The Dynamic Traveling Salesman Problem: An Examination of Alternative Heuristics

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ABSTRACT

In this paper we examine a version of the dynamic traveling salesman problem in which a single mobile server provides service to customers whose positions are known. Service requests are generated according to a Poisson process which is uniform across customer locations. We assume that the mean service time is known and that its variance is bounded. Service time is independent of customer location. We first examine a special case of the problem in which the optimal TSP tour and minimum spanning tree involves only links of equal length and then discuss the case for a general graph. We show that such a tour applies to many grid networks. The goal of this work is to develop algorithms which minimize the average waiting time of each customer. For this special case we show that the cyclic polling algorithm commonly discussed in the literature of queuing theory provides a solution which is very close to optimal for many problem instances. For general graphs we develop a lower bound for the optimal algorithm and an upper bound for a heuristic algorithm. Under very light demand intensity we show that locating the server in the median of the graph and traveling to the demand location to provide service and returning to the median immediately after completing service is approximately optimal.

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INTRODUCTION

The dynamic traveling salesman problem concerns the development of a routing policy for a single mobile server providing service to customers whose positions are known. Service requests are generated according to a Poisson process which is uniform across customer locations. We assume that the mean service time is known and its variance is bounded. Service time is independent of customer location. This problem, called the Dynamic Traveling Salesman Problem (DTSP), was first introduced by Psarafitis(1985). Bertsimas and van Ryzin(1991) studied a similar problem, the Dynamic Repairman Problem (DTRP), in which customer locations are either uniformly distributed in a bounded area in the Euclidean plane or distributed according to a distribution with probability density function $f(X)$.

In this paper we begin by examining a special case of the DTSP. The special case involves networks in which the optimal TSP tour and minimum spanning tree across customer locations involve only links of equal length (see Figure 1 for some examples). For this special case, through analysis of a related queueing system, we show that the average waiting time when the server follows the a priori tour generated by the well known “cyclic polling” algorithm is approximately bounded by $\frac{2-\rho_1}{1-\rho_1}$ times the average waiting time of the optimal algorithm where $\rho_1$, the fraction of time spent in on-site service time for a single node which is less than $\frac{1}{n}$.

Note that when $n$ is large $\frac{2-\rho_1}{1-\rho_1}$ is close to 2, and when $n$ is small $\frac{2-\rho_1}{1-\rho_1}$ is bounded by 3. We also identify circumstances under which our bound is very tight. This implies that under certain conditions the cyclic polling algorithm is close to optimal.
Next, we introduce a heuristic algorithm for the DTSP on a general graph. We provide a lower bound on the waiting time for the optimal algorithm. From Cooper, Niu and Srinivasan (1996), we know the average waiting time of the cyclic polling algorithm. This provides us with an upper bound for the average waiting time under the optimal algorithm. Finally, when the arrival rate is very low, we provide an alternative heuristic and show it is approximately optimal as the arrival rate approaches zero.

Example network with two rows

Example network with 2*K rows (K = 1,2,3…)

Figure 1. Example networks
RELEVANT LITERATURE

As mentioned in the introduction, our problem is related to the DTRP and the DTSP (see for example, Bertsimas and van Ryzin, 1991, Psaraftis, 1985). It also draws upon related research in queueing theory in which cyclic polling algorithms are examined. Much of the research on cyclic polling systems has as its focus the analysis of the waiting time or queue length under various service policies. For example, Cooper, Niu and Srinivasan (1996) developed an explicit expression for the average waiting time under gated or exhaustive policies\(^1\). Srinivasan, Niu and Cooper (1995) extended those results to describe the relationship between the waiting time distributions in zero and non-zero switchover costs when a gated or exhaustive service discipline is enforced. Recently, Borst and Boxma (1997) extend that research to the general case in which algorithms satisfy a more general discipline referred to as a branching property. Eisenberg (1994) analyzed the polling system in which the server comes to a stop when the system is empty rather than continuing to cycle. That work examines a variety of stopping and starting rules. Later, Srinivasan and Gupta (1996) consider the circumstances under which the server should be patient (which means to stop moving when the system is empty). We mention this literature here because much of this paper refers to a recent paper in which we examine the M/G/1 queueing model with switchover costs. In that paper we develop a lower bound for the waiting time in these systems under any arbitrary algorithm, including those that are optimal (Lu, Regan and Irani, 2001). This paper which is concerned with the DTSP relies on some of the results proved in that paper.
THE DTSP ON NETWORKS IN WHICH THE OPTIMAL TSP TOUR AND MINIMUM SPANNING TREE INVOLVES ONLY LINKS OF EQUAL LENGTH

