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Effects of Less-Equilibrated Data on Travel Choice Model Estimation

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ABSTRACT

Most econometric models assume steady state conditions and a fully equilibrated system when estimating unknown coefficients from real-world data. However, the estimated model can be biased when the data set used for the model estimation was drawn from not or less equilibrated traveler behavior. The resulting biased model could lead to a misunderstanding of the system. This paper examines such effects on discrete choice model estimation, by performing Monte Carlo simulation experiments. We employ a day-to-day dynamic evolutionary framework in order to observe changes in traveler's choice and compare the estimated results during the adjustment process with the true behavior parameters.

INTRODUCTION

One of the major assumptions behind the estimation of econometric models, particularly in discrete choice analysis, is to suppose that the system under study has reached a certain level of equilibrium. In fact, most econometric models assume steady state conditions and a fully equilibrated system when estimating unknown coefficients from real-world data. That is, it is assumed that the data utilized for estimation are drawn from a fully equilibrated behavioral system; however, in reality this is not necessarily true. Sometimes, it is not clear whether the sample of observed choices made by decision makers when confronted with a choice situation has been taken from a fully equilibrated system or not. If the model were estimated using data drawn from a less-equilibrated state (quite far from the real equilibrium in the economic sense), the model estimation could be biased. The resulting biased model could seriously cause a misunderstanding of the system that should be explained by the model, and such a bias would not be completely associated to inherent model specification problems, but rather due to the utilization of unreliable data during the estimation procedure.

In this paper we examine the effects of using less-equilibrated data on discrete choice model estimation, by performing Monte Carlo simulation experiments. Specifically, we explore a proper day-to-day dynamic evolutionary framework in order to observe changes in traveler's choice. In addition, we investigate the conditions under which the bias in the estimation, due to relying on less-equilibrated data, could be minimized.

In order to generate less-equilibrated data, we rely on the travelers' evolving behavior in their travel choice from day to day. All intermediate states generated in the simulation are assumed to be less-equilibrated states until equilibrium conditions are reached. Consequently, data drawn from the intermediate states are regarded as less-equilibrated data. Finally estimated results are compared with the true model used in day-to-day dynamics framework. Two numerical tests are conducted: one for departure time choice, the other for both departure time and route choice.

TRAVELER BEHAVIOR AND DAY-TO-DAY EVOLUTION APPROACH

There have been extensive studies on traveler's choice behavior during the last few decades. Regarding traveler's choice behavior, (1) shows a hierarchical structure in travel choice behavior as a sequence of destination, mode, and route choices. (2) and (3) analyzed commuter's departure time choice behavior by examining traveler's scheduling costs and random utility.

With regard to the network problem (4) assumes that travelers select their choice in order to minimize their perceived costs, which gives rise to the stochastic user equilibrium model. The stochastic user equilibrium model was further developed for departure time choice and day-to-day variation (5). (6) and (7) proposed a stochastic process approach for analyzing day-to-day dynamics in a transportation network. The author assumes that the traveler's choice is time homogeneous.

As a different approach, a day-to-day dynamics framework was first proposed by (8) for an experimental analysis of dynamic interaction. The day-to-day dynamics

approach in (8) considers the choice of departure time and route according to the schedule delay. The schedule delay is governed by traveler's daily learning process. The simulation program, DYNASMART (9) has been extensively used for simulating these types of processes under several conditions. Recently, (10) investigated the day-to-day evolution of network flows under real-time information and responsive signal control. Unlike other analytical equilibrium models, the day-to-day dynamic framework enables the analysis of traveler choice changes over a time horizon based on traveler's updated knowledge.

DESIGN OF THE EXPERIMENT

A simulation framework based on day-to-day dynamics is applied for observing changes of traveler's choice. Five modules comprise our experiment: trip choice, network performance, perception update, data generation, and model estimation. We suppose that users perceive the travel time in a certain manner, and according to that perception, they take a certain decision of travel in the way we will explain as part of the trip choice module. Users update their perception of travel time and the next time they perform the same trip, they could make a different choice based upon their previous experience, as in any learning process. If we repeat the experiment several times, we may realize that after the n^{th} repetition (in our context, this could be the n^{th} day), users do not change their choice anymore, since they have accumulated enough knowledge about that specific trip. At that point, we will assume that the system has reached equilibrium, and the n^{th} day is what we call the equilibrium threshold. Any trip executed before the threshold day is part of the learning process, and therefore any information drawn from such instance will be considered as less-equilibrated data. The overall experiment framework is as shown in Figure 1.

