Bi-Level Formulation for Optimal Traffic Information Dissemination

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ABSTRACT

With the fast-growing telematics market and maturing traffic information services, the pervasive usage of telematics devices provides a feasible mean to manage traffic more efficiently. The provision of traffic information to travelers usually involves different parties that have distinctive objectives: travelers are concerned with benefits of travel time saving, private information service providers (ISPs) seeks to provide Marketable information services from which they can derive a profit, and traffic management centers (TMCs) have the responsibility to maintain and improve performance, i.e. the minimization of the total system travel time. How transportation system managers can leverage this new traffic information flow diagram to improve system performance involving the tradeoff among these objectives remains an open question.

The goal of this paper is to study the tradeoffs among the conflicting objectives of different parties and the resulting traffic system performance. In this paper, we formulate the complex traffic network a bi-level program. The upper level can be formulated using various objective functions such as the objectives for ISP and TMC. The lower level is a multi-class dynamic traffic assignment (DTA) model, which determines dynamic traffic flows in the network by considering the information dissemination strategies provided by the upper level model. Numerical results of a small network are provided to illustrate the behavior of this model.

KEY WORDS
Dynamic Traffic Assignment, Multi-class Dynamic Network Modeling, Bi-level Programming
1. INTRODUCTION

Advanced Traveler Information Systems (ATIS) enhance travelers’ knowledge of traffic condition in the road network, therefore improve traveler’s decision making. Especially with telematics and on-vehicle devices equipped in more and more vehicles, travelers can obtain more real-time traffic information and better route guidance service. Hence, they can change from more congested routes to alternate routes to reduce their route travel times and contribute to the better system performance. To provide these services to travelers, an Information Service Provider (ISP) should respond to various needs of different types of travelers with regard to information gathering, processing and disseminating. However, it is far less understood that whether and to what extent the interaction between the travelers and ISP would both parties. Using simulation experiments, Mahmassani and Chen (1991), and Mahmassani and Jayakrishnan (1991), found that the overall system performance may be reduced if more than 20% of the drivers equipped with motorist information system due to concentration and overreaction (Ben-Akiva et al, 1991). They suggested that provision of coordinated information is necessary. Other studies, such as Waltting and Vuren (1993), and Oh and Jayakrishnan (2002) supported these findings.

To model the effect of traveler information services, one must consider the perspectives of different parties involved. Usually, the provision of traffic information to travelers involves three parties that have distinctive objectives: travelers are concerned with benefits of travel time saving, private ISPs seek to provide marketable information services from which they can derive a profit, and traffic management agencies have the responsibility to maintain and improve performance, i.e. the minimization of the total system travel time.

In this paper, we present a bi-level program approach to solve this three parties’ optimization problem. The upper level can be formulated as the objectives of Traffic Management Center (TMC) and ISP. Here we can get a clear idea about their relationships in Fig 1. TMC mainly concerns about the whole dynamic transportation network system.
performance and its objective is to minimize the total system cost. ISPs usually are private sectors and their objective is to maximize their profit. The lower level is Dynamic Traffic Assignment (DTA), to assign traffic flows to different links/routes in terms of different traveler types. For travelers themselves, their objective is to save travel time at an affordable service charge.

Figure 1: The relationships of TMC, ISP and Travelers

To describe and capture the different behaviors of various types of travelers, dynamic traffic network models should stratify travelers into multiple classes. One way of classifying vehicle types is to differentiate travelers as three groups: fixed or predetermined route travelers, guided travelers, and stochastic route choice travelers. For fixed route ones, they form the habit of route choices and will always make choose fixed routes. For guided travelers, they have the up-to-date real time traffic information and
route guidance and will follow the provided route choices. For unguided travelers, they select routes based on the perceived lowest travel time. Traditionally, the notion of perceived travel time in a static context is modeled through a stochastic generalization of Wardrop’s definition of user-equilibrium (Daganzo and Sheffi, 1977; Sheffi and Powell, 1982). Ran and Boyce (1994) extended this notion to a dynamic context, and established the principle of stochastic dynamic user-optimal (SDUO).

2. BI-LEVEL FORMULATION

In the following, we analyze the objectives of all three parties.

