

A Formulation and Solution Algorithm for a Fuzzy Dynamic Traffic Assignment Model

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Abstract

Traffic assignment has received much attention since 1950s'. Stochasticity, dynamics and fuzziness are three characteristics of traffic assignment. Since drivers usually have no perfect information about traffic conditions, they choose their route based on perceived travel time. Traditional traffic assignment model can only capture the stochastic and dynamic features of traffic assignment and cannot represent the fuzziness of driver's perception over travel time. Thus, we present a fuzzy dynamic traffic assignment model in this paper. By the definition of fuzzy perceived link travel time and fuzzy perceived path travel time, we use a fuzzy shortest path algorithm to find the group of fuzzy shortest paths and assign traffic to each of them using the so-called C-Logit method. We also compare the results of our proposed model with those from traditional Stochastic Dynamic Traffic Assignment (SDTA) model.

1 Introduction and Motivation

In order to realize the real-time traffic network monitoring and management function in the Advanced Traffic Management and Information System (ATMIS), a dynamic traffic assignment (DTA) model that captures travelers' route choice behavior in a dynamic and stochastic transportation network needs to be developed. In this model, it is essential to understand how drivers make route choices, especially in the light of the considerable information that the driver may receive within ITS environment, such as variable message signs (VMS), highway advisory radio (HAR), and in-vehicle navigation systems, etc. Along with the driver's prior knowledge of the traffic network, such a

model should replicate, to the extent possible, the driver's perception of available routes and his or her decision-making in selecting the routes.

Most of the current analytical DTA models^[1,2,3] assume that driver's perception error is probabilistic, with certain forms of distribution. However, using probabilistic distribution to simulate driver's perception is questionable by intuition. Because driver's perception of travel time, for example, "Beltline is fast" or "University Ave. is congested now", is actually a linguistic and fuzzy term that has no equivalent exactly defined expression. A few researchers have applied fuzzy logic to model driver's perception. Akiyama proposed standard fuzzy travel time approach using "Standard Fuzzy Travel Time" and fuzzy shortest path approach using "Fuzzy Goal"^[4,5]. Henn developed a fuzzy route choice model using fuzzy number ranking and his model generalizes the results of logit model but providing more flexibility^[6]. However, both Akiyama and Henn only considered static traffic assignment in their papers.

For the purpose of reflecting stochastic and dynamic features of traffic conditions and the fuzziness of driver's perception over travel time more realistically, we propose a Fuzzy Dynamic Traffic Assignment (FDTA) model (also called Fuzzy Route Choice (FRC) Model) in this paper. We first formulate the fuzzy perceived link travel time and fuzzy perceived path travel time in Section 2 and 3. The fuzzy network loading method will be introduced in Section 4, which includes a fuzzy shortest path algorithm, fuzzy assignment and fuzzy flow propagation. In Section 5, the proposed FDTA algorithm will be discussed. Some numerical examples will be given in Section 6. Conclusions remarks will be addressed in the last section.

2 Fuzzy Perceived Link Travel Time

Before we formulate the fuzzy perceived link travel time, the actual link travel time function needs to be discussed.

2.1 Actual Link Travel Time Function

The link travel time function is crucial to the validation and accuracy of DTA model. In order to simulate the link travel time function more realistically and accurately, varieties of methods have been proposed for freeway segments, signalized intersections and stop-controlled intersections. In this paper, we simply deploy some latest results in this area.

2.1.1 Travel Time Function for Freeway Segment

Travel time for freeway segment can be formulated as:

$$c_a = L / speed \quad (1)$$

where L is the length of the link and $speed$ is the speed of the link determined by the modified Greenshield function:

$$speed = cfvf + (1 - c / c_{max}) * (ffvf - cfvf) \quad (2)$$

where

$cfvf$: jam speed.

$ffvf$: free flow speed, can simply use speed limit.

c : density.

c_{max} : jam density.

2.1.2 Travel Time Function for Signalized Intersection

For signalized intersections, travel time function can be expressed as the summation of cruise travel time and intersection delay:

$$c_a = L / ffvf + D_a \quad (3)$$

where D_a is the intersection delay which can be formulated as a Akcelik function^[7]:

$$D_a = \frac{0.5C(1-u)^2}{1-ux} + 900T[x-1 + \sqrt{(x-1)^2 + \frac{8(x-0.5)^2}{KT}}] \quad (4)$$

where:

d : average overall delay (sec./veh.)

C : signal cycle length (sec.)

u : effective green split

x : volume-to-capacity ratio

T : duration of the flow period (h)

K : capacity (veh./h)

2.1.3 Travel Time Function for Stop-Controlled Intersection

We only consider all-way stop-controlled intersections in this paper. Equation (3) can also be applied for this purpose. However, the intersection delay should be represented by the following equation^[8]:

$$D_a = \exp[3.802(V/C)] \quad (5)$$

where V is the total approach volume and C is the approach capacity which has the following formulation^[14]:

$$C = 201.111Ns - 122.535No + 1022.8Ps + 682.2Po - 280.1Lo - 307.6Lc + 202.8Ro + 249.8Rc \quad (6)$$

where

Ns : number of lanes on the subject approach.

No : number of lanes on the opposing approach.

Ps : proportion of traffic on the subject approach.

Po : proportion of traffic on the opposing approach.

Lo : proportion of left turns in the opposing flow.

Lc : proportion of left turns in the conflicting flow.

Ro : proportion of right turns in the opposing flow.

Rc : proportion of right turns in the conflicting flow.

The conflicting flow is defined as the flow on the cross street (from both directions).

2.2 Fuzzy Perceived Link Travel Time (FPLTT)

2.2.1 Fuzzy Sets for Perceived Link Travel Time

Due to the fuzziness of driver's perception over link travel time, fuzzy sets of perceived link travel time are developed to reflect driver's perception of link travel time under different traffic conditions. In this paper, we use linguistic descriptions to represent these fuzzy sets. Since there are various traffic conditions, we only construct fuzzy sets for most frequent traffic conditions, such as normal, congestion, incident and construction. The linguistic descriptions of these fuzzy sets for link a are listed below:

- "The travel time for link a is normal" --- **NORMAL**

- “Link a is congested” --- **CONGESTED**
- “Link a has incident” --- **INCIDENT**
- “Link a has construction” --- **CONSTRUCTION**

In this paper, we will use the above bold word to represent the fuzzy set. Obviously, more fuzzy sets can be constructed to represent other traffic conditions (like special event) if necessary.