<Insert FIGURE 1 here>

Traveler's choices are updated day-to-day based on their utilities that are captured by an assumed true model until their perceived travel costs are equalized to the actual costs. Traveler's behavior data are collected after each repetition (hereafter, iteration), and these are regarded as intermediate data that may not have been equilibrated. These less-equilibrated data obtained as intermediate products are used for model estimation. The estimated model coefficients are expected to be different from the true model assumed at the beginning.

The objective of this experiment is to compare the estimated results under less-equilibrated conditions, and analyze the difference (bias) between the estimated models and the assumed true model. The details and assumptions of each module of this experiment are highlighted next:

Trip Choice Model

Trip choice is made based on traveler's previous experience. The discrete-choice model utilized here is the additive random-utility model of (11), which supposes a decision

maker n facing discrete alternatives $j = 1, \dots, J$ who chooses the one that maximizes utility as given by

$$U_{jn} = V(z_{jn}, s_n, \beta) + \xi_{jn} \quad (1)$$

where V is a function known as the systematic utility, z_{jn} is a vector of attributes of the alternatives as they apply to this decision maker, s_n is a vector of socioeconomic or demographic characteristics of the decision maker, β is a vector of unknown coefficients, and ξ_{jn} is an unobservable component of utility which captures the dispersion of choices made by observationally identical decision makers. The model becomes complete by specifying the joint probability distribution for ξ_{jn} , $j = 1, \dots, J$.

Traveler's choice is made based on traveler's perceived costs, such as perceived travel time, schedule delay, toll, etc. The specific attributes of the alternatives included in our experiment are the following

TIME: Travel time in minutes

SD: Schedule delay (Arrival time for a given alternative minus official work-start time)

SDE: Schedule delay early = -SD if SD < 0

SDL: Schedule delay late = SD if SD > 0

TOLL: Amount of toll imposed in cents

In the context of this paper, two different trip choice structures are experimented with. In the first case the model includes only travel time choice problem while the second model includes both travel time and route choice problem.

In the first case, we are dealing with just a departure time choice problem. We address the alternatives regarding the arrival time at work; specifically we estimate a model of choice among discrete alternatives, each representing arrival at work within a particular five-minute interval (2). The choice set consists of 12 intervals centered from 40 minutes before the 15 minutes after the official work-start time for the individual. A multinomial logit model is assumed, implying that the J random terms are independently and identically distributed (iid) with the extreme-value distribution. The systematic utility V_{in} is written as a linear function of TIME, SDE and SDL. Analytically, the choice probability for each alternative and the systematic utility specification is as follows:

$$P_{in} = \frac{e^{V_{in}}}{\sum_{j=1}^J e^{V_{jn}}}; \quad V_{in} = \beta_1 TIME_{in} + \beta_2 SDE_{in} + \beta_3 SDL_{in} \quad (2)$$

where the variable TIME for each alternative is obtained by asking how long would it take to travel to work if one were to arrive within each of the aforementioned discrete intervals.

In the second case, on the other hand, we are interested in studying the joint decision of departure time and route choice. In this case, it seems that a model like that

defined above is not appropriate, since we surmise that the independence from irrelevant alternatives (IIA) holds for some pairs of alternatives but not all. In other words, there exists some correlation among alternatives. A GEV specification has been designed to handle situations like these, and it will be assumed in this case (nested specification, as defined by (12)).

We use the notation introduced by (12) for partitioning the whole set of alternatives into two levels: at a superior level (node level) we set the route choice, including regular road and toll road, and at a lower level (alternative level) we set the departure time choice, considering the same departure time alternatives as those described above. Graphically,

<Insert FIGURE 2 here>

Analytically, the utility can be decomposed into two pieces: the first portion is constant for all alternatives attached to a node s , W_s , and a portion that varies over both nodes and alternatives, denoted by V_{ks} associated to alternative k , node s . The probability of choosing alternative k attached to node s ($s=1$ regular road, $s=2$ toll road) is

$$P_k = P(k/s)P(s) \quad (3)$$

where

$$P(k/s) = \frac{e^{V_{ks}/\rho}}{\sum_{j \in B_s} e^{V_{jk}/\rho}} = \frac{e^{V_{ks}/\rho}}{e^{I_s}} \quad (4)$$

$$P(s) = e^{W_s + \rho I_s} / \sum_r e^{W_r + \rho I_r} \quad (5)$$

and $I_s = \log \sum_{j \in B_s} e^{V_{jk}/\rho}$ (inclusive value of node s). Note that we assume a parameter ρ common to both nodes 1 and 2. In addition,

$$\begin{aligned} V_{ks} &= \beta_1 TIME_{ks} + \beta_2 SDE_{ks} + \beta_3 SDL_{ks} \\ W_1 &= 0 \\ W_2 &= \beta_4 TOLL_2 \end{aligned} \quad (6)$$

Network Performance

The network performance is calculated based on the flow on the route during certain time interval Δt . In this simulation framework, the network travel time is calculated based on a cost function known as BPR (Bureau of Public Road) cost function. The analytical specification of such a function is as follows:

$$T = T_0 \times \left(1 + 0.15 \cdot (V/C)^4 \right) \quad (7)$$

where, T = average travel time (min/user)
 T_0 = free flow average travel time (min/user)
 V = traffic volume
 C = capacity of route

Note that a static link performance function is being used, so it may result in unrealistic travel times that violate the first in first out (FIFO) conditions. To prevent such cases, travel time associated to later departure is constrained to be greater than travel time corresponding to earlier departure.

Perception Update

This section deals with the learning procedure explained above. This procedure is fundamental in order to mimic the real process of approaching from less-equilibrated states towards a final equilibrium when acquiring experience from repeated events. In this experiment, travel times are updated in a day-to-day evolution manner.

Notice that the role of this module in the experiment is fundamental. In fact, the way in which we update the travel time could significantly affect the model's convergence. In this paper, to ensure convergence, the well-known method of average success (MSA) is applied in order to update the perceived travel time from information of previous perceived travel times. The updated estimation of travel time is given by the following expression:

$$T(\alpha + 1) = (\alpha \cdot T(\alpha) + T_e(\alpha)) / (1 + \alpha) \quad (8)$$

where $T(\alpha + 1)$ is the *updated* perceived travel time after the trip on day/iteration α , $T(\alpha)$ is the *perceived* travel time before the trip on day/iteration α , and $T_e(\alpha)$ is the *experienced* travel time during the trip of day α . Here, α represents the number of iterations, repetitions or days, which can also be interpreted as the relative importance of previous travel time measures in this model.

Data Generation

Data are collected at every iteration using Monte Carlo simulation techniques. The idea is to generate a dataset based on the assumed true modes for each case (for both, the multinomial and nested models). A uniform distribution is assumed for generating random values. The independent variables as well as the probability of selecting each alternative are constructed from simulated data using Monte Carlo. We experiment with 50 replications of three different sample sizes (100, 300, and 500) so that the effects of both sample size and number of replications can be empirically examined.

Model Estimation

Using the simulated data, discrete choice models are estimated, by maximizing the Log-Likelihood function at any case. The model structure and specification are assumed to be same as those assumed for the true models. The commercial software EVIEWS is used for model estimation. In the estimation of the GEV specification, the analytical expression of the specific log-likelihood function was coded in the software in order to get consistent standard error estimate for $\hat{\rho}$, avoiding the uncorrected estimate generated when applying the two stages sequential estimation method (12).

EMPIRICAL EXAMPLES

Problem Scenarios

As we mentioned in the last section, this experiment is based on the assumption that the true traveler's behavior model is known in advance and that the choice behavior of individuals is consistent with the true specification over time. In this experiment departure time choice and route choice problem are examined.

In the first scenario only departure time choice problem is dealt with and all travelers are assumed to be commuters whose work start times and routes are the same. Departure time choices are evenly divided into six 10-minute intervals over a one hour period. The route is assumed to be a long stretch with a 30-minute free flow travel time. All travelers select just one departure time interval among these six, and the network is flow dependent according to the model in equation (1). Total demand is fixed and equal to 60 units, and link capacity is assumed to be 15 units per 10 minutes.

The second scenario includes both departure time and route choice decisions. In the second scenario, it is assumed that a toll road is added to the network described for the first scenario. The free flow travel time on the toll road is 25 minutes, and the monetary cost of the toll is assumed to be 200 cents.

As we described in the previous section, the true models are assumed to be multinomial logit for the first scenario and nested logit model for the combined scenario. The model specification and coefficients are shown in table 1. Model coefficients for travel time, schedule delay early, and schedule delay late are drawn from (2).

<Insert TABLE 1 here>

Day-to-day Evolution in Departure Time and Route Choice

Every iteration (day) travelers' travel times are updated based on their experience, which is reflected in a potential change in their choices of departure time according to the new value of perceived travel time. As shown in Figure 3, the departure time pattern is stabilized iteration by iteration. The first three iterations are somewhat oscillating, but from iteration 4 or 5 onwards, departure time converges to a stable pattern. The stabilized pattern could be regarded as an equilibrium state. Figure 4 shows changes in departure time

pattern at specific iterations (days). We report iterations 1, 4, 6, 8, 12, and 20. Notice that at the beginning (iteration 1) we assume a homogenous departure time pattern among alternatives.

<Insert FIGURES 3 and 4 here>

In the second scenario, a toll road is added and incorporated into the system. The initial departure time pattern was assumed to be same as that obtained at the 20th iteration when simulating the first scenario, so the system converged faster than in the first case. Figure 5 shows changes of toll road demand by iteration. Toll road demand was almost stabilized to 16.7 units after the 6th iteration, which is 27.9 % of the total demand.

<Insert FIGURE 5 here>

The corresponding changes in the departure time pattern are shown in figure 6. The travelers departed 1.5 minutes later as the new toll road was added. As shown in the figure, the shape of departure time pattern was slightly shifted to the right when compared to the pattern in figure 4.

<Insert FIGURE 6 here>

Effects of Sample Size

The experiment procedure illustrated in figure 1 involves collecting simulated data drawn from the true model at every iteration, using Monte Carlo simulation techniques based on the aforementioned assumptions. The model coefficients are re-estimated based the dataset drawn from the simulation to see if the dataset can represent the true behavior. In addition, this procedure is repeated in order to get a more accurate measure of the expected value as well as an estimate of the asymptotic variance of such coefficients. In order to evaluate the quality of the re-calibrated models estimated for day j , we compute the Absolute Percentage Error associated with variable ν (APE_ν), as a function of the number of replications R . Analytically:

$$APE_\nu = \left| \frac{\left(\sum_{r=1}^R \text{coef}E_\nu^r / R \right) - \text{coef}T_\nu}{\text{coef}T_\nu} \right| \quad (9)$$

where $\text{coef}E_\nu^r$ and $\text{coef}T_\nu$ are the estimated coefficient of variable ν (replication r), and the true coefficient of ν , respectively. The iteration index has been suppressed in equation (9) for simplicity.

The APE's by sample size and variable type, for $r = 30$, are compared in Table 2 and Figure 7. As the sample size increases, the APE's decrease. In other words, the model estimation improves significantly as the size of the sample increases, which reflects a certain asymptotic tendency. This property is observed over all variables. However, no empirical evidence was observed for finding a relation between number of replications and the accuracy of model estimation considering this indicator.

<Insert TABLE 2 here>

<Insert FIGURE 7 here>

Effects of Less-Equilibrated Data on Model Estimation

In this sub-section we analyze the effect of the above-discussed learning procedure in the accuracy of the estimation for different sample sizes. The first 4 to 6 days, as it is seen in figure 8 and table 3, corresponds to clear transition states (adjustment period), far from being equilibrated. After the sixth day it is possible to observe a clearer convergence towards the true behavioral model. From that, iterations earlier than the fourth will not be taken into account for calibration purposes. After the fourth iteration, the system seems to converge towards an equilibrium state. The model was iterated up to 20 iterations (days) to ensure the convergence.

Table 3 shows the effects of less-equilibrated data on model estimation. APE drops after the fourth iteration in all three cases with different sample size. After the eighth iteration, the differences in coefficient values between the true and the estimated model turn out to be very small. From Table 3 and Figure 8, it is observed that the model estimations before eighth iteration are largely biased, confirming our original premise that using less-equilibrated data may result in biased estimations.

<Insert TABLE 3 here>

<Insert FIGURE 8 here>

In order to investigate in more detail the bias due to less-equilibrated data, the estimated coefficients are compared in Table 4. The coefficients are the result of averaging 30 replications out of a sample of 500. Therefore, the standard deviations represent variations over 30 replications. As pointed out before, coefficients up to the eighth iteration are considered biased. If less-equilibrated data (such as the fourth or the sixth iteration in this experiment) are used for model estimation, the results may be biased, and the bias may lead to a misunderstanding of traveler's behavior.

<Insert TABLE 4 here>

COMMENTS AND CONCLUSIONS

Estimation of discrete choice models is a technique largely used in traveler's behavior analysis, and the resulting estimated models are usually used for future prediction and policy evaluation. Considering that discrete choice models are used to capture point elasticity measures, a proper and unbiased estimation of model coefficients plays a fundamental role in analyzing user behavior and policy issues. However, much attention has not been paid to the importance of the state and nature of the data used for model estimation.

This paper quantitatively investigates the possible effects of using less-equilibrated data on model estimation through a Monte Carlo simulation and a day-to-day evolution approach. The data generated here may not correctly capture real system evolution towards the equilibrium; however, it is acceptable since it captures the real tendency of traveler behavior moving towards a certain equilibrium state. In this paper, we experiment by using two kinds of choices: departure time and route choice problems. All data generated in the process of adjusting traveler departure time are treated as less-equilibrated data.

The experimental results show that the use of less-equilibrated data causes bias in the estimation of model coefficients, as expected. The bias was quantified by comparing the estimated and assumed true coefficients. The scope of this paper was to identify estimation bias through the realization of simulation experiments; however, two fundamental questions can be raised in order to prevent modelers from estimating biased choice models.

- How can one know if the dataset has been obtained from a fully equilibrated state?
- Can the model be corrected if it were estimated using less-equilibrated data?

This study intends to shed light on the often-ignored issues discussed above rather than attempt to answer those context-specific questions for them. From a theoretical point of view, the answer to both questions may be “yes”. Less-equilibrated systems may be observed by doing more than two consecutive surveys with a reasonable time lag. Comparison of consecutive results would be a good way to judge whether the system is fully equilibrated or not. If the system under study is found to be less-equilibrated, the next question is how to adjust the biased models estimated from unstable data. A possible way to handle this problem could be to extrapolate the coefficients from multiple estimated coefficients obtained under less-equilibrated conditions. However, this is out of the scope of this study and needs to be further studied.

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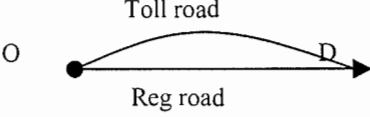
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TABLE 1 Problem Scenarios

	Scenario 1	Scenario 2
	Multinomial Logit	Nested Logit
Model		
TIME	- 0.093	- 0.093
SDE	- 0.067	- 0.067
SDL	- 0.175	- 0.175
TOLL	-	- 0.0093
ρ	-	0.6
Total Demand	60 units	60 units
Network		
Free Travel time	30 minutes (15 unit/10-min)	Reg Road:30 minutes (15 unit/10-min) Toll Road:25 minutes (5 units/10-min) Toll: 200 cents

* TIME: Travel time in minutes
 SD: Schedule delay (Arrival time for a given alternative minus official work-start time)
 SDE: Schedule delay early = -SD if SD < 0
 SDL: Schedule delay late = SD if SD > 0
 TOLL: Amount of toll imposed in cents

TABLE 2 Absolute Percentage Error by Variable and Sample Size (%)

Variable	100 sample	300 sample	500 sample
Average	5.90	5.00	2.68
TIME	12.85	6.92	5.04
SDE	0.05	3.04	0.69
SDL	4.81	5.03	2.31

TABLE 3 Absolute Percentage Error by Iteration and Sample Size (%)

Iteration (day)	100 sample	300 sample	500 sample
4	98.88	99.85	93.71
6	18.14	29.01	15.68
8	8.79	7.33	8.04
12	7.82	6.42	14.78
20	4.53	5.90	5.00

TABLE 4 Coefficient Comparison by Iteration

Iteration (day)	TIME		SDE		SDL	
	Coefficient	Std Dev	coefficient	Std Dev	coefficient	Std Dev
4	-0.24382	0.031451	-0.11732	0.014860	-0.30096	0.023606
6	-0.11241	0.056649	-0.06747	0.019614	-0.18836	0.046870
8	-0.07717	0.070872	-0.06022	0.020102	-0.16479	0.059019
12	-0.09088	0.042861	-0.06437	0.012681	-0.17399	0.039920
20	-0.09769	0.040321	-0.06654	0.011887	-0.17904	0.039231
True Model	-0.09300	-	-0.06700	-	-0.17500	-

* Estimation result with 30 replication of 500 sample size.

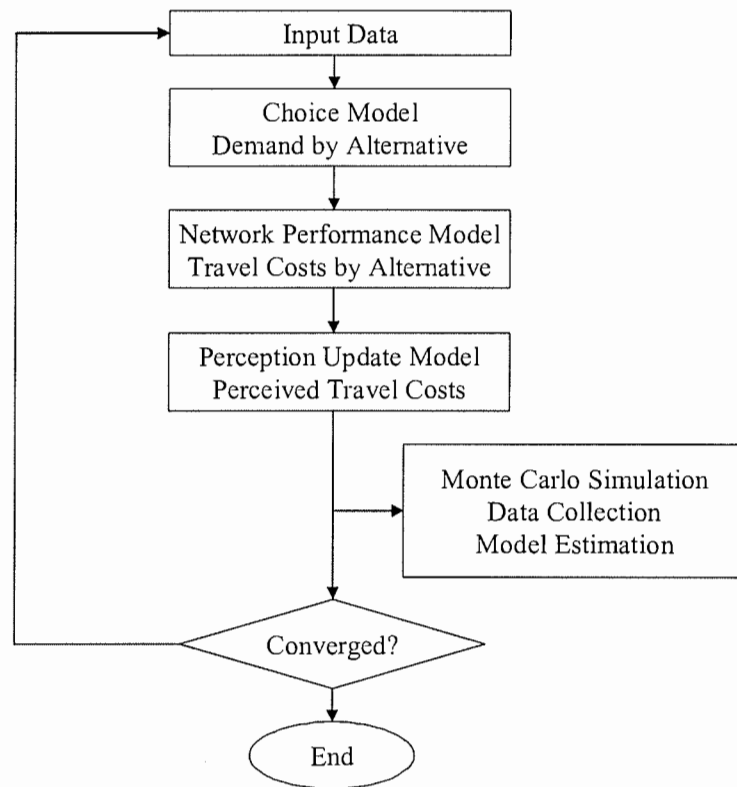


FIGURE 1 FRAMEWORK OF EXPERIMENT

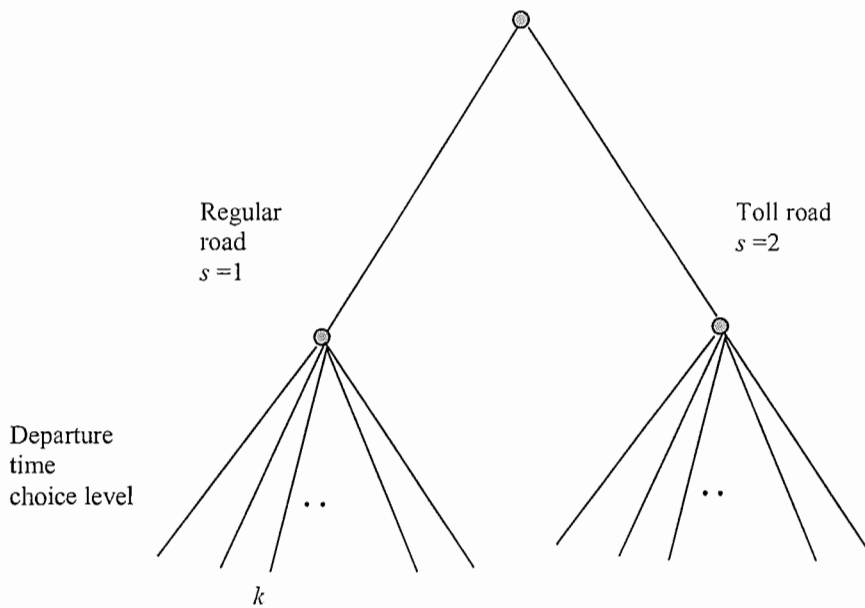


FIGURE 2 NESTED CHOICE MODEL DIAGRAM

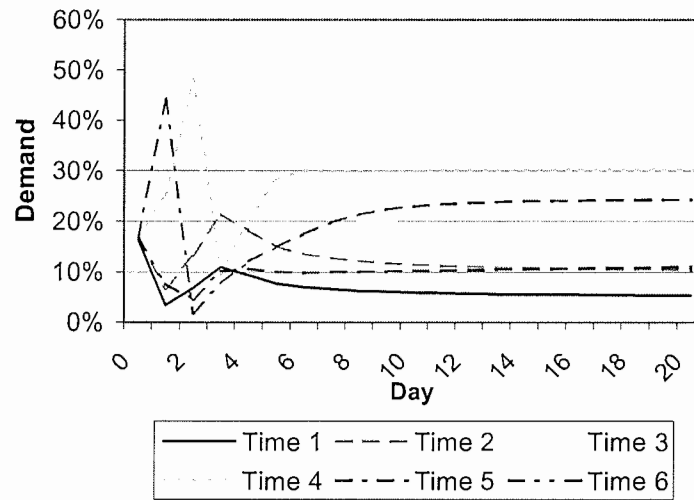


FIGURE 3 CHANGES IN DEPARTURE TIME PATTERN BY ITERATION

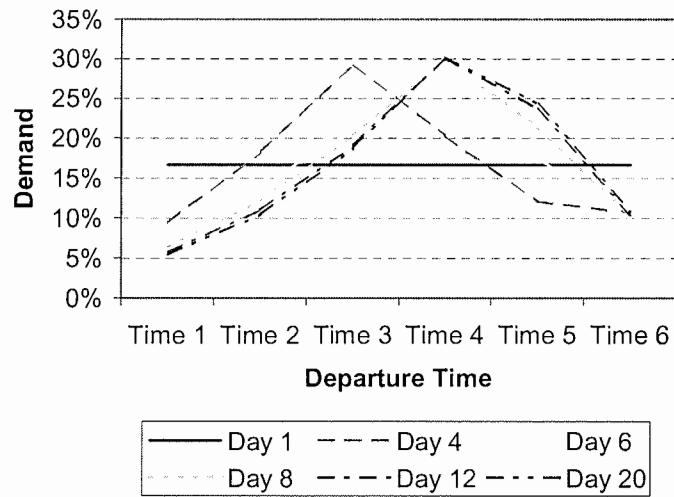


FIGURE 4 CHANGES IN DEPARTURE TIME PATTERN

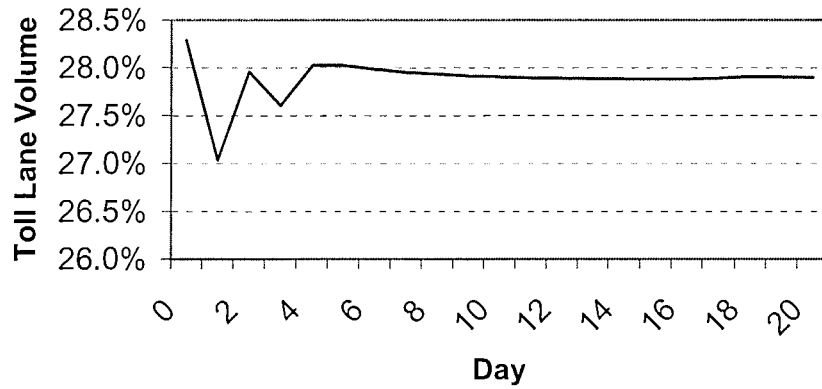


FIGURE 5 CHANGES IN TOLL ROAD VOLUME

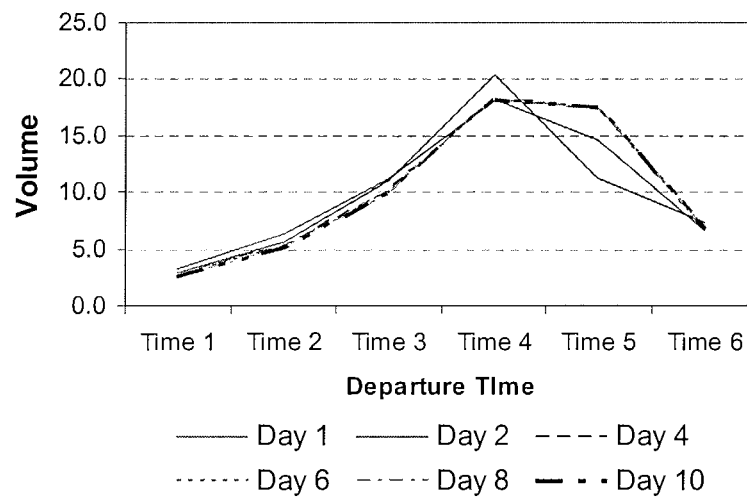


FIGURE 6 DEPARTURE TIME PATTERN WITH ADDITIONAL TOLL ROAD

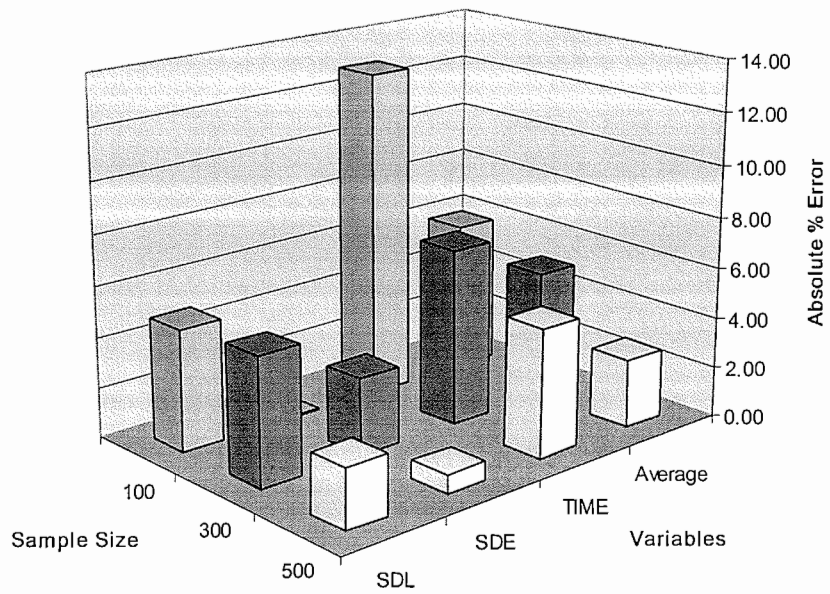


FIGURE 7 ABSOLUTE PERCENTAGE ERROR BY VARIABLES AND SAMPLE SIZE

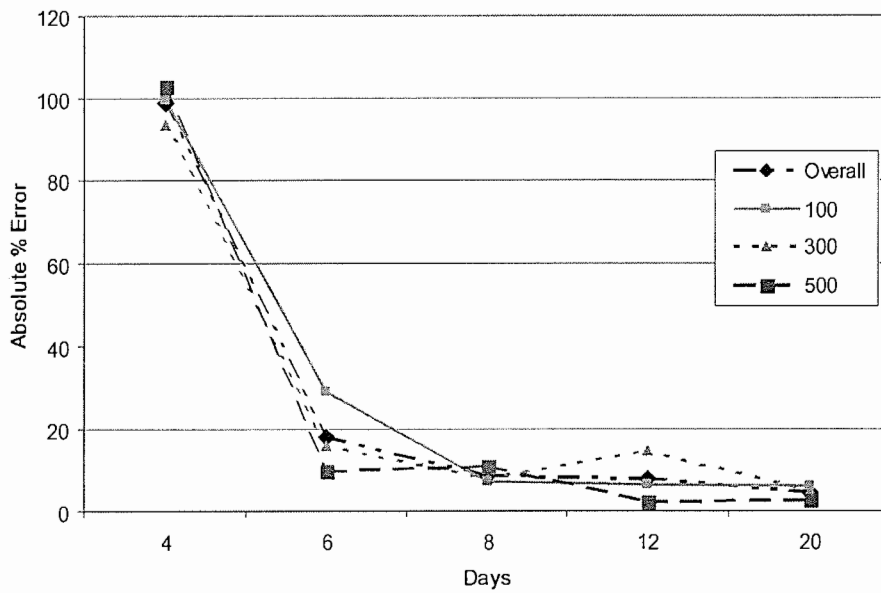


FIGURE 8 ABSOLUTE PERCENTAGE ERROR BY NUMBER OF ITERATIONS AND SAMPLE SIZE