2.1 Upper level: Dynamic Traffic Information Disseminator

Determine optimal traffic information dissemination strategies based on the predicted dynamic flows.

2.1.1 ISP’s Objective: to Maximize its Profit.

ISPs obtain real time traffic data, signal timing, VMS and other traffic information from TMC or by themselves. They will disseminate the real-time or predicted link/route travel time based on their own model with taking special events, incidents, weather, etc into consideration. There are two major parameters for each ISP: information quality $\theta(t)$ and the service charge $c_s(t)$. Because these two items will keep the same in quite a long time, we can consider them as a constant regarding to time t. Since user’s subscription has the same property, the total number of subscribed users will not change over a short time.

ISPs always have different customer service levels to accommodate various clients’ demands. Most of ISPs charge their customers based on three kinds of services:

1. Basic service: to provide real time travel information report such as congestion, incident etc.
2. Enhanced service: to give users access to query routes of OD pairs.
Here we assume the ISP predicted link travel time

\[ t_s(t) = r_s(t) + Accu_s(t) \]  \hspace{1cm} (1)

Where

- \( t_s(t) \) - Actual link travel time.
- \( Accu_s(t) \) - Information error. We assume it follows a normal distribution \((0, r_s(t) \cdot (1 - \theta)^2)\).

ISP’s profit function \( P \) is:

\[ P = Q^* \cdot c_s(t) - [\sigma + \gamma \cdot \theta(t)] + \int_0^Q (\lambda + e^{-\mu x}) dx \]  \hspace{1cm} (2)

Where

- \( Q^* \) - The total number of the subscribed users.
- \( c_s(t) \) - The actual charge of ISP service.
- \( \sigma \) - The static cost of collecting, processing data and disseminating.
- \( \gamma \) - The cost of per dynamic information quality.
- \( \theta(t) \) - Dynamic information quality.
- \( \lambda \) - The ultimate cost per user with a large user base.
- \( \mu \) - The economy of scale of providing the service.

Since \( \frac{\partial^2 P}{\partial Q^2} = \mu e^{-\mu Q^*} > 0 \), ISP has its maximum profit when it can have most subscribed users and charge the highest service fee. This upper level of ISP is influenced by decision variable \( Q^* \) from the DTA lower level.

**2.1.2 TMC’s objective: to minimize the system cost.**

TMC’s goal is to minimize total travel time of all three kinds of travelers
\[
\min \int \left( \sum \nu_a(t) \tau_a(t) \right) dt
\]

Where
- \(T\) - The time period.
- \(\nu_a(t)\) - Inflow rate into link \(a\) at time \(t\)
- \(\tau_a(t)\) - Actual travel time in link \(a\) at time \(t\).

This upper level is subjected to decision variable \(\nu_a(t), \tau_a(t)\) from the DTA lower level.

We can measure the relative reduction (\(RT\)) in total system cost, to demonstrate the change in before and after the implementation of guided services. That is,

\[
RT = \frac{\text{Total cost}^b - \text{Total cost}^a}{\text{Total cost}^a} \times 100\%
\]

The superscripts \(b, a\), respectively, mean “before” and “after” the guided service implementation. A positive \(RT\) indicates total system cost reduction due to the service.

### 2.2 Lower level: Dynamic Traffic Predictor

Here we should consider three classes of travelers: fixed route user, guided users and unguided users.

#### 2.2.1 Objective of Guided users: to minimize route travel cost.

For subscribed users, first we define they are 100% compliance rate users. That is, they will completely follow what they get route choice instructions from their ISP. So they make route choices based on the travel-cost-based ideal dynamic user-optimal state (DUO). If, for each O-D pair at each instant of time, the actual travel costs experienced by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a travel-cost-based ideal dynamic user-optimal state.

In the formulation of the link travel cost, we should consider Value of Travel Time (VOT) as well as Value of Reliability (VOR). For travelers, they prefer to subscribe the
service which has less variance of the predicted link travel time despite the same mean of link travel time. Although Value of Travel Time (VOT) has been thoroughly studied, the Value of Reliability (VOR) was always paid much less attention. Lam (2001) proposed a best-fitting model to measure the VOT and VOR in Orange County, California. In their best model, the VOT is $22.87 per hour, while VOR is $15.12 per hour for men and $31.91 for women.

