$ is the solution of (6) for $\lambda(t) = \lambda(t)$. Assume $G(\lambda(t), F(t), t)$ satisfies the local monotonicity condition and the local Lipschitz condition for any $F_1(t), F_2(t)$ in $B(\overline{F}^*(t))$. Suppose $\overline{F}^*(t, \lambda)$ is continuous (or Lipschitz continuous) in $\lambda(t)$ at $\lambda(t)$ and that for any $\overline{Y} \in B(\overline{F}^*(t))$, the map

$$\lambda(t) \mapsto \Pi_{F(\lambda(t)) = \overline{F}^*(t)}(\overline{Y})$$

is continuous (or Lipschitz continuous) at $\lambda(t) = \lambda(t)$. Then there exists a neighborhood $l(t) \in \Lambda(t)$ of $\lambda(t)$ such that for every $\lambda(t) \in l(t)$, the VI (6) has a unique solution $F^*(t, \lambda)$ in the interior of $B(\overline{F}^*(t))$, and $F^*(t, \lambda)$ is continuous (or Lipschitz continuous) at $\lambda(t) = \lambda(t)$.

In the above theorem, $B(\overline{F}^*(t))$ is the closure of a ball in $R^n \times R$ centered at $\overline{F}^*(t)$. And the definitions of monotonicity, Lipschitz continuity and projection are referred in Nagurney’s book (1999).

Rather than providing the general formulation (equation 2) for DTA sensitivity analysis, the theorem above identifies the upper bound of the solution perturbation over given perturbation of parameters. In other words, if the VI formulation (6) and the parameter perturbation can both satisfy the conditions in theorem 1, it can be guaranteed that a small perturbation of parameters can only cause a small perturbation in the solution.
2.3 Perturbation Propagation

Similar as the flow propagation of DTA, we can formulate the perturbation propagation as the following:

\[ \Delta x_{\alpha p}^r(t) = \sum_{p} \left( \Delta x_{\alpha p}^r(t + \tau_a(t)) - \Delta x_{\alpha p}^r(t) + \Delta E_{\alpha p}^r(t + \tau_a(t)) - \Delta E_{\alpha p}^r(t) \right) \]  

(7)

where \( \alpha \) is a link in the network, \( p \) is the path link \( \alpha \) belongs to, \( \tilde{p} \) is the sub-path from end node of link \( \alpha \) to destination \( D \), \( \tau_a(t) \) is the link travel time at time interval \( t \) for link \( \alpha \) and \( \Delta x_{\alpha p}^r(t) \) is the perturbation of link flow for link \( \alpha \) in path \( p \) connecting origin \( r \) to destination \( s \).

3. Three Scenarios of Sensitivity Analysis in DTA

Sensitivity analysis in DTA can be categorized into three scenarios, each of which has different characteristics. We solely consider VI models in DTA for these three scenarios in this part.

3.1 Perturbation of OD Demand

If we only have small changes of OD demands, then only the \( K(t) \) in equation (6) need to be changed with parameter \( \lambda \) and \( G(t) \) will remain unchanged. So Equation (6) becomes:

\[
\begin{align*}
\langle G(F(t), t), F(t) - F^*(t) \rangle & \geq 0 \\
F(t) & \in K(\lambda(t), t)
\end{align*}
\]

(8)

3.2 Perturbation of Driver’s behavior

In real world, several cases may be included in this scenario and each of which deals with the subjective factors in driver’s decision making regarding route choice. For instances,
change of disutility function and change of driver’s perception over travel time are both treated as the perturbation of driver’s behavior.

In this scenario, only $G(t)$ will vary with $\lambda$ and $K(t)$ will not, so (6) becomes

\[
\begin{bmatrix}
<\delta\lambda(t), F(t), \tau, F(t) - F^*(t)> \\
F(t) \in K(t)
\end{bmatrix} \geq 0
\]

(9)

3.3 Perturbation of Network Supply

This scenario includes adding new links to the network or removing existing links from it. On the other hand, change of the cost function of the network, like improving the link travel time function by adding more lanes, is also contained in this scenario. Baraess’s Paradox may happen in this scenario (Murchland, 1970; Frank, 1981).