We assume here that the length of each link involved in the minimum spanning tree is 1 and the travel speed is $v$, so $\frac{1}{v}$ is the minimum switchover time between any two nodes.

Notation

Let

$n$ represent the number of nodes in the network,

$\bar{s}$ and $\bar{s^2}$ represent the first and second moments of the on-site service time for each demand served, respectively,

$\lambda$ the parameter for the Poisson process at each node which is uniform over all nodes,

$\rho = n\lambda\bar{s}$ and $\rho_i = \lambda\bar{s}$ the fraction of time the server spends providing on-site service to all nodes and the fraction of time spent in on-site service for a single node, respectively,

$\bar{W}^*$ = the average waiting time for the optimal algorithm,

$\bar{W}_{cyclic}$ = the average waiting time for cyclic algorithm,

First, we introduce a related M/G/1 queueing model with switchover costs in the following subsection.

\footnote{Under a gated strategy the server will provide service to all customers waiting at a queue when it arrives but not those arriving during service to these customers. Under an exhaustive strategy the server will not depart a queue while there are any unserved customers present.}
The M/G/1 Queue with Switchover Costs

This model involves n identical independent queues, each fed by Poisson arrivals with identical parameter \( \lambda \). A single server provides service to all customers. The service time is a random variable with first and second moments \( \overline{s} \) and \( \overline{s^2} \), respectively. The server can provide service to only one queue at a time. A constant switchover cost, \( \frac{d_i}{v} \), is incurred each time the server switches to node \( i \). Neither the service of a job nor the execution of a set-up can be interrupted prior to its completion. Because the application of interest to us is the DTSP, we view the transfer cost as the travel cost from one location to another.

A decision strategy or policy for this problem specifies the action to be taken at each decision instance or epoch. The server may remain working at the present queue, remain idle at the present queue or switch to another queue. Decision epochs include the arrival of a customer, the completion of service, and may also occur any time the server is idle.

For the case of uniform switchover costs over all the nodes, from Liu, Nain and Towsley (1992) we know that the best algorithm will remain at its current location. An algorithm satisfying this property is said to be exhaustive. When it transfers to another queue, it will choose the queue with the largest number of waiting customers, this property is known as longest queue criteria. For the general case, from Duenyas, and van Oyen (1996), we know that the best algorithm will remain working instead of idling in the current location. Algorithms satisfying these conditions are differentiated only by the switchover decision rule employed. That is, the server must decide whether to wait at a node after it has completed all the jobs waiting at that node or whether to switch to another node. We refer to the following condition as the continuous condition: if there
are demands in the system when the server completes service at a single arrival process, the
server will depart its current location for a location where there are unserved demands Without
providing proof, we conjecture that as $\rho$ approaches 1.0, the optimal algorithm will have this
property. (In this case the probability that the system is idle is very small.) If this conjecture is
ture, the optimal algorithm is exhaustive, continuous and satisfies the longest queue criterion.
Taken together, these completely specify the optimal algorithm. In addition, we note that under
the special case of zero switchover costs, our system reduces to an $M/G/1$ queueing model.

*Analysis of the $M/G/1$ Queue with Switchover Cost*

In this section we present results that are proven in Lu, Regan and Irani (2001). Because these
proofs are extremely detailed and quite lengthy we simply present the results in this paper. Here,
we use the notation defined in the previous section.

Let $W'_1 = \frac{n\lambda s^2}{2(1-\rho)}$, be the average waiting time in an $M/G/1$ queue with arrival rate $n\lambda$ and
service rate $\mu = \frac{1}{s}$ (with no switchover costs), this is known as Pollaczek-Khinchin(P-K)
formula, please see Bertsekas and Gallager (1992) for a discussion of the P-K formula.

Let $W_{nc}$ and $W_c$ represent the average waiting time for algorithms that do not necessarily satisfy
the continuous condition, and those that do satisfy the continuous condition, respectively.
For algorithms that do not obey the continuous condition we must make some observations and define two new variables. First we observe that in these systems, a server may arrive at a node, provide continuous service to customers until the sub-queue is empty. We call this the initial busy period. The server may then remain idle at the current location until a new customer arrives. At that time it enters into what we refer to at a subsequent busy period. In principle, a server may have many of these subsequent busy periods (later we show that only poor algorithms would allow this). Let \( p \) represent the fraction of demands that arrive during the initial busy periods and \( 1 - p \) represent the fraction that arrive during the either the idle periods or the subsequent busy periods.

**Lemma 1:** \( \bar{W}_{uc} \geq \max \left\{ W_1, \frac{np(1-\rho_1)^2}{2v(1-\rho-(n-\rho)(1-p))} \left( \frac{n}{\sum_{i=1}^{\infty} d_i} \right) - \frac{P(1-\rho_1)}{2\lambda} \right\} \) and

\[ 1 - \rho > (n - \rho)(1 - p). \]

When \( p = 1 \) the algorithm obeys the continuous condition, we obtain the following lower bound,

\[ \bar{W}_c \geq \max \left\{ W_1, \frac{(1-\rho_1)^2}{2n v (1-\rho)} \left( \sum_{i=1}^{\infty} d_i \right)^2 - \frac{(1-\rho_1)}{2\lambda} \right\}. \]

**Lemma 2:** When the cost to switch between nodes is a constant, which is \( d \),

\[ \bar{W} \geq \max \left\{ W_1, \frac{nd(1-\rho_1)^2}{2v(1-\rho)} - \frac{(1-\rho_1)}{2\lambda} \right\}. \]
In fact, we can interpret \( \frac{nd(1 - \frac{\rho}{n})^2}{2(1 - \rho)\nu} \) as the contribution of the switchover time to the total waiting time and \( W_i \) as the contribution of the randomness of arrival process and the service time.

\[
\text{Lemma 3 (Cooper, Niu and Srinivasan, 1996): } \overline{W}_{\text{cyclic}} = \overline{W}_1 + \frac{(1-\rho_i)\sum_{i=1}^{\infty} d_i}{2\nu(1-\rho)}.
\]

Proof: Proof is provided in theorem 1 of the reference mentioned.

In the next sub-section, we examine the DTSP on graphs in which the TSP tours and minimum spanning trees include only links with equal length.

**Examining the DTSP on the Special Graph**

**Theorem 1:** \( \overline{W}_{\text{cyclic}} = W_i + \frac{n(1-\rho_i)}{2\nu(1-\rho)} \) and \( \overline{W^*} \geq \max \left\{ W_i, \frac{n(1-\rho_i)^2}{2\nu(1-\rho)} - \frac{1-\rho_i}{2\lambda} \right\} \).

Proof: If we regard the switchover time as the time between the time when the server leaves a node until it reaches another node then the switchover time must be at least \( \frac{1}{\nu} \). From lemma 2, we have a lower bound for the average waiting time for the optimal algorithm. From lemma 3, we know the average waiting time for the cyclic algorithm. Together these give us theorem 1.

QED
For the case in which \( d_i = 1 \), letting \( x = \nu \lambda s^2 \) (the ratio of \( \lambda s^2 \) to the switching cost for a single switchover) we compare the average waiting time under cyclic polling with that of the optimal algorithm using the following corollaries, where corollary 2 explains corollary 1 in words.

**Corollary 1:** If \( d_i = 1 \), for all \( i \) and if \( x \leq 1 \), then

\[
\lim_{\rho \to 1} \frac{W_{cyclic} - W^*}{W} \leq \frac{\rho_i - x}{1 - \rho_i}
\]

If \( x > 1 \),

\[
\lim_{\rho \to 1} \frac{W_{cyclic} - W^*}{W} \approx \frac{1 - \rho_i}{x}.
\]

**Corollary 2:** For the special case in which switchover costs are uniform, as \( \rho \) goes to 1, if \( x \) is very large, the cyclic polling algorithm approximately optimal; if \( x \) is very small, when \( n \) is very large, the cyclic polling algorithm also approximately optimal.

**THE DTSP ON A GENERAL GRAPH**

**A Heuristic Algorithm**

First, find a TSP solution for all nodes in the network and then visit the nodes along the TSP tour, providing exhaustive service at each node and skipping nodes with no demands. This is essentially a cyclic polling algorithm, we use \( W_{cyclic} \) to represent the average waiting time for this algorithm.
Analysis of the Heuristic Algorithm

We develop a lower and upper bounds for the average waiting time for our heuristic algorithm and we obtain the average waiting time in our system from the work of Cooper, Niu and Srinivasan (1996).

Letting $\omega_1 = \frac{(1-\rho)}{2n\nu(1-\rho)} \left( \sum_{i=1}^{n} \sqrt{d_i} \right)^2 \frac{1-\rho}{2\lambda}$ where $\sqrt{d_i} = \min_{j \neq i} \{d_{ji}\}$ and $d_{ji}$ is the distance between nodes j and i. We obtain the following lemma.

Lemma 4: (A lower bound for any algorithm satisfying continuous condition)

$$\bar{W}_c > \max \{W_i, \omega_1\}.$$  

Proof: We let $\frac{\sqrt{d_i}}{\nu}$ be the minimum distance traveled to reach node $i$, therefore it provides a bound on the switchover cost to node i. From lemma 1, we know this result holds. QED

Let $f(p) = \left( \frac{np^2(1-\rho)}{2\nu(1-\rho-(n-\rho)(1-p))} \right) \left( \frac{n}{\sum_{i=1}^{n} \frac{1}{d_i}} \right) \frac{p(1-\rho)}{2\lambda}$, $\omega_2 = \min_p \{f(p)\}$.

Because $\frac{np^2}{2\nu(1-\rho-(n-\rho)(1-p))} \geq \frac{n}{2\nu(1-\rho)}$ holds for $n \geq 2$, $\omega_2$ is minimized when $p = 1$. So we have $\omega_2 = \frac{n(1-\rho)^2}{2\nu(1-\rho)} \left( \frac{n}{\sum_{i=1}^{n} \frac{1}{d_i}} \right) \frac{1-\rho}{2\lambda}$.
Lemma 5: $\overline{W}_{ne} \geq \max \{W_1, \omega_2\}$.

Proof: Because $\frac{d_i}{v}$ is the minimum distance traveled to reach node $i$, it therefore provides a bound on the switchover cost to node $i$. From lemma 1, we know that $\overline{W}_{ne} \geq f(p)$ for any fixed value of $p$. This implies that $\overline{W}_{ne} \geq \min_p \{f(p)\} = \omega_2$. QED

Theorem 3: $\overline{W}_{cyclic} \leq W_1 + \frac{(1-\rho_1)L_{TSP}}{2v(1-\rho)}$ where $L_{TSP}$ is the length of the optimal TSP tour over all the nodes and $\overline{W}^* \geq \max\{W_1, \omega_2\}$.

Proof: We can see the switchover time for a whole cycle is $\frac{L_{TSP}}{v}$, from lemma 3, we know the first part of the theorem. Because $\omega_2 \leq \omega_1$, so we have $\overline{W}^* \geq \max\{W_1, \min\{\omega_1, \omega_2\}\} = \max\{W_1, \omega_2\}$. QED

Note that when $d_i$ does not vary much, in this case $\omega_1 = \omega_2$, the two lower bounds are almost the same.
THE DTSP ON THE GENERAL GRAPH UNDER LIGHT TRAFFIC INTENSITY

First we define a median of a graph as a location such that the average distance to all nodes is minimized, let $B$ represent the bounded region: \[
\frac{1}{n} \sum_{i=1}^{\infty} \|X_0 - X_i\| = \min_{X \in B} \left\{ \frac{1}{n} \sum_{i=1}^{\infty} \|X - X_i\| \right\}
\]
where $\|X_i - X_0\|$ is the Euclidean distance between $X_i$ and $X_0$.

If the arrival rate $\lambda$ is very low, we introduce a heuristic algorithm that locates the server on the median of the graph, whenever there is demand, the server leaves the median and goes directly to the node to provide service, after the demand is served, it returns immediately to the median. Let the average waiting time for this algorithm be $W_H$.

**Theorem 2:** $\overline{W} \geq \min_{\{X \in B\}} \left\{ \frac{1}{n} \sum_{i=1}^{\infty} \|X_0 - X_i\| \right\}$ and $\overline{W}_H \rightarrow \min_{\{X \in B\}} \left\{ \frac{1}{n} \sum_{i=1}^{\infty} \|X_0 - X_i\| \right\}$ as $\lambda \rightarrow 0$.

Proof: We consider the waiting time for demand $i$ and we divide the waiting time into two components: the first is the waiting time due to the server's travel prior to serving $i$, and the second is the waiting time due to the on-site service times of demands served prior to demand $i$. Essentially, the proof is saying that we have chosen $X_0$ to minimize travel distance and since demand is low, there will be no demands waiting the queue when a new request arrives.

Here we follow the notation found in Bertsimas and van Ryzin (1991) because we use ideas very close to theirs to obtain our first bound. $\overline{W}^d, \overline{W}^i$ represent the two components, respectively.
have the following relationship, $\overline{W}_i = \overline{W}_i^d + \overline{W}_i^s$. Taking expectations and letting $i$ approach infinity $\overline{W}^d = \lim_{i \to \infty} E\left[\overline{W}_i^d\right]$ and $\overline{W}^s = \lim_{i \to \infty} E\left[\overline{W}_i^s\right]$, we have $\overline{W} = \overline{W}^d + \overline{W}^s$.

A lower bound for $\overline{W}^d$ is the expected travel time between the optimal location of the server and the location of a random demand, $\overline{W}^d \geq \min_{\{X_0 \in \mathcal{B}\}} \left\{ \frac{1}{nv} \sum_{i=1}^{i=\infty} \|X_0 - X_i\| \right\}$, therefore

$\overline{W}^d \geq \min_{\{X_0 \in \mathcal{B}\}} \left\{ \frac{1}{nv} \sum_{i=1}^{i=\infty} \|X_0 - X_i\| \right\}$.

In order to complete our proof we need to calculate the average waiting time for the heuristic algorithm. Because the server returns to the median every time it finishes a demand and it begins service from the same location every time, we can use an M/G/1 queue model to calculate the average waiting time due to the on-site service time of demands served prior to demand $i$ (This idea is due to Berman, Larson and Chiu, 1985).

Because the on-site service time is independent of the travel time, the first and second moments of the service time are bounded from above by $s + \frac{2 \sum_{i=1}^{i=\infty} \|X_i - X_0\|}{\nu}$, $s^2 + \frac{2 \sum_{i=1}^{i=\infty} \|X_i - X_0\|^2}{\nu}$ respectively, using the classical result of the P-K formula: $\overline{W} = \frac{\lambda s^2}{2(1-\rho_c)}$, where the subscript c represents the classical definitions for the second moment of the service time and the utilization factor. Let $\overline{W}^d_{n}$ represent the average waiting time due to the on-site service time of customers served prior to the selected customer.
\[ \bar{W}_{H}^t \leq \frac{n\lambda s^2}{2(1 - \bar{\rho})} + \frac{2n\lambda \left( \sum_{i=1}^{\infty} \|X_i - X_0\| \right)}{\nu(1 - \bar{\rho})} \] where \[ \bar{\rho} = \rho + \frac{2n\lambda \sum_{i=1}^{\infty} \|X_i - X_0\|}{\nu}. \]

We observe that as \( \lambda \to 0 \), \( \bar{W}_{H}^t \to 0 \).

Next we examine the average waiting time due to the server's travel prior to serving the selected customer. Let \( \bar{W}_{H}^d \) represent this average waiting time. \( \bar{W}_{H}^d = \min_{\{X_i\in B\}} \left\{ \frac{1}{nv} \sum_{i=1}^{\infty} \|X_0 - X_i\| \right\} \).

Because \( \bar{W}_{H} = \bar{W}_{H}^d + \bar{W}_{H}^t \) as \( \lambda \to 0 \), \( \bar{W}_{H} \to \min_{\{X_i\in B\}} \left\{ \frac{1}{nv} \sum_{i=1}^{\infty} \|X_0 - X_i\| \right\} \). QED
CONCLUSION

In this paper we examine a very difficult problem, the Dynamic Traveling Salesman Problem. We examine both a special and the general case. For the special case we examine the performance of a specific algorithm. For the general case we provide bounds on the performance of the best on-line algorithms by providing bounds on the performance of all algorithms.

Psaraftis (1985) discusses some basic unanswered questions about the DTSP. We repeat these here and then partially address each of these. (1) If the on-site service time is zero, what is the best algorithm? (2) Under what circumstances is the myopic policy, which optimizes over known demands only, best? (3) Does it make sense to let demands accumulate before the vehicle departs?

We address question (1) for the special case of networks in which both the optimal TSP tour and the minimum spanning tree contains only links of equal length. We show that when the demand arrival rate is relatively high and when both the on-site service time and its variance are very small and when the number of locations \( n \) is large, the cyclic polling algorithm will be very close to optimal. That is, the server travels along the optimal TSP tour and provides service to each node as it passes the node in its tour. This is a direct result of corollary two.

We address question (2), in part by showing that for these graphs, when the arrival rate is very low, a myopic algorithm is close to optimal. We show this by providing proof for Psaraftis' conjecture that for the case of low demand, the best algorithm is to locate the server on the
median and to provide service by traveling from the median to the customer locations. The case of moderately heavy or moderately light traffic remains an open question.

We address question (3) also in part. Though we do not prove conclusively that algorithms obeying the continuous condition perform better than those which do not (hence, allowing demands to accumulate), we do provide lower bounds on the waiting time for service for both types of algorithms and show that the lower bound for algorithms obeying this condition dominates the lower bound for those that do not. Therefore, we conjecture that these perform better.

In a dynamic or on-line system decisions are made over time, based on current information. In a static or a priori system decisions are made before demand is realized. The fact that under certain condition(s) the a priori solution is optimal for the dynamic problem provides another indication of the connection between the static and dynamic solutions to dynamic or on-line problems. This connection has been mentioned by many researchers during the last two decades.

Finally, we make the following observation for the more general case in which there exist $i$ and $j$ for which $\lambda_i$ is not equal to $\lambda_j$. Without analysis, we suggest the following algorithm. Use a clustering algorithm\(^2\) to identify partitions that hold approximately the same expected demand; second, select the center of each partition as a representative; third, obtain an a priori tour over the representatives; finally, travel according to the a priori tour, providing service to each group according to a TSP tour over that group. We conjecture that if implemented correctly this

\(^2\) One algorithm involves solving a capacitated m-median problem, several other methods are suitable as well.
algorithm should have very good performance. Analyzing its behavior is a topic of on-going research.

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