Link travel cost:

\[ \tau_a(t) = \alpha + \beta \tau_a(t) + \gamma \sigma_a(t) + c_a(t) \]  

(5)

\[ \int_0^T \sum_{a} \left( \tau_a(t) + \tau_a(t + \Delta t) \right) \left( u_a(t + \Delta t) - u_a(t) \right) dt = 0 \]  

(6)

Where

- \( \tau_a(t) \) - ISP predicted link travel time.
- \( \sigma_a(t) \) - Variance of link travel time which results from ISP’s information quality.
- \( c_a(t) \) - Cost of disseminate information of link a at time t
- \( \alpha \) - Constant value.
- \( \beta \) - Value of Time (VOT).
- \( \gamma \) - Value of Reliability (VOR).
- \( u_a(t) \) - Inflow to link a.
- \( x_a(t) \) - No. of vehicle on link a.
- \( \tilde{\tau}_a(t) \) - Minimal mean actual route travel cost between (t, s) for flows departing origin at time t.

2.2.2 Objective of Unguided users: to minimize perceived route travel time.
For unsubscribed users, they will follow Stochastic Dynamic User Optimal (SDUO). Travel-time-based ideal SDUO state: If, for each O-D pair at each instant of time, the perceived travel times by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a travel-time-based ideal dynamic stochastic user-optimal state. Travelers make route choice decisions in terms of perceived expected route travel time. The perceived link travel time can be formulated as the actual link travel time plus a traveler’s perception error. Travelers’ perception error is assumed normally distributed across the population.

\[ T_a^m(t) = \tau_a^m(t) + \xi_a^m(t), \quad a \in p, \quad p \in R(t), \quad \forall r,s,i,m \]  \hspace{1cm} (7)

Where

\( \tau_a^m(t) \) : Actual link travel time for link a.

\( \xi_a^m(t) \) : Perception error for individual i from group m for link a.

For each route p and each O-D pair rs, define an auxiliary cost term as follows:

\[ F_p^r(s) = [f_{rs}^p(t) - f_{rs}^p(t)] \frac{\partial P_{rs}^p(t)}{\partial f_{rs}^p(t)} = 0 \forall r,s,p \]  \hspace{1cm} (8)

It is obvious that the above equality states the SDUO route choice conditions, since \( \partial P_{rs}^p(t)/\partial f_{rs}^p(t) > 0 \). As shown in Nagurney (1993), the above system of equations is equivalent to the following variational inequality for each time instant \( t \in [0, +\infty) \):

\[ \sum_{s} \sum_{p} F_p^r(s) [f_{rs}^p(t) - f_{rs}^p(t)] \geq 0 \]  \hspace{1cm} (9)

Since \( F_p^r(t) = 0 \), the above inequality is also equivalent to the integral form:

\[ \int \sum_{s} \sum_{p} F_p^r(s) [f_{rs}^p(t) - f_{rs}^p(t)] dt \geq 0 \]  \hspace{1cm} (10)

### 2.2.3 The Dynamic Network Constraint Set
The constraint set for our DTA problem is summarized for each class of travelers.

**Route Flow Assignment Constraints:**
\begin{align}
    f_{ps}^{m}(t) &= f_{ps}^{t}(t)P_{ps}^{m}(t) \quad \text{where} \quad f_{ps}^{t}(t) \quad \text{is given,} \quad \forall r,s,p,m \\
    f_{ps}^{m}(t) &= u_{ps}^{m}(t) \quad \forall t,s,p,m; a \in A(r); a \in p;
\end{align}

**Other Constraints for all traveler classes:**

**Relationship between state and control variables:**
\begin{align}
    \frac{dx_{sa}^{m}}{dt} &= u_{sa}^{m}(t) - v_{sa}^{m}(t) \quad \forall m,a,p,r,s \\
    \frac{dE_{sa}^{m}(t)}{dt} &= j_{sa}^{m}(t) \quad \forall p,m,r,s \neq r
\end{align}