Under this particular scenario, both $K(t)$ and $G(t)$ will change with $\lambda$, we have to solve equation (6) directly to perform the sensitivity analysis.

4. Measurement of Sensitivity Analysis in DTA

4.1 Mathematical Definition of Measurement

Although we gave the mathematical definition of sensitivity for DTA above, it is generally very difficult to obtain $S(t, \tau)$, the doubly time-dependent partial derivative of solution over perturbation. Therefore, we define some measurement of sensitivity for DTA model.

4.1.1 Sensitivity Index (SI)

In static case, denote:

\[
a_s = \left[ \left\{ V, F \right\} \right]
\]

(10)
where $F_j$ is the $j$th element of vector $F$, and $p$ denotes $p$ norm. In practice, 1-norm, 2-norm or $\infty$-norm is simply deployed.

Then we define Sensitivity Index (SI) as following:

$$SI = \|a\|_p$$  \hspace{1cm} (11)

where $a$ is the vector of all $a_i$.

Under dynamic traffic assignment, we denote the sensitivity index for perturbation at time $t$ over the solution at time $t'$ as:

$$SI(t,t') = \nabla F(t,t')$$  \hspace{1cm} (12)

Furthermore, we define the overall SI for perturbation over solution during the time period $[0, T]$ as:

$$SI = \frac{\int_0^T \int_0^T \nabla F(t,t') dt'dt}{T^2}$$  \hspace{1cm} (13)

Hence, the sensitivity index we defined here is kind of “overall” or “integrated” sensitivity performance of the DTA solution over perturbation rather than the individual impact as expressed in equation (2).

4.2 Sensitivity Measurement for Dynamic VI Model

For the VI model of DTA (6), we denote:

$$\Delta \lambda(t) = \lambda(t) - \lambda(t)$$  \hspace{1cm} (14)

$$\Delta F(t) = F^*(t) - F^*(t).$$  \hspace{1cm} (15)

Normalize $\Delta \lambda(t)$ and $\Delta F(t)$, we have:

$$\tilde{\delta \lambda(t)} = \Delta \lambda(t) / \lambda(t) = \left( \frac{\Delta \lambda_1(t)}{\lambda_1(t)}, \frac{\Delta \lambda_2(t)}{\lambda_2(t)}, \cdots, \frac{\Delta \lambda_n(t)}{\lambda_n(t)} \right)^T$$  \hspace{1cm} (16)
\[ \delta F(t) = \Delta F^*(t) \cdot \frac{\Delta F^*_1(t)}{F^*_1(t)} \cdot \frac{\Delta F^*_2(t)}{F^*_2(t)} \cdot \ldots \cdot \frac{\Delta F^*_m(t)}{F^*_m(t)} \]  

(17)

Finally, we have the sensitivity index defined for dynamic VI model as:

\[ SI = \frac{\int_{0}^{T} \left\| \delta F(t) \right\|_p dt}{\int_{0}^{T} \left\| \delta \lambda(t) \right\|_p dt} = \frac{\int_{0}^{T} \left\| \delta F(t) \right\|_p dt}{\int_{0}^{T} \left\| \delta \lambda(t) \right\|_p dt} \]  

(18)

Therefore, the level of the sensitivity property for the DTA solution can be represented by SI. Smaller SI indicates that the solution is less sensitive with the perturbation of given parameters. While SI becomes bigger, the solution becomes more sensitive. Thus we can pre-define some threshold value \( \sigma \), if \( SI \leq \sigma \), we call the solution is not sensitive with the given parameters, otherwise, it is sensitive.

5. Numerical Experiments

In this section, we present some numerical results for sensitivity analysis in different scenarios of dynamic traffic assignment to verify the aforementioned theorem and measurement. We first present briefly on the DTA model in which our sensitivity analysis will build upon, and then give the numerical results based on the testing network.

5.1 Analytical DTA Model

5.1.1 Ideal SDUO Model

In this paper, the ideal stochastic dynamic traffic assignment model (SDUO model) (Ran, et al.1996) is selected for our testing purpose. Each driver will choose his/her route based on his/her perceived route travel time at each time interval. Due to variations in travelers’ perceptions of travel times, travelers do not always end up picking the correct minimum travel time route. Route choice models proposed under this approach can have different
specifications according modeling assumptions on the random error term. The two commonly used random error terms are Gumbel and normal distributions, which result in the logit- and probit-based route choice models. Normally distributed travel time perception is chosen in our experiments. That is, the perceived link travel time for link $a$ at time $t$ is:

$$
\bar{\tau}_a(t) = \tau_a(t) + N(\mu_a(t), \theta_a^2(t))
$$

(19)

where $\tau_a(t)$ is the estimated actual link travel time for link $a$ at time $t$, $\mu_a(t)$ and $\theta_a(t)$ are mean and standard deviation of the perception error.

In practice, it is reasonable to assume the mean of travel time perception error is zero and its standard deviation is proportional to the estimated actual link travel time. Thus, equation (19) becomes:

$$
\bar{\tau}_a(t) = \tau_a(t) + N(0, \tau_a^2(t) \cdot \sigma_a^2(t))
$$

(20)

5.1.2 Link Travel Time Function

In this paper, we use following travel time functions for freeway segments.

$$
c_a = L / speed
$$

(21)

where $L$ is the length of the link and $speed$ is the speed of the link determined by the following modified Greenshield function:

$$
speed = cf sf + (1 - c / c_{max}) \cdot (ffsf - cf sf)
$$

(22)

where

$cf sf$: jam speed.

$ffsf$: free flow speed, can simply use speed limit.

$c$: current density.

$c_{max}$: jam density.
5.2 Testing Network

The testing network is indicated in Figure 1 with seven nodes and eight links. The length of each link is 2.5 miles with detailed link information depicted in Table 1. Some cases for the three scenarios mentioned in section 3 will be investigated in our experiments. All the experiments here share the following common input characteristics:

- Origin is node 1 and destination is node 2.
- The O-D flows are 30 vehicles for each of the five 60-second periods (equivalent to a flow of 1800 vehicles per hour). The total flows from Origin to Destination for the whole analysis period is 130.
- All the links are simulated as freeway and the free flow speed is 50 miles per hour.
- The original standard deviation for perception error of link travel time, $\sigma_{o}(t)$, is shown in Table 2. $\sigma_{o}(t)$ will not vary over different time interval.
- Each perturbed parameter is tested individually, and the range of perturbation is from -50% to +50% in order to get a converged value of SI.
- Stopping criterion for DTA is 1% (maximum difference of link inflow between two adjacent iterations).
- We set 1 as the threshold of SI. That is, if SI is larger than 1, the DTA solution is sensitive over the perturbation, otherwise, the solution is not sensitive.

![Diagram of Testing Network](image)

(The underlined number is the link number)

**Fig.1 Testing Network**
Table 1. Link Information

<table>
<thead>
<tr>
<th>Link Number</th>
<th>Start Node</th>
<th>End Node</th>
<th>Length (miles)</th>
<th>Capacity (% of Vehl.)</th>
<th># of Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2.5</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>2.5</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>5</td>
<td>2.5</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>7</td>
<td>2.5</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>6</td>
<td>2.5</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>7</td>
<td>3.1</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>2</td>
<td>2.5</td>
<td>4400</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>2</td>
<td>2.5</td>
<td>4400°</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2. Standard Deviation of Travel Time Perception for Each Link

<table>
<thead>
<tr>
<th>Link Number</th>
<th>Standard Deviation (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
</tr>
</tbody>
</table>

5.3 Experiments

5.3.1 OD Perturbation

OD perturbation is tested for all the 5 time intervals separately. The results are shown in Figure 2. SI of OD perturbation varies tremendously when the perturbation is small (less than 10%) due to the computational error and discrete error of DTA model. While perturbation becomes larger, the SI also comes to a converged state. For example, SI is roughly 0.4 for OD perturbation at time interval 3 and Figure 3 gives the converged SI for each time interval.
These figures are all symmetric, which means that the OD perturbation on each side of the original OD volume has roughly the same effect over the final DTA solution. Note that this conclusion is valid also for supply perturbation and perception of driver’s behavior which we will investigate later. For all the five cases, the converged value of SI is within the range of 0.4 ~ 0.6.

An interesting observation from Figure 3 is that SI reaches its minimum at time interval 3. It is easy to explain why SI decreases from interval 1 to interval 3 due to the fact that OD perturbation is unpredictable perturbation (UPP). Since the OD perturbation of later time interval (say 3) has no effect on former time interval (1 and 2), it is reasonable that the SI for time interval 3 is less than those for interval 1 and 2. This explanation should also hold for time interval 4 and 5, which should in turn lead to the result that SI at interval 3 is larger than those of interval 4 and 5, and SI at time 5 is the minimum. However, since the O-D flows are only for the 5 time intervals in the beginning, perturbations in the last interval 4 and 5 will be more sensitive than that in the interval 3.

5.3.2 Supply Perturbation
We only consider the variation of link capacity for this scenario, which corresponds to many real world cases, like incident, construction, etc. For the purpose of simplification, we solely vary the capacity of link 2 from -50% ~ +50% as indicated above. Figure 4 shows the sensitivity analysis results for this case.

From Figure 4 we can see that SI for capacity perturbation converges to some stable state on both negative and positive sides of the original value (0), the converged value is about 0.2.

5.3.3 Driver Behavior Perturbation
In this scenario, we just consider the sensitivity index of our DTA solution over the perturbation of the standard deviation of perceived link travel time distribution, $\sigma_g(t)$, as indicated in equation (20). Figure 5 shows the sensitivity analysis results for this case.
5.3.4 Analysis of the experiments

From the results of our experiments, we can see firstly that the perturbation has almost the same results on both the negative and positive sides of the original parameter. This symmetric property of the impacts of perturbation is partially due to the symmetry of our network layout. Also, it indicates that SI is a valid measurement for sensitivity analysis in DTA.

Secondly, the value of SI is not fixed over different values of perturbation. This result verifies the complexity and difficulty of sensitivity analysis for dynamic traffic assignment. Since SI varies with perturbation, it is hard for us to judge whether the solution of DTA is sensitive upon a given input parameters. To solve this problem, in this paper, we just simply choose the value of SI when perturbation is 10% as our benchmark. Thus, we can conclude that our DTA solution (or algorithm) is not sensitive with OD perturbation at time interval 3, 4 and 5, while it is sensitive when OD perturbs at time interval 1 and 2. On the other hand, the DTA solution is not sensitive with the perturbation of the capacity of link 2, neither with the standard deviation of perceived link travel time distribution $\sigma_t(t)$. Therefore, the SDUO algorithm we are testing is pretty stable in term of the parameters’ perturbations.

However, the sensitivity results achieved for the experiments can not be applied directly to other networks or DTA algorithms. Similar investigation must also be deployed to them before you draw any conclusion.

6. Conclusion and Future work

In this paper, we defined the sensitivity analysis in dynamic traffic assignment problem which investigates the impacts of perturbed parameters upon DTA solution in both space and time dimensions. One measurement method, sensitivity index (SI), is proposed to measure the level of sensitivity. Experimental results indicate the validation of our definition and method of SI. The proposed framework can help researchers of DTA to further understand the solution algorithm behavior of DTA and focus on the most critical input parameters.
More DTA models and network layouts need to be tested to verify further the validation of our proposed theorem, definition and measurement method. Most importantly, theoretical investigations are still needed to derive more generalized theorems and formulations of sensitivity analysis to fit into more scenarios. The goal of sensitivity analysis is to achieve the time-dependent partial derivative of DTA solution over perturbed parameters, although in reality it is very difficult to obtain if not impossible.

![SI for OD Perturbation](image)

**Fig. 2 SI for OD Perturbation**
Fig. 3 Converged SI for OD Perturbation

Fig. 4 SI for Capacity Perturbation
Fig. 5 SI for Perturbation on the Variance of Perception Error

References:


