Temporal Control of Variable Message Signs Towards Achieving Dynamic System Optimum

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ABSTRACT

This paper explores how close the dynamic flow splits resulting from the temporal control of variable message signs (VMS) can come to a dynamic system optimum. While VMS is generally considered not useful in providing flow splits across alternate paths at any given time, temporally changing VMS messages can achieve certain desirable flow splits over time, though this depends substantially on the driver compliance behavior. The paper focuses on how useful this approach is. First, a dynamic system optimum traffic assignment problem for temporal VMS route guidance is formulated as an integer programming problem and a heuristic algorithm is presented. As opposed to the path flow-based solution that presents the optimal solution as a form of path flow, the algorithm provides a set of time dependent optimal paths for a given time period. Every time step an optimal path is selected from a set of mutually exclusive alternatives that can be expressed by VMS messages. The algorithm is tested and compared with various VMS guidance methods via simulation experiments incorporating a reasonable driver’s compliance model. Our special interest is to investigate path flow pattern of temporal VMS guidance and to explore how much the pattern differs from that of dynamic system optimum. The results show that the path flow pattern and network performance somewhat close to a dynamic system optimum can be achievable via temporal VMS guidance.

Key Words: Variable message signs; Route guidance; Advanced traveler information systems (ATIS); Dynamic traffic assignment; Dynamic system optimum.
INTRODUCTION

Advanced Traveler Information Systems (ATIS) are considered as a promising technology to improve traffic condition by helping travelers to use efficiently existing transportation facilities. Recently there has been plenty of research on this field, and ATIS are in transition moving from laboratory to real world. It is expected to improve travel times for the drivers and reduce the total cost for the system under ATIS environment. As a type of ATIS, variable message signs (VMS) have been widely installed for freeway traffic management in most metropolitan areas. Even though they have not achieved their full capabilities yet due to various reasons, particularly lack of algorithm in seeking alternative routes, and deficiencies of the surveillance systems, VMS have potential to improve traffic condition to a significant extent.

This paper explores how close the dynamic flow splits resulting from the temporal control of variable message signs (VMS) can come to a dynamic system optimum. While VMS is generally considered not useful in providing flow splits across alternate paths at any given time, temporally changing VMS messages can achieve certain desirable flow splits over time, though this depends substantially on the driver compliance behavior. The paper focuses on how useful this approach is.

First, a dynamic system optimal traffic assignment problem needs to be solved to compare any results from temporal VMS guidance to a benchmark. Dynamic optimal route guidance solution can be obtained by solving dynamic traffic assignment problem (DTA). With need of dynamic traffic management in intelligent transportation systems (ITS), much attention has paid to DTA. There have been many studies on DTA (1-7) since the first flow-based dynamic traffic assignment model was developed as a discrete nonlinear programming formulation by Merchant and Nemhauser (8, 9). DTA problems have been tackled by either analytical models or simulation-based heuristic schemes. Most analytical models have limitations in capturing realism mainly due to network simplification though they often posses certain desired analytical properties to guarantee optimality. In contrast, the simulation-based approaches have been able to capture traffic conditions more realistically and have shown superior abilities in incorporating signal controls and detailed behavior models (4).

A dynamic optimal solution for VMS route guidance can also be found by solving the DTA problem from a VMS location. The problem can be formulated as a mathematical problem and heuristically solved (10). The decision variables in the problem are path flows from the VMS location to target
destination. That is, the solution is represented as a form of path flow fraction between alternative routes. However, due to the limitations of VMS control, achieving such a pattern is not a trivial task. Such pattern can be achieved only by forcing traffic to split according to the desired optimal fraction across the alternatives. However, such enforcement is impossible in VMS route guidance. Therefore, it is necessary to develop a new algorithm providing one path at a time for VMS route guidance. This paper introduces a practically applicable method providing a set of such time dependent optimal paths for a given time horizon. Every time step an optimal path is selected from a set of mutually exclusive alternatives that can be expressed by VMS. This predictive approach becomes more realistic when incorporating drivers’ compliance behavior model is incorporated. The predictive model with compliance allows more plausible prediction. This temporal VMS control is similar to 0-1 control as in Sawaya et al. (11); however, our framework incorporates an explicit compliance model to derive plausible traffic pattern from such VMS guidance.