**Flow conservation constraints:**
\begin{align}
    \sum_{s \in \delta(i)} v_{sa}^{m}(t) &= \sum_{s \in \delta(i)} u_{sa}^{m}(t) \quad \forall j,p,m,r,s; i \neq r, s \\
    \sum_{s \in \delta(i)} v_{sa}^{m}(t) &= \sum_{s \in \delta(i)} u_{sa}^{m}(t) \quad \forall m,r,s; i \neq r
\end{align}

**Flow propagation constraints:**
\begin{align}
    v_{sa}^{m}(t) &= \sum_{i \in B(j)} [E_{ai}^{m}(t + \tau_{ai}(t)) - E_{ai}^{m}(t)] + [E_{ai}^{m}(t + \tau_{ai}(t)) - E_{ai}^{m}(t)] \quad \forall \alpha \in B(j); j \neq r, p, r, s
\end{align}

**Definitional constraints:**
\begin{align}
    \sum_{sa} u_{sa}^{m}(t) &= u_{a}(t), \quad \sum_{sa} v_{sa}^{m}(t) = v_{a}(t), \quad \sum_{sa} x_{sa}^{m}(t) = x_{a}(t), \quad \forall a
\end{align}

**Nonnegativity conditions:**
\begin{align}
    f_{ps}^{m}(t) &\geq 0, \quad u_{sa}^{m}(t) \geq 0, \quad v_{sa}^{m}(t) \geq 0, \quad \forall m,a,p,r,s \\
    E_{sa}^{m}(t) &\geq 0, \quad E_{sa}^{m}(t) \geq 0, \quad \forall p,m,r,s
\end{align}

**Boundary conditions:**
\begin{align}
    E_{sa}^{m}(0) &= 0, \quad \forall p,m,r,s \\
    x_{sa}^{m}(0) &= 0, \quad \forall a, p, m, r, s
\end{align}

**3. THE SOLUTION ALGORITHM**
The problem is to determine information quality $\theta(t)$, the service charge fee $c_s(t)$ and the market penetration $\rho(t)$ to find the optimal solution for these three objectives while they usually conflict with each other. Since information quality is highly correlated with service charge fee, we can formulate one as a function of the other to simplify the complexity. For the upper level, in the outmost iteration, we can fix the information quality, then for every inner iteration to change market penetration, we can optimize the lower level in the DTA model to satisfy the objectives of both guided and unguided travelers. Then pass the traffic inflow rates and link travel time to the upper level to minimize the total system cost. ISPs are profit-driven and they will reach their optimal when they have the largest market penetration, which is usually not consistent with the system optimal.

![Diagram](image)

**Figure 2** The decision variables from lower level to upper level

To solve the VI problem in lower level, we need to convert our continuous time VI problem into a discrete time VI problem. The time period $[0, T]$ is subdivided into $K$.
small time intervals. Each time interval is considered as one unit of time. In this combined algorithm, we define the travel time approximation procedure (relaxation) as the outer iteration and the Franke-Wolfe/MSA procedure as the inner iteration. The algorithm for solving our multi-class route choice model can be summarized as follows:

**Step 0:** Initialization. Initialize all link flows \( \{x_{kl}^{\infty}(k)\}, \{u_{kl}^{\infty}(k)\}, \{v_{kl}^{\infty}(k)\} \) to zero and calculate initial time estimates \( t_0^*(k) \), regardless of traveler classes. Set the outer iteration counter \( j=1 \).

**Step 1:** Relaxation. Set the inner iteration counter \( n = 1 \). Find a new approximation of actual link travel times: \( \tilde{t}_l^{\infty}(k) = \tilde{t}(x_l^{\infty}(k)) \), where (*) denotes the final solution obtained from the most recent inner problem. Solve the route choice program.

*For Fixed Route Travelers: Predetermined Routes*

[Step 1.1]: Fixed Route Assignment. Assign O-D departure flow \( f_{ kl}^{\infty}(k) \) of class 1 travelers to each prespecified route with prespecified route flow share. Use flow conservation and propagation constraints to calculate the resulting link flows \( \{u_{kl}(k)\}, \{v_{kl}(k)\}, \{x_{kl}(k)\} \).

*For Unsubscribed Travelers: Stochastic Subproblem*

[Step 1.2.1]: Stochastic Dynamic Network Loading. Perform Monte Carlo simulation by sampling random route travel times. Compute minimal perceived route travel time \( \bar{t}_l^{\infty}(k) \) and assign all departure flows \( f_{ kl}^{\infty}(k) \) of unsubscribed travelers to these routes during each Monte Carlo iteration. Let the temporary link flow vector resulted from the all-or-nothing loading be called \( \{\tilde{p}, \tilde{q}, \tilde{r}\} \) at Monte Carlo iteration \( i \).

Then the stochastic network loading is solved by the recursive equations (23)-(25). As \( i \) equals a prespecified number, stop. The vector \( \{\tilde{p}, \tilde{q}, \tilde{r}\} \) is used as the converged link flows \( \{p^*, q^*, r^*\} \) at inner iteration \( n \), determined by the stochastic network loading of the O-D flows according to the specified multinomial probit route choice density function.
\[ p'_{\omega_a}(k) = \frac{(i-1)p_{\omega_a}^{\omega_{\omega a}}(k) + \hat{p}_{\omega_a}(k)}{i} \quad \forall a \]  
\[ q'_{\omega_a}(k) = \frac{(i-1)q_{\omega_a}^{\omega_{\omega a}}(k) + \hat{q}_{\omega_a}(k)}{i} \quad \forall a \]  
\[ y'_{\omega_a}(k) = \frac{(i-1)y_{\omega_a}^{\omega_{\omega a}}(k) + \hat{y}_{\omega_a}(k)}{i} \quad \forall a \]  

[Step 1.2.2]: Method of Successive Averages (MSA). Using the predetermined step size \(1/n\), yield a new MSA main problem solution through equations (26)-(28).

\[ u'_{\omega_a}^{\omega_{\omega a}}(k) = u_{\omega_a}^*(k) + \frac{1}{n}[p_{\omega_a}^{\omega_{\omega a}}(k) - u_{\omega_a}^*(l)] \quad \forall a \]  
\[ v'_{\omega_a}^{\omega_{\omega a}}(k) = v_{\omega_a}^*(k) + \frac{1}{n}[q_{\omega_a}^{\omega_{\omega a}}(k) - v_{\omega_a}^*(k)] \quad \forall a \]  
\[ x'_{\omega_a}^{\omega_{\omega a}}(k) = x_{\omega_a}^*(k) + \frac{1}{n}[y_{\omega_a}^{\omega_{\omega a}}(k) - x_{\omega_a}^*(k)] \quad \forall a \]  

For Subscribed Travelers: Deterministic Subproblem

Taking partial derivatives of the objective function \(Z\) with respect to the major decision variables, a linear subproblem can be constructed:

\[
\begin{aligned}
\min_{u',v',x'} & \sum_{k} \sum_{i} \sum_{\omega} \{ t_i'_{\omega_a}(k) p_{\omega_a}^{\omega_{\omega a}}(k) + t_i'_{\omega_a}(k) q_{\omega_a}^{\omega_{\omega a}}(k) + t_i'_{\omega_a}(k) v_{\omega_a}^{\omega_{\omega a}}(k) + (k + 1) \} \\
\text{s.t.} & \text{ constraints (11)-(22)}
\end{aligned}
\]

where the subproblem variables \(p_{\omega_a}^{\omega_{\omega a}}(k), q_{\omega_a}^{\omega_{\omega a}}(k), y_{\omega_a}^{\omega_{\omega a}}(k)\) are used to represent the main problem variables \(u_{\omega_a}^*(k), v_{\omega_a}^*(k), x_{\omega_a}^*(k)\), respectively. The cost terms are defined as follows:
\[ r^*_a(k) = \frac{\partial J^*}{\partial u^*_a(k)} = \tau_a[\bar{u}_a(k), \bar{u}_{a^*}(k), u_{a^*}(k), \bar{x}_a(k), \bar{x}_{a^*}(k), x_{a^*}(k)] - \bar{y}^*_a(k) \quad \forall a, r; k = 1, \ldots, K \]  
(30)

