Optimization Models for Auctions for Transportation Service Contract Procurement

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Abstract

In this paper, we consider the bid analysis problems faced by shippers with sophisticated business constraints in combinatorial auctions for the procurement of transportation services. For many years, large shippers have deployed a variety of B2B auctions to procure transportation services from common carriers based on periodically renewed contracts. Among them, combinatorial auctions have recently arisen many interests due to its economic efficiency in handling the economies of scope property in freight transportation operations. However, the bid analysis problem in combinatorial auctions is doomed to be a very difficult problem and a shipper’s non-price business constraints further complicate this matter. This paper examines this problem by formulating the shipper’s business rules as side constraints in an integer-programming model. A Lagrangian relaxation based algorithm is developed to provide near optimal solutions. The experimental performance of this approach is further analyzed with empirical benchmarking on a set of randomly generated problems.
Introduction

B2B procurement auctions have become a dominant price discovery mechanism to assist shippers to develop strategic contractual relationships with transportation service providers (carriers). In these auctions, shippers supply a request for quote (RFQ) for each lane or package of lanes and carriers bid for contracts to serve these lanes.

Using the truckload industry as an example, a lane is a move from an origin to a destination. The contracts are used to fix the price that shippers pay carriers to move truckloads of freight on each lane. The shipper will estimate the number of loads that will require service on each lane in a typical week. The shipper does not guarantee its demand estimates and the carrier does not guarantee that it will have the capacity to move all loads. Its simply an agreement to move the loads and the predetermined price if the demand and capacity can be matched. The contracts are typically re-negotiated every year or so.

In recent years, shippers have found it beneficial to use combinatorial auctions for transportation service contract procurement and to allow carriers bid for packages of lanes instead of individual lanes. In a combinatorial auction, shippers will not pre-define packages of lanes to bid and carriers have flexibility to combine multiple lanes into their preferred packages and to bid for conditional bids. As a result, there might be overlapping between different carriers' packages, as well as between a single carrier's own packages. Furthermore, carriers' valuations on different packages are often not additive. That is, the sum of the prices for two separate bid packages may not be equal to the price for a package including all of the lanes included in the two separate packages.

This sheer number of possible combinations creates a very difficult problem for shippers who must determine winners and assign lanes so as to minimize procurement costs. As a matter of fact, this bid analysis problem, also called the winner determination problem (WDP) in general combinatorial auctions, is NP-complete in most cases.

Further, shippers often have to consider certain non-price business requirements when they analyze the bids. For instance, a shipper often has an idea of approximately how many carriers they would like to have as partners. These side constraints further complicate the bid analysis problem.

In this paper, after providing a general introduction to auction mechanisms, we first review the bid analysis problem in transportation procurement auctions, particularly in combinatorial auctions. Then we discuss how to incorporate shipper's business constraints into the bid analysis problem and present an integer-programming model for this problem. Further, we propose a Lagrangian relaxation based algorithm to solve this problem. Numerical results are also presented to analyze the empirical behavior of our algorithm. Finally, we offer conclusions and discuss future research directions.

In an earlier paper (Song, Regan and Nau, 1994) we examine a simpler problem in which the shipper identifies mutually exclusive and collectively exhaustive
bundles of lanes a priori. In practice many companies prefer these so called unit auctions because of their relative simplicity for bidders and auctioneers and because they allow shippers more control over the structure of the contracts awarded. We successfully employed Lagrangian relaxation in that research. Therefore, it seemed natural to extend that framework to this more difficult problem.

Research Background

According to McAfee and McMillan’s (1987) definition, an auction is “a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants.” Auctions are increasingly becoming a viable means of economic trade (or negotiation protocols) especially when the seller does not know the value of the good with certainty. Advances in technology have also made the process of conducting auctions much easier. The rapid growth of electronic market places, especially popular sites like eBay have made auctions a widely accepted form of market clearing. Multi-attribute auctions relate to items that can be differentiated on several non-price attributes such as quality, delivery date etc. Transportation auctions are multi-attribute auctions in the contest that bidding items (a “lane”) have other non-price attributes for e.g. service quality and time windows for delivery.

An auction is characterized by its bidding rules, market clearing rules and the information revelation policy. Market clearing rules specify the allocation of items to bidders and what the bidders payment. Information revelation policy determines the process of disclosing the information externality during the course of the auction. In general the auction protocols can be categorized mainly in two forms depending on whether the current bid price and other bidders identity is known at every round. When prices and identities are known to all we refer to the auction as open. Otherwise the auction is known as closed.

The most famous types of open auctions are i) English auctions ii) Dutch auctions. The best-known closed auctions are the i) First price auctions ii) Second price Vickery auctions. Combinatorial auctions are those in which multiple items are put out to bid simultaneously and in which bidders can bid on combinations of these items. These are considered especially useful when the bidder has non-additive preferences among the goods being auctioned. In the transportation industry, the items put out to bid are lanes, with specified demands.

In an English auction, bidders are free to raise their prices until they reach their valuation. In the end only the highest bidders remain and if the bid is at or higher than the seller’s reserve price, then the item is awarded. In the descending price or Dutch auction, the auctioneer begins by asking a certain price and gradually lowers it until some bidder agrees to pay the current asking price. In the first-price sealed bid auctions, the bidder with the highest bid receives the object at a price equal to the amount of the highest bid. The second-price sealed bid auction is similar except that the winning bidder has to pay an amount equal to the second highest bid. In double auctions buyers make bids and
sellers makes asks and the item is traded when the current bid for the item is higher than
the current ask. If the auctioneer has more objects to sell, traditionally these are sold in a
series of single object auctions or as a whole in a single auction. The second-price sealed
bid auction is also called Vickery auction. Freight-traders.com uses Vickery auction for
their closed bid auction.

The outcomes of the auctions are influenced by the strategic behavior of the bidders and
the sellers, the presence of asymmetries and independence of the private information.
Each bidder has a private valuation of the objects that they are bidding for based on their
valuation of the utility of the object. This information can either be dependent on either
some knowledge about the valuations of other bidders or completely independent of other
bidders. Symmetry in an auction implies that all the private valuations are drawn from the
same common probability distribution whereas asymmetry implies that each bidder has a
different probability distribution from which he chooses his valuation. See Milgrom,

In combinatorial auctions, using the Vickery-Clarke-Groves (VCG) scheme is the
dominant strategy for agents to report their true valuations (Milgrom, 2004). The
auctioneer solves the WDP for optimal allocation and solves a set of winner
determination problems for optimal allocation excluding a single agent each time. The
payment a bidder or an agent has to make is the difference in “welfare” of the other
bidders without him and the welfare of the others when he is included in the allocation.
In order to find this difference in welfare n separate winner determination problems, each
excluding one bidder must be solved (where n is the number of participants). VCG
auctions are impractical to implement when the number of bidders are large.

The valuation problem in combinatorial auctions is hard as there are an exponential
number of packages to consider. iBundle (Parkes 2000a) is an iterative combinatorial
auction solves this problem to an extent. In the initial phase all the prices set to zero and
in the bidder evaluation phase, given a current set of package prices each agent
determines the set of packages that are within a bid increment of epsilon of maximizing
utility. The winner determination is a primal allocation using the winner determination
problem to maximize his revenue. The pricing phase (dual pricing) determines the
package prices and is reported back to the bidders. The bidders evaluate the package
prices and resubmit their bids. The bidders at this stage can generate different packages
and submit a bid for them. The fundamental assumption in iBundle is that the bidders
employ myopic best response where only utility maximizing packages are submitted.

Ledyard et al. (2002) conducted a multi-round combinatorial reverse auction for the
procurement of contracts of serving over eight hundred lanes (delivery routes) for Sears
combinatorial auction for long term contracting at Home Depot. Logistics.com,
Accesstransporta.com (Canada), Translogistica.com (UK) have reported the use of
combinatorial auction methods for long term contracting. Song and Regan (2003a,
2003b, 2004) develop optimization based bidding strategies for carriers bidding in these
auctions.
Shipper’s business constraints

As discussed in Caplice and Sheffi (2003), shippers have a variety of business constraints when they assign bids to carriers. These include:

- Minimum / maximum number of winning carriers: a shipper could limit the size of carriers that can win in procurement auctions – at the lane, facility, or system wide levels. This is to control the risk of service unavailability and/or the overhead cost of too many carriers.
- Favoring of incumbents: shippers often favor particular incumbents in their core carrier group at the lane, facility or system level, or restrict particular carriers from serving certain portions of the network. Caplice and Sheffi (2003) noticed “incumbents are often favored by 3% to 5% - especially on service-critical lanes to key customers or time-sensitive plants”. This constraint can be modeled by associating penalty cost for non-incumbent carriers.
- Back up concerns: a shipper may require carriers to submit both bids as a primary and backup service provider.
- Minimum / maximum coverage: a shipper may want to ensure the amount of traffic that a carrier wins within certain bounds, at a system, lane or facility level.
- Threshold volumes: a shipper can specify that if a carrier wins any freight (on a lane, from or to a facility, or system wide) that it is of either a certain minimum threshold amount, or they win nothing at all.
- Complete regional coverage: a shipper can require every carrier be able to cover all lanes from a certain location or in a particular region. This constraint can be addressed by combing all traffic from that location or region into a single package in the bid preparation stage.
- Performance factors: shippers may have a trade-off between cost and service. Some shippers conduct certain screening activity in bid preparation stage to ensure the minimum service level of bidders. Another way to model this constraint is to modify the cost coefficients by either a multiplicative or an additive factor.

The complete incorporation of all these constraints requires building a sophisticated decision support system and is beyond the scope of our paper. In this work, we only consider the business constraints which apply across the system, not on individual lanes or at the facility level. The service backup issue can be illustrated in the bid preparation stage by requiring each carrier to submit both primary and alternate bids and hence is not considered here. We assume each lane includes a single unit and is not separable; it is further assumed that performance factors are already addressed in bid preparation stage or incorporated into the coefficient cost.

Sandholm and Suri (2001) present the side constraints and other non-price attributes in real world electronic markets and discusses computational complexities of the market clearing mechanisms. Guo et al (2003) discusses how to incorporate some of these constraints into their carrier assignment models in unit procurement auctions for
transportation services. In the formulation, items are lanes and the business constraint considered is shipper preference for specific carriers (expressed as penalty costs for carriers that are not preferred.) These penalties are modeled as negotiation costs at points of transit in their formulation. The bid analysis problems were solved using meta-heuristics and experimental results are presented in their paper. As mentioned earlier, Song, Regan and Nandiraju (2004) also develop carrier assignment formulations in unit procurement auctions incorporating the side auctions in which the items are to assigned are packages of lanes.

In the next section, we discuss how we incorporate these constraints into the bid analysis problem in a combinatorial procurement auction of transportation services.

Bid Analysis Problem with Shippers’ Business Constraints

In this paper we consider those shipper’s business constraints as discussed in Caplice and Sheffi (2003): maximum / minimum number of winning carriers, incumbent preference, maximum / minimum coverage, performance factors. The service backup issue also discussed in that article can be addressed in the bid preparation stage by requiring each carrier to submit both primary and alternate bids and hence is not considered here. Similarly, the complete regional coverage constraint can be addressed by combining all traffic lanes from that location or within that region into a single bid package at the bid preparation stage. With respect to performance factors, some shippers conduct pre-screening activities on bidder’s qualifications at the bid preparation stage to ensure minimum level of services (Ledyard et al., 2002); another way to model this constraint is to use an adjusted price instead of pure bid price for the cost function in the bid analysis problem.

In our analysis, several assumptions are made without loss of generality. First, we assume that the freight volume on each lane is not separable. Further, XOR bids are not considered and the bidding vocabulary is OR bid only. Ledyard et al (2002) noticed in practice carriers can easily transfer XOR bids into OR bids by adding a lane with small value into both XOR bids.

Now, the bid analysis problem in combinatorial transportation procurement auctions is modeled as:

- 6 -
\[
\min \sum_{j} \sum_{i} c_{ij} x_{ij} + \sum_{k} p_{k} y_{k} \tag{0}
\]
\[
\text{s.t.} \quad \sum_{j} \sum_{i} a_{ij} x_{ij} = 1, \quad \forall i \tag{1}
\]
\[
K_{\text{min}} \leq \sum_{k} y_{k} \leq K_{\text{max}} \tag{2}
\]
\[
T_{\text{min}} y_{k} \leq \sum_{j} \sum_{i} a_{ij} x_{ij} \leq T_{\text{max}} y_{k}, \quad \forall k \tag{3}
\]
\[
y_{k}, x_{ij} \in (0,1) \tag{4}
\]

where the input parameters are:

\(i\): index of a lane in set \(I\);
\(j\): index of a bid package in set \(J\) which may include multiple lanes
\(k\): index of a bidding carrier in set \(K\);
\(c_{ij}\): is the shipper's cost to select carrier \(k\) to serve package \(j\);
\(a_{ij}\): = 1 if carrier \(k\)'s package \(j\) includes lane \(i\); \(a_{ij}\) = 0 otherwise;
\(p_{k}\): is the penalty cost for carrier \(k\) to be selected as a winner, \(p_{k} \geq 0\);
\(K_{\text{min}}\): is the minimum number of carriers to be selected as winners;
\(K_{\text{max}}\): is the minimum number of carriers to be selected as winners;
\(T_{\text{min}}\): is the maximum number of packages (lanes) assigned to carrier \(k\) if it wins;
\(T_{\text{max}}\): is the minimum number of packages (lanes) assigned to carrier \(k\) if it wins;

We also have the following decision variables:

\(x_{ij}\): a binary variable indicating whether a carrier \(k\) wins package \(j\);
\(y_{k}\): a binary variable indicating whether a carrier wins anything at all;

In this model we have \(T_{\text{min}} \geq T_{\text{max}} \geq 1\) and \(K_{\text{min}} \geq K_{\text{max}} \geq 1\). Also note that without constraint (2), (3) and the penalty cost, this BAP problem reduces to a Set Partitioning problem which is well known to be NP-complete (Balas and Padberg, 1976).

The objective function of this formulation minimizes total procurement costs including the actual "bid" price and the penalty cost. The first constraint ensures each lane can only be assigned to one package. Constraint (5) limits the number of winners that shippers want to have in their core carrier base. Constraint (6) models the coverage constraint which restricts the number of lanes (amount of traffic) that a carrier can win and is also the coupling constraint between \(x_{ij}\) and \(y_{k}\);
\[ y_i = \begin{cases} 1, & \text{if and only if } \sum_j x_{ij} \geq 1 \\ 0, & \text{otherwise} \end{cases} \]

Further, note that each bid package is not defined by shippers; instead, they are constructed and submitted by individual carriers themselves. As a result, different packages may be inclusive of common lanes.

**Proposition 1:** BAP is NP-hard.

**Proof:**
Combinatorial reverse auction consists of objective function without the penalty terms with constraints (1) and (4) is a classical weighted set partitioning problem, which is NP-hard (Balas and Padberg, 1976).

First note that if we add another redundant coupling constraint to the BAP problem:

\[ x_{ij} \leq y_i \quad \forall k, j \in J \quad (5) \]

By properly choosing the parameters for the maximum and minimum number of lanes to be won by a carrier, they can be relaxed so that they do not constrain the set of feasible allocations. Suppose the carrier can win any number of lanes or none at all, then problem reduces to combinatorial reverse auction problem with a constraint on number of winners which is NP-hard (Sandholm and Suri, 2001). Hence BAP is also NP-hard.

**Lagrangian Relaxation for BAP**

1. Relax constraint (1)

The problem structure makes it natural to consider relaxing constraint (1) in the BAP formulation. Let \( \mu_k \in \mathbb{R} \) be the Lagrangian multipliers, we have:

\[
\min_{x, y} \sum_k \sum_j c_{kj}x_{kj} + \sum_k p_k y_k + \sum_i \mu_k \left( \sum_j d_{ij}y_{ij} - 1 \right)
\]

subject to

\[
K_{\text{min}} \leq \sum_i y_i \leq K_{\text{max}}
\]

\[
T_{\text{min}} \leq \sum_j d_{ij}y_{ij} \leq T_{\text{max}}, \quad \forall k
\]

\[
y_i, x_{ij} \in (0,1)
\]
where: \( c_y = c_y + \sum_{i} a_{iy} \), note \( c_y \) can be any real number.

2. Relax constraint (3)

Given two Lagrangian multipliers \( \lambda_l \geq 0 \) and \( \mu_l \geq 0 \), we can rewrite BAP to the following:

\[
\max_{\lambda, \mu} \min_{x, y} \sum_{i} \sum_{y} c_{y} y_{i} + \sum_{i} p_{i} y_{i} \\
\quad + \sum_{i} \lambda_{i} \left( \sum_{j} a_{ij} x_{i} - T_{m_{i}} y_{i} \right) + \sum_{i} \mu_{i} \left( T_{m_{i}} y_{i} - \sum_{j} a_{ij} x_{i} \right) \\
= \max_{\lambda, \mu} \min_{x, y} \sum_{i} \sum_{j} \left( c_{y} + \sum_{j} a_{ij} \lambda_{j} + \sum_{j} a_{ij} \mu_{j} \right) x_{i} \\
\quad + \sum_{i} \left( p_{i} - \lambda_{i} T_{m_{i}} + \mu_{i} T_{m_{i}} \right) y_{i} \\
\text{(BAP-LR2)}
\]

\[ \sum_{j} a_{ij} x_{i} = 1, \quad \forall i \in L \quad (9); \]

\[ K_{\min} \leq \sum_{i} y_{i} \leq K_{\max}, \quad (10); \]

\[ x_{i}, x_{i} \in (0, 1) \quad (11). \]

The choice of constraints to relax depends on the complicating constraints shown in formulations BAP-LR1 and BAP-LR2. The next step is to decide which formulation to use to present the lagrangian heuristic and this can be decided by finding whether these formulations satisfy the integrality property (IP). If the solution to the lagrangian dual is unchanged by relaxing the integrality constraints then the Lagrangian dual has the integrality property. The consequences of integrality property are that the maximum lower bound attainable by the lagrangian dual is equal to the lower bound achieved by the linear programming solution. Hence the prudent method is to relax the constraints that do not result in a Lagrangian dual with this property.

**Proposition 2:** BAP-LR1 does not satisfy the integrality property.

**Proof:**

The problem BAP-LR1 is decomposed into any problems between kMin and kMax. After doing this it is decomposes into knapsack problems. The sub-problem does not have the integrality property since the linear programming (LP) relaxation of knapsack problems do not always have integral solutions. The lagrangian relaxation (LR) scheme hence in this case yields a stronger bound than the LP bound. Also finding an LP bound is cumbersome. We could provide similar evidence that BAP-LR2 also has the attractive
feature of not satisfying the integrality property. However, constraint set (9) presents some formidable solution hurdles. Therefore we begin our analysis of the suitability of Lagrangian relaxation for this problem using the BAP-LR1 formulation.

**Lagrangian Relaxation based Algorithm**

The Lagrangian relaxation algorithm is generalized as this: first, given a specific $\mu$, we find an optimal solution for the BAP-LR1 problem; then we search for a feasible solution for the original bid analysis problem from this optimal solution; next we examine whether the stopping rule is satisfied, if not, we update $\mu$, and continue this procedure to improve the lower bound.

**Finding an optimal solution for a relaxed Lagrangian problem**

First note that the BAP-LR1 problem is similar to a network flow problem as follows:

In the above network, s and t are dummy source node and end node respectively. The problem is to push as much flow as possible from s to t via intermediate node k and j. However, only a number of nodes (carriers) between bound $[K_{min}, K_{max}]$ can be used for the first step. Each edge between s and k, j and t has a zero cost, and each edge between k and j is associated with both a cost $c_{ij}$ and a weight $w_{ij}$, where $c_{ij}$ can be any real number and $w_{ij}$ is the number of lanes included in carrier k’s package j. For each node j (package), there is also a capacity bound $[T^d_{j}, T^u_{j}]$ if carrier k is selected in the solution. To send a flow through k to j, the total weight $\sum_j w_{ij}$ for k must satisfy this capacity bound.
To solve this relaxed problem, suppose for each carrier we can find a list of packages $J'_k$ such that $\sum_j c_{j} x_{j}$ can be minimized while satisfying constraint (9) (we call it volume constraint), now we compute $p_k + \sum_j c_{j} \mid j \in J'_k$ for each $k$ and make a list out of them. Then we select $K_{\text{min}}$ number of carriers from the decreasing order of the list. If there are still carriers in the remaining of that list such that $p_k + \sum_j c_{j} \mid j \in J'_k < 0$, then we continue adding those carriers as long as the constraint $\sum_j y_{j} \leq K_{\text{max}}$ is satisfied.

For $x_{j}$, we set $x_{j} = 1$ only for those lanes that are already picked by carriers in the last step, that is $x_{j} = 1$, $\forall j \in J'_{k}$.

Now we discuss how to find those packages in $J'_{k}$. We transform this problem into two Knapsack sub-problems. The Knapsack problem is still NP-complete but many efficient pseudo-polynomial algorithms are known (Martello and Toth, 1990).

First note when $c_{j} \leq 0$, we should continue to add package $j$ into $J'_{k}$ for each carrier as long as the number of lanes assigned to this carrier $T_{k} = \sum_{j \in J'_{k}} x_{j} \leq T_{\text{max}}^{j}$. For the second sub problem, if $T_{k}$ is still less than $T_{\text{max}}^{j}$ after all packages with $c_{j} \leq 0$ are added, we have to select from packages with $c_{j} > 0$ and add them into $J'_{k}$ until $T_{k} \geq T_{\text{max}}^{j}$.

For the first sub problem, let $f'_{j} = c_{j} \mid c_{j} \leq 0$ and weight $w_{j} = \sum_{j \in J'_{k}} s_{j} x_{j}$ (this weight is actually the number of lanes contained in that package), the problem is to put as many packages as possible to a knapsack while the total weight satisfies that knapsack’s capacity constraint $T_{\text{max}}^{j}$. That is:

$$\begin{align*}
\min & \quad \sum_{j \in J} f'_{j} z_{j} \\
\text{s.t.} & \quad \sum_{j \in J} w_{j} z_{j} \leq T_{\text{max}}^{j}, \quad (\text{KP1}) \\
& \quad z_{j} \in \{0,1\}
\end{align*}$$

Note $f'_{j} \in R$ and $w_{j}, T_{\text{max}}^{j} \in N^{+}$. If we rewrite the objective function in KP1 as $\max \sum_{j \in J} (-f'_{j}) z_{j}$, it becomes a standard form of a binary Knapsack problem.
After we solve the first sub problem, we count whether the number of lanes $T_k$ is equal to or greater than lower bound $T_{\text{min}}$. If it is, we stop here and the optimal solution is found. Otherwise, we continue to solve the second sub problem.

Let $f_j^* = c_{u_j} | c_{u_j} > 0$, the second sub problem can be written as:

$$\begin{align*}
\min & \quad \sum_{j \in \mathcal{J}} f_j^* z_j \\
\text{s.t.} & \quad \sum_{j \in \mathcal{J}} w_j z_j \geq T_{\text{min}} - T_k \quad \text{(KP2)} \\
& \quad z_j \in \{0,1\}
\end{align*}$$

This is a minimization version of the binary Knapsack problem. We can easily change it to a standard maximization version by letting $z_j = 1 - z_j$.

$$\begin{align*}
\max & \quad \sum_{j \in \mathcal{J}} f_j^* \bar{z}_j \\
\text{s.t.} & \quad \sum_{j \in \mathcal{J}} w_j \bar{z}_j \leq \min \{ \sum_{j \in \mathcal{J}} w_j - T_{\text{min}} + T_k, T_{\text{max}} \} \quad \text{(KP3)} \\
& \quad \bar{z}_j \in \{0,1\}
\end{align*}$$

After we find a list of candidate packages for each carrier by solving these Knapsack problems, we can solve the relaxed Lagrangian problem easily by following procedures described at the beginning of this section.

**Find a feasible solution for the original bid analysis problem**

We present an outline of the greedy algorithm used to find the optimal solution. The algorithm works as an adjustment heuristic in a lagrangian relaxation as at every iteration $c_{u_j}$ and $u_j$ are the input for this procedure to generate a feasible solution.

**Feasible solution heuristic($c_{u_j}, u_j$)**

- Sort the lanes in the descending values of $u_j$.
- Do until $L = \emptyset$
  - Pick the lane $i \in L$ with the highest $u_j$.
  - Then pick the package containing lane $i$ with the lowest $c_{u_j}$ and making sure lane constraints are not violated.
  - Remove all the lanes from this package $j \in J$ from $L$. $L = L / (\forall i \in j)$
Calculate the Upper bound.

The feasible solution obtained with this procedure provides an upper bound for the original bid analysis problem, and we can further improve the solution by iterating the above Lagrangian procedures with an update on Lagrangian multipliers. There are alternative ways to do this, among them is the well-known subgradient search method. Let \( Z_0(u^n) \) be the optimal solution from the Lagrangian problem BAP-LR1 (lower bound) and \( x', y' \) be the optimal assignment at iteration step \( n \), and let \( Z' \) be the feasible solution (upper bound), the subgradient search method starts with an initial value \( u^0 \) for the Lagrangian multipliers and updates them over the iterations as:

\[
u^n = u^n + t_n (\sum_i x_{ij} - 1)
\]

where:

\[
t_n = \frac{\lambda_n (Z' - Z_0(u^n))}{\sum_j (\sum_i x_{ij} - 1)^2}
\]

In the above equation, \( t_n \) is a scalar satisfying \( 0 < t_n \leq 2 \), normally we have \( t_n = 2 \) and it will be halved whenever \( Z_0(u^n) \) has failed to increase in a fixed number of iterations.

The iterative search for optimal solutions will stop when certain rules are satisfied. These rules normally include: optimal or near-optimal solution is found; there are too many iterations; the scalar \( \lambda_n \) is too small.

**Empirical Benchmarking**

In this section we give some empirical results in order to compare the new approach to optimal winner determination based on the integer programming formulation of the bid analysis problem (and consequently solved with commercial software CPLEX).

Input data for each problem includes each carrier’s bids, bid prices, penalty cost, minimum and maximum number of lanes if this carrier is a winner, minimum and maximum number of winners. In our experiments, a carrier’s bid price \( c_{ij} \) is randomly distributed between 15 and 35, and the penalty cost is randomly distributed between 0 and 10. Please note that this method of generating test problems is for preliminary tests of our solution method. Future data sets will be generated with real transportation networks. Those tests will be conducted in Fall, 2004. We set \( K_{min} = 1 \) and \( K_{max} \) to the number of bidders. In addition, each carrier has a \( T_{max} \) that is uniformly distributed over \([1, 0.33^{\#bidders}]\) and \( T_{max} \) is set to the number of lanes in the auction.
Preliminary Empirical Results

In our preliminary investigation we test the computational efficiency of the solution procedure, we generate two test suites and apply Lagrangian relaxation solution procedure. The numerical results are summarized in Table 1 and Table 2. The lower bound in numerical results is the best lower bound obtained by solving the Lagrangian dual BAP-LR1. The upper bound is found using the greedy algorithm presented above. The tables also list the optimal solution and time by commercial software CPLEX.

Test suite 1

We generated problems with 4 carriers, 8 lanes for bidding and 18 bids from the carriers.

<table>
<thead>
<tr>
<th>Prob #</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Gap(%)</th>
<th>LR Time(minutes)</th>
<th>Cplex solution</th>
<th>Cplex time</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>102.5570668</td>
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Table 1

Test suite 2

In this test, we generated problems with 6 carriers, 12 lanes for bidding and 32 bids from the carriers.

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<th>Prob #</th>
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<th>Upper Bound</th>
<th>Gap(%)</th>
<th>LR Time(minutes)</th>
<th>Cplex solution</th>
<th>Cplex time</th>
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We can see that our results are mixed. In some cases we obtain very tight bounds and in others, we do not. We believe that we have evidence that we are on the right track, but that we must improve the methods used to move from the solution to the relaxed problem to a feasible solution.

Conclusions and future research

The paper presents solution methodology to solve carrier winner determination problem in the presence of shippers' business side constraints. A mathematical programming formulation is provided and a lagrangian based heuristic procedure is outlined.

As a part of on-going research we are looking at the possibility of applying GRASP heuristics or other meta heuristics to obtain a better upper bound. Future research will focus on improving our adjustment heuristic and fine-tuning the parameters. We expect this to improve the convergence of the sub-gradient optimization.

Unfortunately, real-world data for these problems are not readily available. In the future experiments, we try to use the test data developed by Leyton-Brown, Pearson and Shoham (2000). In addition we will construct realistic data from national and regional transportation networks.
References:


- 16 -
Song, J and Regan, A.C., 2003b, Combinatorial Auctions for Transportation Service procurement: An Examination of Carrier Bidding Policies, Proceedings of the 10th International Conference on Travel Behavior Research, Lucerne