This paper tests the proposed algorithm and compares with the other route guidance methods via off-line simulation experiments. We investigate performance of the route guidance systems and path flow patterns. Our special interest is to investigate path flow pattern of temporal VMS guidance and to explore how much the pattern differs from that of dynamic system optimum. The study results indicates that temporal VMS guidance is more promising than is perhaps generally assumed.

This paper is outlined as follows. The next section provides DTA formulation and solution procedure for VMS route guidance, and Section 3 presents a driver’s en-route diversion model. Section 4 shows results of the simulation experiments and discusses findings. Finally Section 5 concludes the paper.

**DYNAMIC SYSTEM OPTIMAL SOLUTION FOR VMS ROUTE GUIDANCE**

**Path flow based DSO Problem for VMS Route Guidance**

Achieving a dynamic system optimal (DSO) state is often preferred goal in VMS route guidance even though the state may not be easily achievable. The purpose of this section is to present a methodology for determining time-dependent system optimal solution. Maymassani and Peeta (4) have formulated the DSO problem with deterministic full information, and Valdes-Diaz et al (10) have re-formulated the problem for
the VMS routing case. The objective function is to minimize total system cost for all vehicles in the network. In the VMS routing formulation, drivers are classified into two classes: those who influenced by VMS and those who are not. In its objective function, cost terms for vehicles routed by VMS consist of two terms, before VMS and after VMS. Decision variables in the problem are time-dependent path flow pattern from VMS location to corresponding destinations. Refer to Valdes-Diaz et al (10) for the complete formulation.

As addressed by Ghali and Smith (11), the dynamic system optimal may generally not have a unique solution and may be a non-convex and non-differentiable problem. However, the dynamic system optimal problem can be approximately solved for given time-dependent O-D demand during a finite time period. Ghali and Smith (13) have shown that it is beneficial to route vehicles according to the local marginal delay, and Mahmassani and Peeta (4) have obtained a quasi DSO solution by assigning vehicles to the minimum time-dependent marginal travel paths. Likewise, the quasi DSO solution for VMS route guidance is heuristically found. In our study, to obtain dynamic network conditions, a macroscopic traffic simulation program, DYNASMAINT (14), is used. The solution procedure based on the traffic simulation as follows

Step 1: Set iteration counter \( i = 0 \). Assign the given time-dependent O-D flows to a time-dependent initial set of paths.

Step 2: Simulate vehicles with VMS route guidance (a set of time dependent path splits between VMS and corresponding destinations). Obtain time-dependent link travel times and delays from the simulation.

Step 3: Compute marginal costs for links and turn movements, and find a new VMS route guidance scheme (a set of minimum time-dependent marginal travel paths between VMS and corresponding destinations).

Step 4: Update the VMS guidance scheme (a set of time dependent path splits, \( s \), between VMS, \( v \), and corresponding destinations, \( j \), during time step, \( t \)) for the next iteration, \( i+1 \), using the method of successive averages (MSA).

\[
S_{vjk}^{(i+1)} = \frac{i \cdot S_{vjk}^{(i)} + \delta}{i + 1}, \text{ where } \delta = \begin{cases} 1, & \text{if optimal at iteration } i \\ 0, & \text{otherwise} \end{cases}
\]
Step 5: Check convergence with criteria, $|\delta_{jk}^{r+1} - \delta_{jk}^{r}| \leq \varepsilon$. Count the number of violations. Stop if the number of violation is greater than upper bound; otherwise, update iteration number $i = i+1$, and go to Step 2.

**DSO Problem for Temporal VMS Route Guidance**

The aforementioned path flow based DSO for VMS route guidance provides time-dependent optimal splits between alternative routes associated a VMS. The time-dependent optimal splits may be used as target value; however, achieving such splits is not a trivial task. It is not possible for VMS to provide multiple path guidance at a time since VMS display hardware can often show only one path at a time. That is, the time-dependent optimal path splits cannot be directly translated into a VMS message. A possible way of using such splits may be to recommend a path with highest split.

This section introduces a DSO problem for temporal VMS route guidance, given path flows within a finite time, $T$, in a sub-network associated with VMS. In the temporal VMS route guidance, the route recommendation is same during a time period, but can change only over time. That is, the temporal route guidance selects the optimal path for a time period. The problem is formulated as a mixed integer programming problem where decision variables are a set of time-dependent optimal paths. VMS is located on a VMS link with downstream node $v$ as the decision node (VMS node). The VMS operation is for routing the traffic to the respective destination $d$. The objective function of the problem is formulated as equation (1) minimizing the total travel time for all vehicles on the network. The first and second terms are for vehicles traversing VMS locations. The first term represents the total travel time from origins to respective VMS nodes for those who traverse the VMS location. The second term represents the total travel time for the same vehicles from the VMS nodes to the respective destinations via one of the alternatives. The third term represents the total travel time for the rest of vehicles in the network. The time-dependent optimal path is represented by 0-1 binary format and alternatives are mutually exclusive due to the constraint (2). Other constraints for the DTA problem are skipped in this paper. They are formulated similar manner as in Peeta (1994) and other DTA references.
\[ Z = \min \sum \sum \sum \left( r_{i,j} \cdot T_{i,j,k} \cdot \sigma_k \right) + \sum \sum \sum \left( r_{i,d} \cdot T_{i,d,k} \cdot \sigma_k \right) \] 
\[ + \sum \sum \sum \left( r_{i,j} \cdot T_{i,j,k} \cdot (1 - \sigma_k) \right) \]  

subject to

\[ \sum \sigma_k = 1 \]  

where

\[ V = \text{set of VMS nodes} \]
\[ K_{i,j} = \text{set of paths considered between node } i \text{ and } j \]
\[ r_{v,j} = \text{number of vehicle leaving from VMS node } v \text{ to node } j \text{ during time period } \tau \]
\[ r_{i,j} = \text{number of vehicle leaving from node } i \text{ to node } j \text{ via path } k \text{ during time period } \tau \]
\[ T_{i,j,k} = \text{travel time from node } i \text{ to node } j \text{ via path } k \text{ for traffic leaving during time period } \tau \]
\[ \sigma_k = \begin{cases} 1, \text{if path } k \text{ includes a VMS, } v, \text{ and is destined to the corresponding destination, } d \\ 0, \text{otherwise} \end{cases} \]
\[ u_k = \begin{cases} 1, \text{if VMS path } k \text{ is chosen to recommend} \\ 0, \text{otherwise} \end{cases} \]

Difficulty of the problem arises from the all-or-nothing nature in assigning the path flows from VMS to corresponding destination. In contrast to the path flow based DSO, the iterative approach with MSA update cannot be applied for the problem. This paper develops a heuristic algorithm that spreads flow over time rather than spreading flow over alternatives within discrete time interval. The heuristic method seeks the best path that can be expressed by a VMS message for each time step. The solution procedure for the problem is as follows:

Step 0. Initialization

Step 0.1 Run a simulation without VMS route guidance, and then store time dependent link travel times and delays.

Step 0.2 Find the best VMS path, \( M_{i,j}^{\tau} \), for VMS \( v \) at time step \( \tau \) based on the time dependent marginal path costs, and set the number of fixed time step, \( N = 1 \).
Step 0.3 Set the initial set of VMS paths for all time steps as same as the VMS path at time step 1, i.e.,

\[ M_{i,0} = M_{i,1}^*, \ \forall v \in V, t \in T. \]

Step 1. Simulation

Run a simulation with a set of VMS route guidance, \( M_{ij}, \forall v \in V, t \in T \), and store time dependent link travel times and delays.

Step 2. Selection of best paths

Calculate time-dependent marginal path costs and find a new set of best VMS paths,

\[ M_{ij}^*, \forall v \in V, t \in T \]

Step 3. Update

Step 3.1 Continue if \( M_{i,x}^* = M_{i,x+1}^*, \forall v \in V \) and \( N+1 < T \); otherwise, go to Step 3.3

Step 3.2 Set \( M_{i,x+1}^* = M_{i,x+1}^*, \forall v \in V \) and \( N=N+1 \); go back to Step 3.1

Step 3.3 Set the rest of VMS paths as same as the VMS path at time step \( N+1 \),

i.e., \( M_{ij} = M_{i,x+1}^*, \forall v \in V, t \in [N+1, T] \), and \( N=N+1 \).

Step 4. Stopping Criteria

Stop if the fixed time step reaches the total number of time steps \( (N+1 = T) \); otherwise go to step 1.

The heuristic fixes optimal paths from the beginning time step, and continues to the next as order of time sequence. That is, this algorithm finds solutions time step by time step, but it does not need to iterate all time steps. When the optimal path for the first undecided time step is same as the previous one, the method automatically moves to the next time step until one of the other paths becomes the optimal. That is, this method seeks the timing to switch the optimal path as well. Once optimal paths are fixed, they are always optimal with the previous time steps' VMS guidance scheme. An example in FIGURE 1 shows how the optimal set of VMS route guidance is determined. This heuristic method can be also applied to dynamic user equilibrium by replacing marginal path costs with average path costs. This heuristic embeds the rolling-horizon approach in a sense that the optimal path for next time period is determined based on the prediction with the current control scheme. Therefore, the algorithm can directly applied to real-time with the feedback control scheme.
VMS for 10 time steps:

# 0 : 0 0 0 0 0 0 0 0 0 0 => No VMS assumed
1 1 1 1 2 2 2 3 3 3 => Best paths under no VMS
# 1 : 1 1 1 1 1 1 1 1 1 1 => VMS guidance 1 (fixed time step = 1)
1 1 2 2 3 3 3 3 3 3 => Best paths under VMS 1
# 2 : 1 1 2 2 2 2 2 2 2 2 => VMS guidance 2 (fixed time step = 3)
1 1 2 2 1 3 3 2 3 3 => Best paths under VMS 2
# 3 : 1 1 2 2 2 1 1 1 1 1 => VMS guidance 3 (fixed time step = 6)
1 1 2 2 2 1 1 2 2 2 => Best paths under VMS 3
# 4 : 1 1 2 2 2 1 1 2 1 1 => VMS guidance 4 (fixed time step = 9)
1 1 2 2 2 1 1 2 2 2 => Best paths under VMS 4
# 5 : 1 1 2 2 2 1 1 1 2 2 => VMS guidance 5 (fixed time step = 10); Final Guidance scheme

FIGURE 1. Example of optimal VMS determination procedure

Note: Each number represents a path from VMS to a target destination.

INCORPORATION OF DRIVER’S EN-ROUTE DIVERSION MODEL

Unlike other components of advanced traffic management systems, the effectiveness of ATIS is determined by the drivers’ awareness of the information and their evaluation of its usefulness. There have been many studies on driver’s response to ATIS, and many factors influencing drivers’ decision have identified (15-20). However, most studies have investigated the benefits of ATIS by various levels of guided drivers, in spite of importance of drivers’ compliance model in ATIS evaluation (21, 22, 23). Oh et al. (24) have stressed importance of compliance behavior in evaluating route guidance systems. This study incorporates a general form of drivers’ en-route diversion model into the simulation model for more realistic analyses.

This section introduces a model framework to capture driver’s compliance behavior. The model includes a minimal number of essential variables affecting driver’s decision associated with VMS. It is a
binary logit model that assumes Gumbel distribution for the error components. The model takes three different types of variables into consideration as follows:

- Driver’s characteristic variable: driver’s knowledge on network
- Traffic characteristic variable: traffic condition on the current road
- VMS’s characteristic variable: VMS’s level of reliability

VMS are considered as devices providing prescriptive route guidance rather than descriptive information. In the sense, drivers make their decisions according to their evaluation of the information source. That is, the evaluation is an inherent factor in the driver’s mind, affecting their compliance. This inherent variable is a compound of strength of message and reliability. The message, “ACCIDENT AHEAD / HEAVY CONGESTION” will be stronger than the simple message, “DELAY POSSIBLE”. That is, there exists relative strength of message, and the strength is represented by the inherent variable with reliability of information. This inherent factor associated to the information source can be obtained from the driver’s long-term experience. From the modeler’s point of view, the value can be captured via a day-to-day update approach (25).

There are also other factors affected by driver’s characteristics. One of important driver’s characteristics associated with en-route diversion behavior is driver’s knowledge on network. Many researchers have pointed out this as an important factor affecting driver’s en-route decision. That is, drivers familiar with the network may switch their route more often than those who are not. Drivers unfamiliar with the area tend to stick to their initial route due to fear of losing their ways. Current traffic condition is also another factor affecting driver’s en-route decision. Usually driver’s needs to divert increases in heavy traffic condition, especially those who are familiar with the area or those who are late for their schedule. As a representative variable for current traffic condition, a ratio of current speed with respect to free flow speed is used in the model. The model framework proposed here is a binary logit model, deciding whether or not to divert. The probability that a driver, \( n \), follows the guidance from VMS, \( v \), at the link, \( a \), is expressed as equation (3).
\[ P(n) = \frac{1}{\exp(\alpha + \beta \cdot LOF_n + \gamma \cdot LOC_a + \delta \cdot VMS_v)} \]  

where, \( LOF_n = \) Level of network familiarity for driver \( n \). (range 1-5)  
\( LOC_a = \) Level of congestion on link \( a \) represented by speed / free speed. (range 0.0-1.0)  
\( VMS_v = \) Reliability value for VMS \( v \). (range 0.0-1.0)  
\( \alpha, \beta, \gamma, \delta = \) Parameters

By adjusting the model parameters, we can obtain compliance rates that are reasonable for our studies. Since such a compliance model has not been calibrated in the past, our simulation studies in the next section uses a parameters set (\( \alpha=1.8, \beta=0.5, \gamma=5, \) and \( \delta=-5.6 \)) that shows reasonable compliance behavior over a reasonable range of variable values. The parameter set is an example for demonstration purpose, so the model parameters need to be calibrated for the application of real world. While other variables are internally provided within the simulation model, the reliability value of VMS needs to be externally defined. FIGURE 2 shows changes of compliance rates interacting with changes of other variables. Note that the above compliance model is used only for illustration purpose here, the resulting behavior in the simulation studies appears acceptable for our purposes.
FIGURE 2. Driver’s compliance model

Note: Numbers in legend box indicate levels of familiarity
SIMULATION EXPERIMENTS

The main purpose of experiments is to evaluate performance of the proposed method and to explore how much the resulting path flow pattern differs from the dynamic system optimal pattern. We compare the proposed method with other three methods, such as the path-flow based DSO, temporal average cost routing, and instantaneous feedback routing. First, simulation experiments are conducted under 100% compliance assumption to compare algorithmic performance. Then, for more realistic simulation, simulation experiments are repeated with drivers’ compliance model.

Test Network

We use a simple network with two main corridors, three crossing streets, and three interchanges as shown in FIGURE 3. In the network, a VMS is assumed located between nodes 2 and 3, and its role is to guide drivers to an event place at node 28. There are seven zones in the network. A total of 9,450 vehicles are simulated during one hour period, including 1,600 vehicles from the VMS location to the event place.

![Test Network Diagram]

FIGURE 3. Test network
Performance Comparison under 100% Compliance

First we evaluate performance of temporal VMS route guidance. A compliance rate of 100% is assumed in this experiment. The path flow-based DSO is used as a benchmark solution. There is no guarantee that the approach results in “true” dynamic system optimal solution due to its heuristic nature. However, the method is conceptually advantageous in the sense that the path flow-based DSO algorithm finds an optimal flow pattern between alternatives within discrete time interval unlike the temporal VMS algorithm that seeks an optimal path for the discrete time interval.

Path flow-based DSO vs. temporal VMS DSO guidance

The proposed temporal VMS route guidance with a short VMS update interval shows very close performance to the path flow-based DSO. Even though it may be because of the heuristic nature in the path flow-based algorithm, not because of the excellency in the proposed approach, it shows potential capability that the temporal VMS guidance can be a comparable approach.

The temporal VMS control, as opposed to the path-flow based approach, includes not only the proposed marginal cost route guidance, but also the average cost route guidance and the instantaneous feedback route guidance. In general, the temporal VMS route guidance is sensitive to the update interval while the path flow-based DSO (from now, we call it simply DSO) is insensitive to the VMS update interval since it seeks optimal pattern by spreading flows over alternatives as well as time. As shown TABLE 1 and FIGURE 4, compare performances of route guidance methods with different VMS update intervals. The VMS update interval is found to be an important factor affecting performance of temporal route guidance. As the update interval increases, the performance decreases in all cases. This is mainly due to the time-lag effect that results in inaccuracy of the route guidance. Especially in the case of the instantaneous feedback approach with five-minute update, total travel time is higher than the no guidance case. That is, route guidance systems with long update intervals may result in worse traffic condition as well as deterioration of drivers’ compliance. It is because of over-reaction under the 100% compliance assumption. In the real world where a lower compliance rate is expected, this kind of over-reaction is less likely to occur.
### TABLE 1. Performance Comparison (100% compliance)

<table>
<thead>
<tr>
<th>VMS Update Interval</th>
<th>VMS Guided Vehicles</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. Travel Time (minute)</td>
<td>Saving (%)</td>
</tr>
<tr>
<td>No Guidance</td>
<td>-</td>
<td>18.36</td>
</tr>
<tr>
<td>DSO</td>
<td>1</td>
<td>16.54</td>
</tr>
<tr>
<td>Predictive Average Cost Route Guidance</td>
<td>2</td>
<td>16.78</td>
</tr>
<tr>
<td>Predictive Average Cost Route Guidance</td>
<td>3</td>
<td>17.06</td>
</tr>
<tr>
<td>Predictive Average Cost Route Guidance</td>
<td>4</td>
<td>17.22</td>
</tr>
<tr>
<td>Predictive Average Cost Route Guidance</td>
<td>5</td>
<td>17.45</td>
</tr>
<tr>
<td>Instantaneous Feedback Route Guidance</td>
<td>1</td>
<td>17.57</td>
</tr>
<tr>
<td>Instantaneous Feedback Route Guidance</td>
<td>2</td>
<td>17.95</td>
</tr>
<tr>
<td>Instantaneous Feedback Route Guidance</td>
<td>3</td>
<td>18.02</td>
</tr>
<tr>
<td>Instantaneous Feedback Route Guidance</td>
<td>4</td>
<td>18.15</td>
</tr>
<tr>
<td>Instantaneous Feedback Route Guidance</td>
<td>5</td>
<td>19.62</td>
</tr>
</tbody>
</table>

### FIGURE 4. Performance comparison

- **MC1** = Marginal Cost Routing with 1-minute update
- **AC3** = Average Cost Routing with 3-minute update
- **INS5** = Instantaneous Feedback Routing with 5-minute update
Marginal cost routing vs. average cost routing

Next, we compare the performance difference between route guidance objectives, such as user equilibrium (UE) and system optimum (SO). The proposed method aims at minimizing the total system cost, and guides vehicles' routes based on marginal path costs. The proposed approach can also be used for the dynamic user equilibrium by replacing marginal path costs with average path costs. In the comparison, the marginal cost route guidance (SO) performs better than the average cost routing (SO). However, the difference decreases as the VMS interval increases. There is no significant difference when a long update interval is used. For the case of 5-minute, the average cost route guidance performs even slightly better than the marginal cost routing. It might be because the marginal cost effect disappears when a long update interval is used.

Predictive vs. instantaneous feedback

The proposed method takes a predictive approach that uses predicted travel time for route guidance. In the off-line simulation experiment, the predictive approaches, both the marginal cost route guidance and the average cost route guidance, show better performance than the instantaneous feedback approach as other studies have shown. In the on-line application, however, the performance of the predictive approach of course depends on its prediction accuracy.

As opposed to predictive control in VMS route guidance, feedback control strategies have been considered attractive thanks to their simplicity and robustness to system disturbances, such as in O/D demand, compliance rate, incident occurrence, etc. (26, 27, 28). The approach relies on and responds to real measurements that reflect the consequences of all uncertain disturbances. The main drawback of the approach is that the routes selected by the simple feedback system may not be optimal since it reacts to current traffic measurements without predicting future traffic condition. On the other hand, failure of accurate prediction in predictive the approach may result in even worse condition than simple feedback approach. Therefore, for the real-time application, the prediction algorithms in predictive approach should also be able to adjust their prediction according to system disturbances by incorporating feedback approach
or rolling horizon approach. More discussion on the predictive feedback approach and on-line systems can be found from other studies (11, 29, 30). They also show general framework for on-line and off-line adjustments.

**Route Guidance Schemes and Path Flow Patterns**

This section investigates VMS’s actual path recommendation and the corresponding path flow pattern. FIGURE 5 shows time-dependent VMS messages by methods and update intervals. Each method shows different pattern of recommendations. The predictive marginal cost routing is the most sensitive to the traffic congestion, in that it tends to change recommendations more often than the other methods. This could be because the slope of the marginal cost curve is generally steeper than that of the average cost curve. Response of the instantaneous feedback is the slowest because it does not capture future traffic congestion but react to the observation. Such differences are reflected to the performance of individual methods.

While the temporal VMS recommends a path at a time as in FIGURE 5, the path flow-based approach provides an optimal split for each time step as in FIGURE 6. Even though the temporal VMS guidance methods do not split flows during a time step, path flows are dispersed between alternatives over time and the cumulative flow is ended up a similar split as the path flow-based approach. FIGURE 7 depicts changes in cumulative splits on each alternative routes. Interestingly, after 35 minute, both approaches show similar cumulative path flow splits regardless of the update interval.
FIGURE 5. Time-dependent VMS guidance

FIGURE 6. Dynamic system optimal path flow pattern
FIGURE 7. Comparison of path flow solution

Note: the path flow-based DSO vs. the temporal marginal cost routing by VMS update interval
Performance of Route Guidance with Compliance Model

Now, we incorporate drivers’ compliance model into the simulation model. In VMS route guidance it is almost impossible to achieve a compliance rate of 100%. Incorporation of compliance behavior model is expected to produce more realistic state. We experiment three cases with reliability values of 0.75, 0.5, and 0.25. The resulting compliance rates are approximately 45%, 16%, and 5%, respectively. The compliance rates are resultant aggregated measures endogenously obtained from the simulation. In the simulation experiment, a 1-minute of VMS update interval was used. TABLE 2 and FIGURE 8 reports simulation results.

In overall performance comparison, similarly as the case of 100% compliance, the marginal cost routing performed best and the performance was better at higher compliance rates. While the average cost routing showed similar result, the instantaneous feedback routing performed best at the compliance rate of 45%. This means that there is a range of compliance rate showing best performance in instantaneous information case. In comparison of travel times between complied drivers and non-complied drivers, the benefit of complied drivers over non-complied drivers was more at lower compliance rates.

Next, we compare the resulting path flow splits with the dynamic optimal path flow split. The better performance route guidance showed the closer path flow pattern to the DSO. FIGURE 9 shows changes in overall path flow pattern by routing methods. Comparing the mean square deviations of path fraction with respect to the DSO, the marginal cost routing with high compliance showed the least deviation. The deviation increases as the compliance rate decreases. Even though the temporal VMS guidance cannot make same path flow split as system optimum during a time interval, it can achieve similar path flow pattern in overall when moderate level of compliance rate is achievable.
TABLE 2. Performance Comparison (with Compliance Model)

<table>
<thead>
<tr>
<th>Reliability of VMS</th>
<th>Guidance Method</th>
<th>Overall Travel Time (min)</th>
<th>Travel Time Saving (%)</th>
<th>Vehicular affected by VMS Travel Time (min)</th>
<th>Travel Time Saving (%)</th>
<th>Complied (a)</th>
<th>Non-Complied (b)</th>
<th>Saving (\frac{b-a}{b})</th>
<th>Compliance Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Do Not</td>
<td>13.40</td>
<td>-</td>
<td>18.36</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>DSO</td>
<td>12.32</td>
<td>8.06</td>
<td>16.54</td>
<td>9.91</td>
<td>16.54</td>
<td>-</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>-</td>
<td>MCR(^1)</td>
<td>12.33</td>
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\(^1\)MCR: Marginal cost route guidance, \(^2\)ACR: Average cost route guidance, \(^3\)INR: Instantaneous feedback route guidance

FIGURE 8. Performance comparison (with compliance model)
FIGURE 9. Comparison of path flow pattern

Note 1: the path flow-based DSO vs. the temporal marginal cost routing by level of compliance

Note 2: Mean square deviations of path flow fraction with respect to the DSO were
1.000 at high compliance level, 1.868 at medium level, and 2.521 at low level.
CONCLUSION

This paper has explored how close a dynamic system optimal flow split can be achieved by using temporal variable message signs (VMS). A heuristic method for temporal DSO was developed for temporal VMS route guidance. As opposed to the path flow-based solution that presents optimal solution as a form of path flow fraction, the algorithm provides a set of time dependent optimal paths for a given time horizon though only one path is displayed during each time step.

The algorithm was tested and compared with various VMS guidance methods via simulation experiments with incorporation of a reasonable driver’s compliance model. Our special interest was to investigate path flow pattern of temporal VMS guidance and to explore how much the pattern differs from that of dynamic system optimum. The temporal VMS guidance has shown its potential capability to be a comparable method towards achieving the dynamic system optimum.

In the comparison, the marginal cost route guidance (SO) performed better than the average cost routing (SO). The difference decreases as the VMS update interval increases because the marginal effect disappears when a long update interval is used. The marginal cost routing was the most sensitive to the traffic congestion, so that it tended to change recommendations more often than the other methods.

While the temporal VMS recommends a path at a time, the path flow-based approach provides an optimal split for each time step. Even though the temporal VMS guidance methods do not split flows during a time step, path flows are dispersed between alternatives over time and the cumulative flow is ended up a similar split as the path flow-based approach. Interestingly, both approaches show similar cumulative path flow splits regardless update interval. The results have shown that the path flow pattern and network performance comparable to the dynamic system optimum can be achievable via temporal VMS guidance. We note, however, that during short periods the flow splits from temporal VMS guidance could be rather different from DSO. Considering that the aggregate splits and travel time averages are close to DSO over longer periods, it remains to be seen if temporal VMS guidance could be driving the system towards any possible alternate DSO states.
REFERENCES