2.2.2 Membership Function for FPLTT

Generation of membership function arouses a lot of interests in the literature. However, no standard method exists yet. The membership function can be constructed based on a survey. However, due to the magnitude of factors that influence driver’s perception (like driver’s personal characteristics, traffic conditions), no effective method has been developed so far to do the survey. Therefore, in this paper, we use flat L-R type fuzzy number to represent FPLTT. L-R type fuzzy number was first proposed by Dubois and Prade^[9]. Typically, if fuzzy number \tilde{m} is a L-R type fuzzy number, then its membership function $\mu_{\tilde{m}}(x)$ has the following form^[10]:

$$\mu_{\tilde{m}}(x) = \begin{cases} L[(\underline{m} - x)/\alpha] & \text{if } x < \underline{m}, \alpha > 0, \\ 1 & \text{if } x \in [\underline{m}, \overline{m}], \\ R[(x - \overline{m})/\beta] & \text{if } x > \overline{m}, \beta > 0. \end{cases} \quad (7)$$

where \underline{m} and \overline{m} are the left-hand mean and right-hand mean, respectively. α, β are the left-hand and right-hand spreads. L and R are reference functions. Thus, a L-R fuzzy number \tilde{m} can be denoted as $(\underline{m}, \overline{m}, \alpha, \beta)_{LR}$.

As shown in Fig. 1 and Fig.2, triangular and trapezoidal fuzzy numbers are two special cases of L-R fuzzy numbers that are used in most applications of fuzzy sets theory. One of the possible reference function used for both triangular and trapezoidal fuzzy numbers is:

$$L(y) = R(y) = \begin{cases} 1 - |y| & , y \in [-1, 1] \\ 0 & , otherwise \end{cases} \quad (8)$$

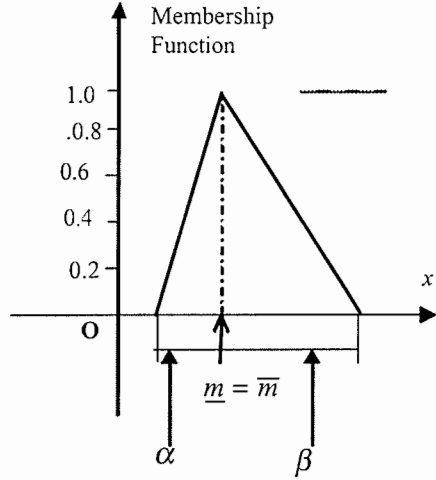


Fig.1 Triangular Fuzzy number

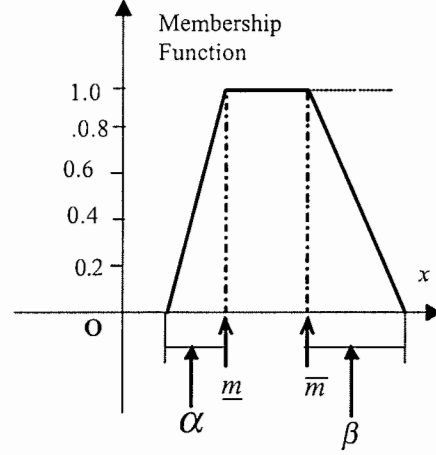


Fig.2 Trapezoidal Fuzzy number

We denote the FPLTT for link a as $\tilde{\tau}_a(t)$. In this paper, we construct the membership function of FPLTT $\mu_{\tilde{\tau}_a}(t)$ using triangular fuzzy number based on actual link travel time τ_a proposed in section 2.1.

1. NORMAL FPLTT

NORMAL FPLTT represents driver's perception over link travel time under normal traffic condition (no congestion, incident, construction or special event, etc.). This is the most common traffic condition in real world. Fig. 3(a) depicts the membership function for NORMAL FPLTT. Lh and Rh in this figure are Left-hand and right-hand spreads which can be generated using estimation or experimental results.

NORMAL FPLTT can be expressed as:

$$\tilde{\tau}_a = (Lh, \tau_a, \tau_a, Rh) \quad (9)$$

2. CONGESTED FPLTT

Driver's perception over link travel time under congested traffic condition is represented as CONGESTED FPLTT. To represent different levels of congestion, Severity Index of Congestion (SIC) is introduced. As shown in Fig. 3(b), the mean of CONGESTED

FPLTT is SIC times normal actual link travel time τ_a . The left-hand spread extends to τ_a , Rh is the right-hand spread.

CONGESTED FPLTT can be expressed as:

$$\tilde{\tau}_a = ((sic - 1) * \tau_a, sic * \tau_a, sic * \tau_a, Rh) \quad (10)$$

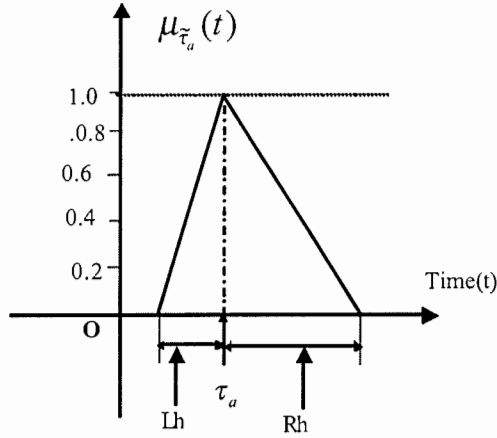


Fig.3(a) Membership function of NORMAL FPLTT

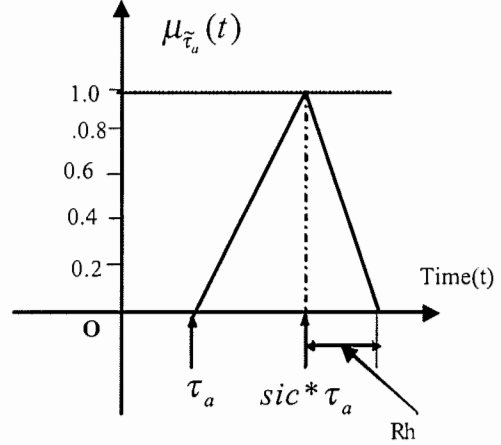


Fig.3(b) Membership function of CONGESTED FPLTT

3. INCIDENT FPLTT

Driver's perception over link travel time under incident condition is represented as INCIDENT FPLTT. Similarly with CONGESTED FPLTT, Severity Index of Incident (SII) is defined for different level of incident. SII can be generated from sources describing the severity of the incident. Fig. 3(c) shows the membership function for INCIDENT FPLTT. Rh in this figure is the right-hand spread.

INCIDENT FPLTT can be expressed as:

$$\tilde{\tau}_a = ((sii - 1) * \tau_a, sii * \tau_a, sii * \tau_a, Rh) \quad (11)$$

4. CONSTRUCTION FPLTT

CONSTRUCTION FPLTT is used for construction condition. Severity Index of Construction (SICO) can also be defined to represent the severity level of construction. Since most of the constructions can be known in advance, SICO can be obtained by acquiring construction information from specific agency or other traffic information sources. Fig. 3(d) is the membership function for CONSTRUCTION FPLTT.

CONSTRUCTION FSPLTT can be expressed as:

$$\tilde{\tau}_a = ((sico - 1) * \tau_a, sico * \tau_a, sico * \tau_a, Rh) \quad (12)$$

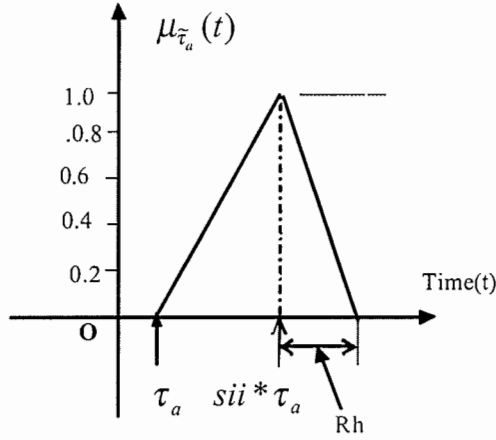


Fig.3(c) Membership function of INCIDENT FPLTT

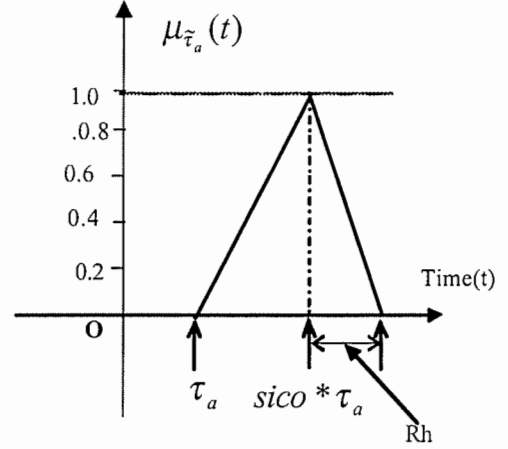


Fig.3(d) Membership function of CONSTRUCTION FPLTT

2.2.3 Dynamic FPLTT

The fuzzy sets shown from Fig.3(a) to Fig.3(d) is for fixed time interval. However, for dynamic traffic assignment, we have to consider a range of time intervals. Therefore, we need to construct the dynamic FPLTT for given link **a**. Generally, we can use the following equation to represent the dynamic FPLTT for link **a** at time t:

$$\tilde{\tau}_a(t) = (Lh_a(t), \underline{m}_a(t), \overline{m}_a(t), Rh_a(t)) \quad (13)$$

where $Lh_a(t), Rh_a(t)$ are the left-hand and right-hand spreads at time t for link **a**, $\underline{m}_a(t), \overline{m}_a(t)$ are the left-hand and right-hand means at time interval t.

Due to the complexity and unpredictable features of traffic conditions, these four terms in equation (13) may vary over different time interval. For example, for link **a**, we have such a traffic condition description: “link travel time was normal from 1:00AM to 8:00AM, but an incident happened at 8:00AM which was cleared at 10:00AM”. Fig. 4 illustrates the FPLTT for this link during 1:00AM to 12:00PM. Please note the changes of FPLTT at 8:00AM and 10:00AM. The FPLTT used for time period from 1:00AM to 8:00AM and after 10:00AM is (0.2, 2, 2, 0.2), and that used for time period from 8:00AM to 10:00AM is (2, 4, 4, 0.4). Also note that we only use fixed left-hand and right-hand

parameters in Fig.4 although in reality they all vary over time which makes Fig.4 much more complex.

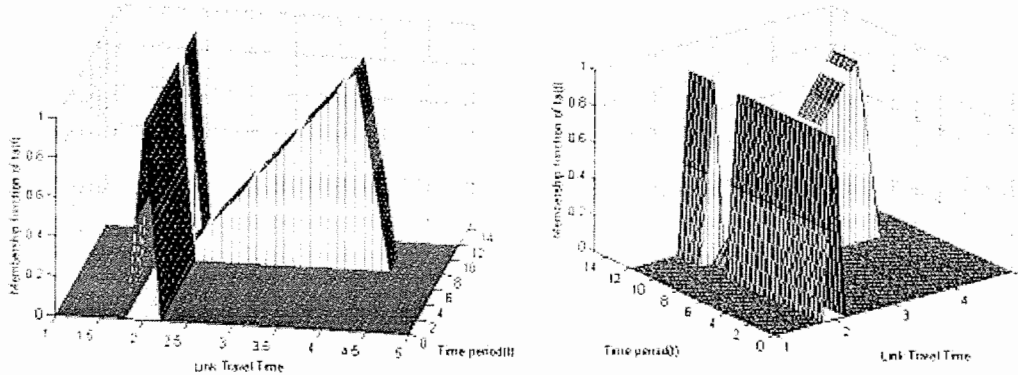


Fig. 4 Illustration of Dynamic FPLTT

3 Fuzzy Perceived Path Travel Time (FPPTT)

3.1 Static FPPTT

In crisp case, static path travel time equals to the summation of the travel time of those links that construct this path ^[11].

$$c_k^{rs} = \sum_a t_a \delta_{a,k}^{rs} \quad (14)$$

where c_k^{rs} is the path travel time for the k-th path from origin r to destination s; $\delta_{a,k}^{rs}$ is the link-path incidence matrix.

For fuzzy case, we can generalize the FPPTT from equation (14). Denote \tilde{c}_k^{rs} as the FPPTT for the kth path from origin r to destination s, $\tilde{t}_a = (Lh_a, \underline{m}_a, \bar{m}_a, Rh_a)$ is the FPLTT for link a, then we have:

$$\tilde{c}_k^{rs} = \sum_a^{\oplus} \tilde{t}_a \delta_{a,k}^{rs} \quad (15)$$

The symbol “ \oplus ” above “ \sum ” in equation (15) denotes fuzzy number addition. Denote $\tilde{m}_1 = (\alpha_1, \underline{m}_1, \bar{m}_1, \beta_1)$ and $\tilde{m}_2 = (\alpha_2, \underline{m}_2, \bar{m}_2, \beta_2)$ are L-R type fuzzy numbers, then their addition can be expressed by:

$$\tilde{m} = \tilde{m}_1 \oplus \tilde{m}_2 = (\alpha_1 + \alpha_2, \underline{m}_1 + \underline{m}_2, \bar{m}_1 + \bar{m}_2, \beta_1 + \beta_2) \quad (16)$$

Thus, equation (15) becomes:

$$\tilde{c}_k^{rs} = \sum_a \delta_{a,k}^{rs} (Lh_a, \underline{m}_a, \bar{m}_a, Rh_a) = \left(\sum_a \delta_{a,k}^{rs} Lh_a, \sum_a \delta_{a,k}^{rs} \underline{m}_a, \sum_a \delta_{a,k}^{rs} \bar{m}_a, \sum_a \delta_{a,k}^{rs} Rh_a \right) \quad (17)$$

Equation (17) can be used for constructing the FPPTT in static transportation networks.

3.2 Dynamic FPPTT

In dynamic case, a recursive formulation can be used to calculate path travel time from origin r to destination s for path k at time t as shown in Fig. 5^[1].

$$\eta_k^{ri}(t) = \eta_k^{r(i-1)}(t) + \tau_a[t + \eta_k^{r(i-1)}(t)] \quad (18)$$

where link $a = (i-1, i)$, $i = 1, 2, \dots, s$. And $t + \eta_k^{r(i-1)}(t)$ is the time interval when the flows departing from origin r to destination s through path k at time t enter link a .

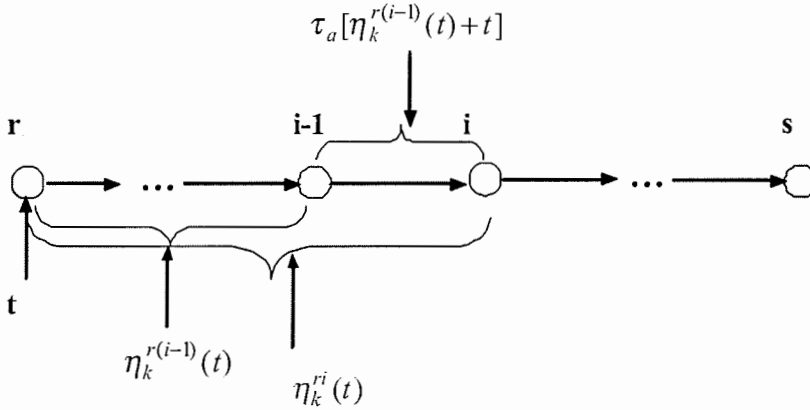


Fig. 5 Dynamic Path Travel Time Illustration

Similarly as in fuzzy case, we can derive the FPPTT from equation (18) as:

$$\tilde{\eta}_k^{ri}(t) = \tilde{\eta}_k^{r(i-1)}(t) + \tilde{\tau}_a[t + \tilde{\eta}_k^{r(i-1)}(t)] \quad (19)$$

where $t + \tilde{\eta}_k^{r(i-1)}(t)$ is the “fuzzy” time interval when the flows departing from origin r to destination s through path k at time t enter link a . Since $t + \tilde{\eta}_k^{r(i-1)}(t)$ is also a fuzzy number, $\tilde{\tau}_a[t + \tilde{\eta}_k^{r(i-1)}(t)]$ is actually a fuzzy number over another fuzzy number. In order to make our computation simple, in this paper, we defuzzify $t + \tilde{\eta}_k^{r(i-1)}(t)$ into a crisp number. Denote $\bar{\eta}_k^{r(i-1)}(t)$ is the crisp number defuzzified from $\tilde{\eta}_k^{r(i-1)}(t)$. Using the Center of Area (COA) method for defuzzification, we have:

$$\bar{\eta}_k^{r(i-1)}(t) = \frac{\int x \mu_{\bar{\eta}_k^{r(i-1)}(t)}(x) dx}{\int \mu_{\bar{\eta}_k^{r(i-1)}(t)}(x) dx} \quad (20)$$

where $\mu_{\bar{\eta}_k^{r(i-1)}(t)}(x)$ is the membership function of $\bar{\eta}_k^{r(i-1)}(t)$ on universal set T(time)
Hence, equation (19) becomes:

$$\tilde{\eta}_k^{ri}(t) = \tilde{\eta}_k^{r(i-1)}(t) + \tilde{\tau}_a [t + \bar{\eta}_k^{r(i-1)}(t)] \quad (21)$$

4 Fuzzy Network Loading (FNL)

Given FPLTT and FPPTT, Fuzzy Network Loading (FNL) method is used to find the fuzzy shortest paths and assign traffic flow from origin to destination to links in the network. Therefore, three steps are involved in FNL: Fuzzy Shortest Path (FSP) algorithm, Fuzzy Traffic Assignment (FTA) and Fuzzy Flow Propagation (FFP).

4.1 Fuzzy Shortest Path (FSP) Algorithm

Finding the shortest path in a road network is a basic algorithm for traffic assignment in both static and dynamic cases. However, under fuzzy case, it is difficult to find the only path that is the “shortest”. Instead, it is more feasible to construct a fuzzy set of paths that are “shortest” in term of fuzzy travel time. We donate this fuzzy set as fuzzy shortest paths. During the past decade, Fuzzy Shortest Path (FSP) algorithm has been explored extensively. Lin and Chern ^[12] developed a fuzzy linear programming approach to calculate the fuzzy shortest path and the most vital arc. Okada and Soper addressed their FSP approach by modifying the multiple labeling method for traditional shortest path algorithm for fuzzy graph whose link weight is L-R type fuzzy number ^[10]. Blue and Bush discussed the classification of fuzzy graphs and divided them into 5 categories^[13, 14]. They proposed a FSP algorithm for Type V fuzzy graph by converting fuzzy graph into two corresponding crisp graphs and then finding the representative shortest paths and also their memberships through solving the k-shortest path problem. In this paper, Blue and Bush’s method is adopted to calculate the fuzzy short paths.

According to Blue's classification, a pure Type V fuzzy graph has crisp nodes and links, but fuzzy weights on the links. Road network is a pure Type V fuzzy graph, and the fuzzy weights in this paper are the fuzzy travel times. Denote G is a Type V fuzzy graph, we can find the fuzzy shortest paths using the following four steps.

Step 1: Converting Fuzzy Graph to Crisp Graphs

We first construct two crisp graphs, \overline{G} and \underline{G} , that are identical to G , except that the link travel time is crisp. And, the link travel time function for \overline{G} has the following form:

$$\overline{\tau}_a = \sup\{\text{supp}(\tilde{\tau}_a)\} \quad (22)$$

Where a is a link in fuzzy graph G (also a link in crisp graph \overline{G} and \underline{G}), $\tilde{\tau}_a$ is the fuzzy travel time of link a , "sup" means least upper bound, and "supp" means support of fuzzy set.

Similarly, the link travel time for link a in graph \underline{G} has the following form:

$$\underline{\tau}_a = \inf\{\text{supp}(\tilde{\tau}_a)\} \quad (23)$$

where "inf" means greatest lower bound.

We call \overline{G} as upper-bound-graph (UBG) of G , and \underline{G} as lower-bound-graph(LBG) of G .

Step 2: Find Shortest Path in Upper Bound Graph \overline{G}

It is a traditional problem to find the shortest path in a crisp graph and various algorithms are available to solve it ^[15]. Denote k is the length (travel time) of this path and k is a crisp value.

Step 3: Find the Support of Fuzzy Shortest Paths

Find all the paths connecting origin to destination and with the length less than k in LBG \underline{G} . Denote \underline{S} is the set of these paths:

$$\underline{S} = \{\underline{p}_1, \underline{p}_2, \dots, \underline{p}_m, \dots, \underline{p}_M\} \quad (24)$$

where \underline{p}_m is the m -th path in \underline{S} and the total number of paths is M .

For each \underline{p}_m in \underline{S} , find p_m , its counterpart in fuzzy graph G . This means that p_m has the exactly same sequence of nodes and links as \underline{p}_m , except that its path travel time is fuzzy rather than crisp. Denote the set of all p_m as S , which has the following form:

$$S = \{p_1, p_2, \dots, p_m, \dots, p_M\} \quad (25)$$

Then S is the support of the fuzzy shortest paths. The critical part in this step is to find \underline{S} and it can be achieved by using modified k -shortest path algorithm^[16,17].

Step 4: Construct Fuzzy Shortest Paths

After obtaining the support of the fuzzy shortest paths S , we then define the fuzzy shortest paths \tilde{P} , which is a fuzzy set defined on S and the membership function for \tilde{P} is $\mu_{\tilde{P}}(p_m)$. Please note that since each p_m is also a fuzzy set, \tilde{P} is actually a fuzzy set defined on a group of other fuzzy sets. To determine $\mu_{\tilde{P}}(p_m)$, we first calculate the membership of k (obtained in step 2 above) in each of the fuzzy path p_m and assign this membership as the membership of p_m in \tilde{P} . That is,

$$\mu_{\tilde{P}}(p_m) = \mu_{p_m}(k) \quad (26)$$

Since k is a crisp number and p_m is a fuzzy set with known membership function, it is straightforward to compute $\mu_{p_m}(k)$.

4.2 Fuzzy Traffic Assignment (FTA)

As shown in section 4.1, each fuzzy path p_m in the fuzzy shortest path set \tilde{P} has the membership $\mu_{\tilde{P}}(p_m)$.

Now the question becomes how to assign the traffic over the support of fuzzy shortest paths, given the traffic flow from origin O to destination D is T_{od} . Historically, stochastic traffic assignment has often been formulated as the multinomial logit (MNL) model. The MNL assumes the error terms are identical and independently distributed (IID). The independence assumption leads to a property known as the independence of irrelevant alternatives (IIA). However, the IIA property of standard MNL model can be violated because of the correlation among the alternatives. Particularly, for urban transportation network, there are often several alternative routes share some common links. Therefore the MNL model does not accurately represent choices among overlapping alternatives, which are bound to comprise any realistic urban transportation network. In this context,

alternative model forms based on MNL has been proposed, such as C-Logit model^[18] or paired combinatorial logit (PCL) model^[19] (Glielie, et al, 1999). The C-Logit overcomes the main shortcoming of MNL for the unrealistic choice probabilities for paths sharing a number links, while keeping a closed analytical structure allowing calibration on disaggregate data and efficient path flow computations when paths are explicitly enumerated. Therefore we use C-Logit for the fuzzy traffic assignment.

In C-Logit, a commonality factor is subtracted from the main utility function to account for the degree of overlapping of a path with other paths in the choice set. The commonality factor reduces the probability of choosing paths that overlap and increases the probability of choosing an independent path. The path choice probability $P(m|M)$ can be expressed as:

$$P(m|M) = \frac{\exp(\mu_{\bar{p}}(p_m) - CF_m)}{\sum_{i \in M} \exp(\mu_{\bar{p}}(p_i) - CF_i)} \quad (27)$$

The term CF_i denoted as “commonality factor” of path p_i , is directly proportional to the degree of similarity (or overlapping) of path p_i with other path belonging to S. Here CF_i is specified as:

$$CF_i = \beta \ln \sum_{j \neq i} \left(\frac{L_{ij}}{\sqrt{L_i L_j}} \right)^\gamma \quad (28)$$

where L_{ij} is the length of links commons to path i and j , while L_i and L_j are the overall length of path i and j respectively. β and γ are scaling parameters.

Thus, we can assign the traffic flow to these paths in S using equation (29):

$$T_{od}^m = P(m|M) * T_{od} \quad (29)$$

4.3 Fuzzy Flow Propagation (FFP)

For dynamic traffic assignment, we have to consider the flow propagation. In this case, equation (30) is used for this purpose^[1]:

$$x_{ap}^{rs}(t) = \sum_{b \in \tilde{p}} \{x_{bp}^{rs}[t + \tilde{\tau}_a(t)] - x_{bp}^{rs}(t)\} + \{E_p^{rs}[t + \tilde{\tau}_a(t)] - E_p^{rs}(t)\} \quad (30)$$

where p is the path link \mathbf{a} belongs to, \tilde{p} is the sub-path from end node of link \mathbf{a} to destination D, $\tilde{\tau}_a(t)$ is the fuzzy link travel time at time interval t for link \mathbf{a} . Similarly as in section 3, we use $\bar{\tau}_a(t)$, the de-fuzzification of $\tilde{\tau}_a(t)$ to represent $\tilde{\tau}_a(t)$ in equation (30). The formulation to calculate $\bar{\tau}_a(t)$ is shown below using COA:

$$\bar{\tau}_a(t) = \frac{\int x \mu_{\tilde{\tau}_a}(t)(x) dx}{\int \mu_{\tilde{\tau}_a}(t)(x) dx} \quad (31)$$

Hence, equation (30) becomes:

$$x_{ap}^{rs}(t) = \sum_{b \in \tilde{p}} \{x_{bp}^{rs}[t + \bar{\tau}_a(t)] - x_{bp}^{rs}(t)\} + \{E_p^{rs}[t + \bar{\tau}_a(t)] - E_p^{rs}(t)\} \quad (32)$$

5 The Solution Algorithm for FDTA

To solve the FDTA problem, we need to convert our continuous time FDTA problem into a discrete time FDTA problem. The time period $[0, T]$ is subdivided into K small time intervals. Each time interval is regarded as one unit of time. Further, $u_a(k)$ represents the inflow into link a during interval k , $v_a(k)$ represents the exit flow from link a during interval k , $x_a(k)$ represents the number of vehicles at the beginning of interval k , and $f_p(k)$ represents the departure flow from path p during interval k .

This discrete FDTA can be solved by using a combination of relaxation, Method of Successive Averages (MSA) and Fuzzy Network Loading (FNL) techniques. In this combined algorithm, we define the travel time approximation procedure (relaxation) as the outer iteration, the MSA procedure as the inner iteration and FNL procedure as the fuzzy loop. For each relaxation(or diagonalization) iteration, we temporarily fix link flow $x_a(k)$, fuzzy travel time $\tilde{\tau}_a(k)$ for each link \mathbf{a} and its defuzzification $\bar{\tau}_a(k)$ in fuzzy path travel time and link flow propagation constraints.

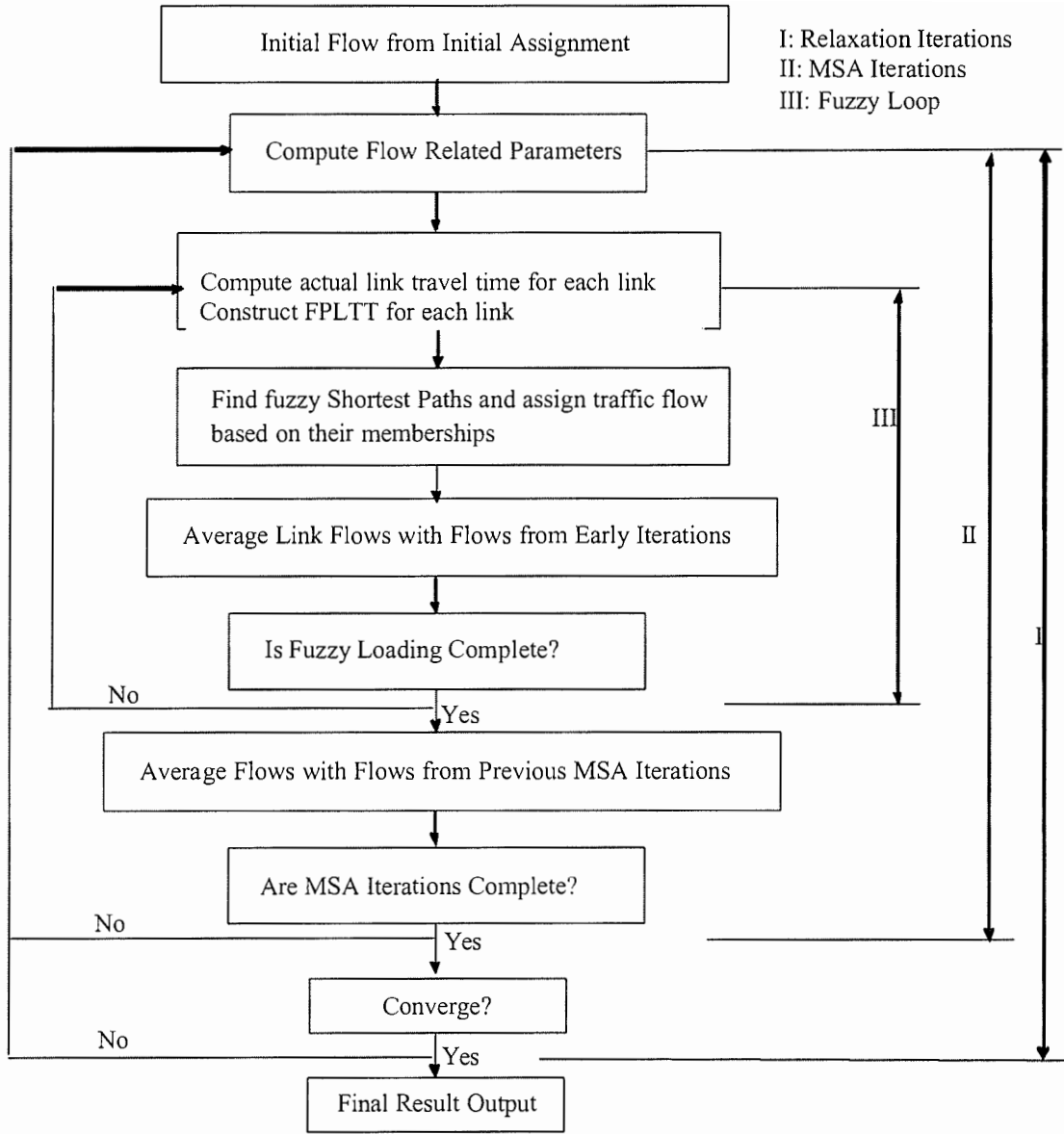


Fig. 6 FDTA Algorithm Flow Chart

The algorithm for solving our proposed FDTA model can be summarized as follows:

Step 0: Initialization. Initialize all link flows $\{x_{am}^{(0)}(k)\}, \{u_{am}^{(0)}(k)\}, \{v_{am}^{(0)}(k)\}$ to zero and calculate initial actual link travel time estimates $\tau_a^{(1)}(k)$. Based on $\tau_a^{(1)}(k)$, calculate the fuzzy link travel time $\tilde{\tau}_a^{(1)}(k)$ for each link \mathbf{a} . Set the outer iteration counter $l=1$.

Step 1: Relaxation. Set the inner iteration counter $n = 1$. Find a new approximation of actual link travel times: $\tau_a^{(n)}(k) = \tau(x_a^{(*)}(k))$, where $(*)$ denotes the final solution obtained

from the most recent inner problem. Construct the FPLTT for each link a based on $\tau_a^{(n)}(k)$. Solve the route choice program for the main problem using fuzzy network loading and method of successive averages.

[Step 1.1]: Sub-problem - Fuzzy Network Loading(fuzzy loop). Perform FSP algorithm and calculate corresponding memberships for each fuzzy path. Assign all departure flows $f^{\alpha}(k)$ to these routes proportionally to their memberships. Let the temporary link flow vector resulted from current assignment be called $(\hat{p}^i, \hat{q}^i, \hat{y}^i)$ at each fuzzy network loading i . Then, the fuzzy dynamic network loading is solved by the following recursive equations:

$$p_a^i(k) = [(i-1)p_a^{(i-1)}(k) + \hat{p}_a^i(k)]/i \quad \forall a \quad (33)$$

$$q_a^i(k) = [(i-1)q_a^{(i-1)}(k) + \hat{q}_a^i(k)]/i \quad \forall a \quad (34)$$

$$y_a^i(k) = [(i-1)y_a^{(i-1)}(k) + \hat{y}_a^i(k)]/i \quad \forall a \quad (35)$$

Set $i = i + 1$. As i equals to a pre-specified number, stop; otherwise, perform step 1.1 again. The final vector (p^i, q^i, y^i) is used as the converged link flows at inner iteration n .

[Step 1.2]: Method of Successive Averages. Using the predetermined step size $1/n$, yield a new MSA main problem solution through the following equations:

$$u_a^{n+1}(k) = u_a^n(k) + \frac{1}{n}[p_a^n(k) - u_a^n(k)] \quad \forall a \quad (36)$$

$$v_a^{n+1}(k) = v_a^n(k) + \frac{1}{n}[q_a^n(k) - v_a^n(k)] \quad \forall a \quad (37)$$

$$x_a^{n+1}(k) = x_a^n(k) + \frac{1}{n}[y_a^n(k) - x_a^n(k)] \quad \forall a \quad (38)$$

If n equals a pre-specified number, go to step 2; otherwise $n = n+1$, and go to step 1.1.

Step 2: Convergence Test for the Outer Iterations. If $|x_a^{(l)}(k) - x_a^{(l-1)}(k)|/x_a^{(l)}(k) \leq \Delta$ for all time interval k and all link a , stop. The current solution $\{u_a(k)\}, \{v_a(k)\}, \{x_a(k)\}$ is in a near optimal state; otherwise, set $l=l+1$ and go to step 1. Δ is the pre-defined threshold(for example, 5%).

The flow chart for the solution algorithm is shown in Fig. 6. The number of inner iterations n and the number of outer iterations l are correlated. If we set l large, then n should be set small and vice versa. The computational convergence of this proposed solution algorithm deserves further study.

6 Numerical Examples

In this section, we present some numerical results from our experiments for a small test network using the proposed FDTA model. We will compare our solution quality with that obtained from stochastic dynamic traffic assignment (SDTA) developed by Ran and Boyce (1996) to illustrate that our proposed fuzzy DTA model can generate reasonable results.

Table 1. Link Information

Link Number	Start Node	End Node	Length (miles)	Capacity (# of Vehi.)	# of Lane
1	1	2	1	2200	1
2	2	3	1	2200	1
3	2	4	1	2200	1
4	4	3	1	2200	1
5	3	5	1.5	2200	1
6	4	6	1.5	2200	1
7	5	6	1	2200	1
8	5	7	1.5	2200	1
9	6	8	1.5	2200	1
10	8	7	1	2200	1
11	7	9	1.5	2200	1
12	8	10	1.5	2200	1
13	9	10	1	2200	1
14	9	11	1	2200	1
15	10	11	1	2200	1

The test network is indicated in Fig.7 with eleven nodes and fifteen links. Detailed link characteristics are shown in Table 1. Three scenarios will be tested as shown in Table 2. These three scenarios share the following common input characteristics:

- Origin is node 1 and destination is node 11.
- The O-D flows are 15 vehicles for each of the five 60-second periods (equivalent to a flow of 900 vehicles per hour). The total flows from Origin to Destination for the whole analysis period is 75.
- Free flow speed is 50 miles per hour.

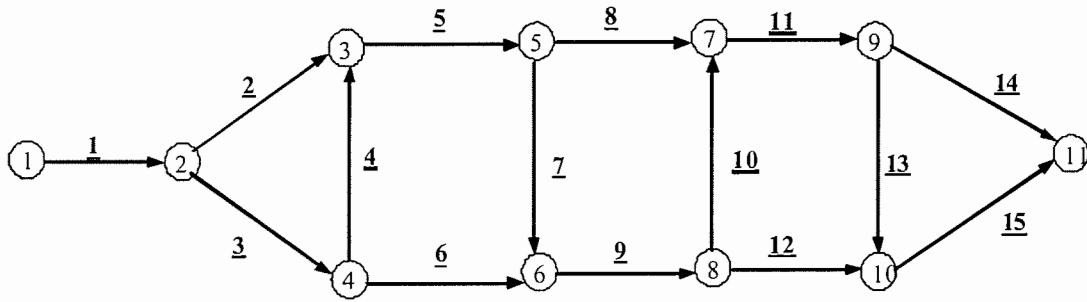


Fig.7 Experimental Network

Table 2. Distinctive Features of Three Scenarios

Scenarios	Distinctive features
#1	All links are NORMAL
#2	Link 8 has incident at time interval 1 and lasts until time interval 6, all other links are NORMAL
#3	Link 8 has incident at time interval 1 and lasts until time interval 9, all other links are NORMAL

In scenarios #1, all links are NORMAL. To better present the results, we accumulate the number of vehicles passing through each link for the entire analysis period. These numbers can be verified by the time-dependent results for each link at every time interval shown as Table 4 and Table 5 in the appendix. Table 3 shows the result of fuzzy DTA and the comparison of this result with SDTA.

From Table 3, we can see that our FDTA method can generate traffic flows very close to those created from SDTA, with most of the differences being less than 10%. Fig. 8 also shows that the difference between these two methods is insignificant. Since all links are NORMAL in this scenario, intuitively, path 1->2->5->8->11->14 and path 1->3->6->9->12->15 are two major routes which will be taken by most drivers (This statement can be verified by the results from both FDTA and SDTA as shown in Table 3). We denote path 1->2->5->8->11->14 as “top path” and path 1->3->6->9->12->15 as “bottom path”. Also, link 4, 7, 10, 13 are referred as “Least-Used(LU)” link in this paper. As shown in Table 3, SDTA assigns zero flow to these LU links, while FDTA assigns a small portion of the total flow(less than 10%) to three of them(4, 7, 10). Therefore, it demonstrates that

FDTA tends to assign traffic to more links and thus generates more reasonable traffic flow pattern than SDTA.

Table 3. Results for Scenario #1

Link #	Flow from SDTA	Flow from FDTA	Flow difference(%)
1 1->2	74.99	75	0.01333
2 2->3	38.46	34.37	11.89991
3 2->4	36.53	40.63	10.0911
4 4->3	0	6.3	/
5 3->5	38.46	40.67	5.43398
6 4->6	36.53	34.33	6.408389
7 5->6	0	7.01	/
8 5->7	38.46	33.66	14.26025
9 6->8	36.53	41.34	11.6352
10 8->7	0	4.55	/
11 7->9	38.46	38.21	0.654279
12 8->10	36.53	36.79	0.70671
13 9->10	0	0	/
14 9->11	38.46	38.21	0.654279
15 10->11	36.53	36.79	0.70671

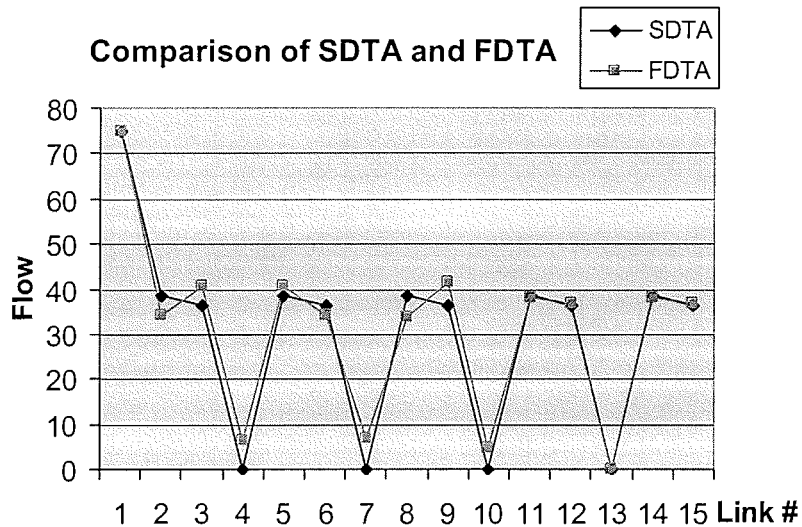


Fig. 8 Comparison of FDTA and SDTA

For Scenario #2 and #3, link 8 has incident and all other links are NORMAL. The only difference between Scenario #2 and #3 is that the duration of the incident for link 8 is different: in Scenario #2, the duration time is 6 time intervals(from 1 to 6), while in Scenario #3, it is 9(from 1 to 9). We should point out here that SDTA can not resolve these two scenarios very well because it has no appropriate mechanism to represent incident traffic condition.

To make Scenario #2 and #3 more realistic, we can assume a VMS (Variable Message Sign) sign was installed on link 1 to indicate the traffic conditions for the entire network, as shown in Fig. 9. In these two scenarios, this VMS sign will display the incident condition on link 8. The duration time is the estimated clearance time for the incident.

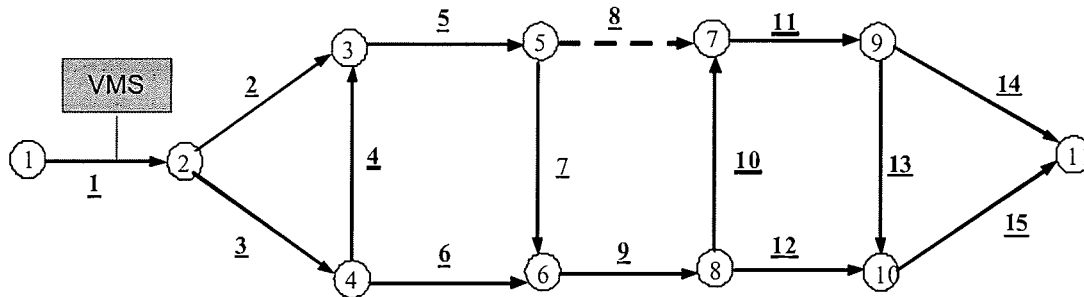


Fig.9 Test Network with VMS Installed

In Scenario #2 and 3, we assume all drivers are familiar with the network, that is, they know there are two major routes from origin 1 to destination 11 and also the approximate travel time for each link. Since all drivers will go through link 1, they will see the VMS sign and their route choice behavior at intersection 2 will be affected by this VMS.

Upon arriving at intersection 2, drivers will choose to go through link 2 or link 3. This will in turn result in top path and bottom path respectively. For the first 15 drivers in Scenario #2, they will arrive at intersection 2 at time interval 2 since the travel time from origin 1 to node 2 is 1 mile / (50MPH) = 0.02 hour \approx 60 seconds = 1 time interval. Because it will need them 3 time intervals (2.5/50=0.05 hours = 3 minutes = 3 time intervals) to travel to node 5, they will get there at time interval 5 when the incident on link 8 will not be cleaned. Therefore, we can predict that most of the drivers within the first 15 will choose bottom path instead of top path. Similar analysis can be applied to the second 15 drivers departing origin 1 at time interval 2. Most of them will also choose bottom path. However, for the last three groups of drivers, they will not be affected by the incident on link 8 since they will arrive at node 5 after time interval 6 if they choose the top path. Hence, travelers in these three groups will choose top path and bottom path almost evenly because the normal travel time for these two paths are nearly the same.

Above predictions can be verified in Fig. 10, which shows the time dependent flow for link 2 and link 3 in Scenario #2. Less than 27% of the drivers (4 out of 15) chose link 2 (corresponds to top path) for the first two groups of drivers. But, starting from time interval 4, this percentage increased to nearly 50% which means the last three groups of drivers chose top path and bottom path almost evenly.

Scenario #3 is very similar to #2 except that the duration of the incident in link 8 will last until time interval 9. Since the fifth group will depart at origin 1 at time interval 5 and arrive at node 5 at time interval 9 if they choose top path, all these five groups of drivers will be affected by the VMS. So, we can predict that the result is most of the drivers in these groups will choose bottom path. Fig. 11 verifies our prediction. Around 27% of the drivers chose link 2 (top path) for all five groups, while other 73% chose bottom path.

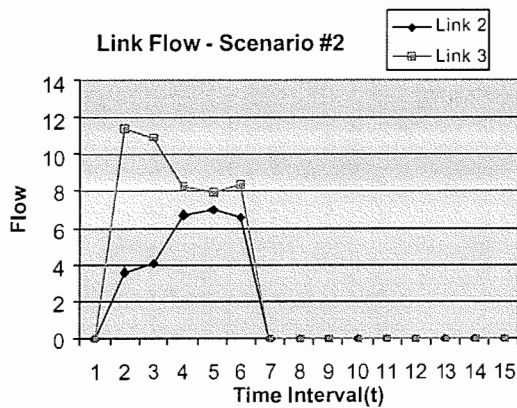


Fig. 10 Flow for Link 2 and 3 in Scenario #2

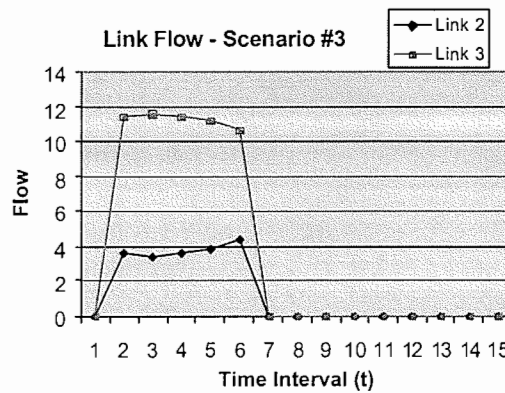


Fig. 11 Flow for Link 2 and 3 in Scenario #3

Fig. 12 depicts the time dependent flow for link 8 in both Scenario #2 and #3. From this figure, we can see that the flows on link 8 remain low for all time intervals in Scenario #3, whereas in Scenario #2 the flows are high (nearly 14) during time interval 8 and 9. Table 6 and 7 in appendix illustrate the time dependent link flow for these two scenarios.

These three scenarios demonstrate that the proposed FDTA model can generate reasonable traffic flows over the network comparing with its traditional SDTA counterpart. Furthermore, it can represent various traffic conditions realistically and

therefore solve decently those dynamic traffic assignment problems which can not be solved very well using SDTA models.

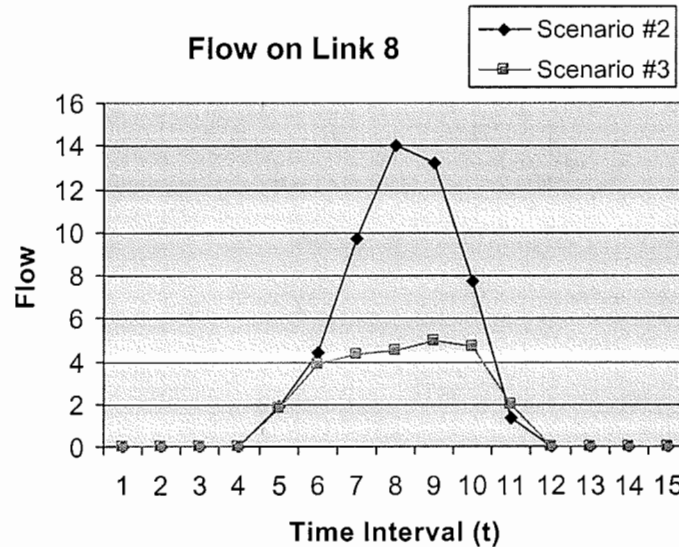


Fig. 12 Flow for Link 8 in Scenario #3

7 Conclusions Remarks

In this paper, we present a new model to apply fuzzy logic theory to dynamic traffic assignment. Fuzzy sets for normal, congested, incident and construction traffic conditions are constructed, and we use L-R type fuzzy numbers to represent these fuzzy sets. It turns out that this method can generate results as reasonable as those generated by the traditional SDTA model under normal traffic conditions. Further, the proposed fuzzy DTA model can solve the DTA problem that cannot be effectively solved by conventional DTA methods, like congested or incident conditions.

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