\[ r^*_a(k) = \frac{\partial J^*}{\partial v^*_a(k)} = \sum_i C^{(i)} \frac{\partial J^*}{\partial v_a(k)} \int_d^\infty \frac{d\omega}{\partial v_a(k)} = 0 \quad \forall a, r, k = 1, \ldots, K \]  
(31)

\[ r^*_a(k) = \frac{\partial J^*}{\partial x^*_a(k)} = \sum_i C^{(i)} \frac{\partial J^*}{\partial x_a(k)} \int_d^\infty \frac{d\omega}{\partial x_a(k)} = 0 \quad \forall a, r, k = 2, \ldots, K \]  
(32)

\[ r^*_a(K + 1) = \frac{\partial J^*}{\partial x^*_a(K + 1)} = 0 \quad \forall a \]  
(33)

[Step 1.3.1]: Update. Calculate \( r^*_a(k), r^*_a(k), \) and \( r^*_a(k) \) using equations (30) - (32).

[Step 1.3.2]: Direction Finding. Based on \( r^*_a(k), r^*_a(k), \) and \( r^*_a(k) \), search the minimal-cost route forward from each artificial origin to the super-destination over an expanded time-space network for each O-D pair \((r, x)\). Perform an all-or-nothing assignment, yielding subproblem solution \( p^*_{a^*}(k), q^*_{a^*}(k), y^*_{a^*}(k) \).

[Step 1.3.3]: Line Search. Find the optimal step size that solves the one-dimensional search problem using a standard line search procedure.

[Step 1.3.4]: Move. Find a new solution by combining \( u^*_{a^*}(k), v^*_{a^*}(k), x^*_{a^*}(k) \) and \( p^*_{a^*}(k), q^*_{a^*}(k), y^*_{a^*}(k) \) using the optimal step size.

[Step 1.3.5]: Convergence Test for Inner Iterations. If \( n \) equals a prespecified number, go to step 2; otherwise, set \( n = n + 1 \) and go to step 1.3.1.

**Step 2**: Convergence Test for the Outer Iterations. If \( \bar{Z}^{(n+1)}(k) \equiv \bar{Z}^{(n)}(k) \), stop. The current solution \( \{u^*_{a^*}(k)\}, \{v^*_{a^*}(k)\}, \{x^*_{a^*}(k)\} \) is in a near optimal state; otherwise, set \( l = l + 1 \) and go to step 1.
3. CASE STUDY

The testing network is indicated in Fig 3. The length of each link is 1 mile with detailed link information depicted in Table 1. All the experiments here share the following common input characteristics:

- The total O-D flows are 20 vehicles for each of the five 15-second periods (equivalent to a flow of 4800 vehicles per hour).
- Free flow speed is 50 miles per hour.
- The $\Delta$ threshold specifying the desired accuracy was set to 0.01.
- We define the relationship between information quality and ISP service charge fee as:
  \[ \theta(i) = 1 - e^{-\alpha r(i)} \]
  \[ \alpha = 0.9 \] listed in Figure 4.

![Figure 3 Experimental Network](image-url)
Table 1. Link Information

<table>
<thead>
<tr>
<th>Link Number</th>
<th>Start Node</th>
<th>End Node</th>
<th>Length (miles)</th>
<th>Capacity (# of Veh.)</th>
<th># of Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4800</td>
<td>2</td>
</tr>
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</tr>
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<td>4</td>
<td>1</td>
<td>4800</td>
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<tr>
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<td>8</td>
<td>9</td>
<td>1</td>
<td>4800</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 4 Information Quality vs. Service Charge Fee
There are two distinctive scenarios for this simple network as in Table 2.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Distinctive features</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Incident happened at link 1 at all time intervals. Capacity reduction = 0.8.</td>
</tr>
<tr>
<td>2</td>
<td>No incident happens. All links have 2 lanes.</td>
</tr>
</tbody>
</table>

First we consider the worst case that there was a serve congestion happened, which is equivalent to 80% capacity reduction. In Greenshield function, jam density \( c_{max} = \frac{eam*lanes*link\_Length}{240\ \text{veh/mile}} \). Then we can calculate the speed of link 1 in the following:

\[
\text{if } (\text{Link}[1].dXa(t) > c_{\text{max}}) \quad \text{speed}=c_fv_f; \quad (34)
\]

\[
\text{else} \quad \text{speed}=c_fv_f+(ffvf-c_fv_f)*\left(1-\text{Link}[1].dXa(t)/c_{\text{max}}\right); \quad (35)
\]

Where

\( c_fv_f \) – jam-speed.

\( ffv_f \) - free flow speed.

There are three approaches to verify the results. First, we can compare the difference of total system cost when service charge fee changes. The second approach analyzes the circulative number of vehicles passed through each link for the entire analysis period \( T \). In last approach, we can observe the route travel time for guided travelers and unguided ones.
When ISP’s service charge fee goes up, usually the provided information quality will improve in some degree. For information quality 86.4%, the system performance gets improved apparently before 30% subscribed users, and maximum relative reduction (RT) 34.1%. If the subscribed rate is higher than 75%, the system cost will fluctuate to a little bit higher. When information quality decreases to 66.4%, the system obtains relatively better performance after 45% travelers subscribe ISPs’ services and maximum RT is 30.4%. There is also a little fluctuation to higher total system cost if the subscribed rate is more than 55%. For the last case when information quality equals 42.8%, over time system performance keeps getting better before 65% users subscribe ISPs’ services and maximum RT is 31.3%. When more than 70% travelers subscribe the ISPs’ services, the system cost goes up. All these three cases indicate total system cost improves in a large degree if appropriate travelers subscribe ISPs’ services when the transportation network is congested. Furthermore, better ISP provided information quality will lead to the decrease of equilibrium subscribed rate.
For the second approach, we will compare circulative percentage of vehicles (underlining number in the Fig. 6 and Fig. 7) passed through each link during the whole analysis period for all guided users and all unguided users. Here we can see about 11% inflow rate reduction in the congested link 1 when all travelers are guided.

Figure 6 Circulative Percentage of Vehicles in every link for all unguided users

In the third approach, we will compare the route travel time for 100% unguided case and 100% guided case first. Here we study the case when the information quality as 86.4%. When all travelers are unguided, the worst route (1-2-5-8-9) travel time is 15 time intervals for the group who departed in the last time interval of the five 15-second periods, while only 4 time intervals for the guided group departing at the same time interval. Meanwhile, when only 1 traveler subscribe ISP service, his best route (1-4-7-8-
9) only last 4 time intervals. In this case, the saving time for the subscribed users is about 73%. When the subscribed rate increases, more subscribed users will choose less congested routes, and in turn will benefit the unsubscribed travelers in congested routes. However, when the subscribed rate is high enough and all the subscribed users will rush into the less congested route, it will even make the system performance worse, and increase the route travel time of themselves. In Figure 8, when more than 40% guided travelers head into the less congested routes, their route travel time will increase, while that of the unguided travelers will decrease from 14 units to 5 units. It also supports that the appropriate guided users can benefit the whole travelers and system performance.

![Route travel time for the last departing group](image)

Figure 8 Route travel time – percentage of guided users

In the second scenario, where there is no congestion happened, it is no obvious change for total system cost whether the travelers subscribe ISPs’ services or not. In this case, unsubscribed travelers can make reasonable route choices based on their driving experiences and perceptions. Even the last departing group of the five 15-second periods needs only 4 time intervals to travel from origin to destination. In this sense, there is no
much travel time saving for subscribed travelers. It will result in that few travelers will pay for the ISPs’ services.

![Figure 9 System Cost - Percentage of Guided Users without congestion](image)

4. CONCLUDING REMARKS

In this paper, we present a bi-level program approach to formulate multi-equilibrium objectives in dynamic transportation network. Since TMC, ISP and travelers have different objectives, it is hard to find the optimal solution for all of them. It is applied to figure out what market penetration and service charge fee are sustainable for TMC, ISP, and travelers. Numerical results of a small network are provided to illustrate the behavior of this model. It turns out that when there are congestions in the dynamic transportation network, appropriate subscribed rate benefit both all the travelers and system performance, while the ISPs’ information influences little without congestion in the transportation network.
REFERENCES:


