

# Estimation of the time-dependency of values of travel time and its reliability from loop detector data

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## Abstract

Although the effects of travel time and its reliability have been addressed in a variety of papers concerning pricing policies, most of the existing research is based on the assumption that travelers' preferences are static over a given time interval, such as the morning commuting period. Here, we relax this assumption, assuming rather that travelers' tastes toward the travel time and its reliability vary with time, and examine their time-dependent effects on traveler's route choice decisions. We adopt a mixed logit formulation of route choice behavior as a function of travel time, reliability, and cost. To uncover the values of travel time and its reliability, we introduce an alternative approach to the use of traveler surveys to estimate the model coefficients by determining the parameter set that produces the best match between the aggregated results from the travelers' route choice model and the observed time-dependent traffic volume data from loop detectors. We apply the methodology to loop detector data obtained from the California State Route 91 value-pricing project, and use a genetic algorithm to identify the parameters. The time-dependent values of travel time and values of reliability for the morning commuting period are estimated and their implications on the toll pricing policy are discussed. The results indicate that, under the time-dependent formulation, travel-time savings may be more important than uncertain travel time when departure time is close to such time constraints as work-start time.

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## 1. Introduction

That travel time and its reliability impact individual traveler's route choice behavior is undisputable, as has been shown in a number of previous research studies (Abdel-Aty et al., 1995; Bates et al., 2001; Lam and Small, 2001; Yan et al., 2002; Liu et al., 2004). Lam and Small (2001) and Liu et al. (2004) have studied the value of time (VOT) and value of reliability (VOR) and uncovered the heterogeneity of the motorists' route choice decisions, and some theoretical analyses on value of time are presented in Jiang and Morikawa (2004).

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Our previous study has indicated that the VOR is higher than VOT in traveler's route choices (Liu et al., 2004). However, these studies employ the same basic assumption: the departure time of a trip is exogenous to traveler's route choice. In other words, commuters' tastes toward travel-time reliability are independent of their departure time; commuters, who start their journeys at different time, have the same degree of aversion to travel-time unreliability. In this work, we challenge the reasonableness of such an assumption. Although this assumption is certainly a modeling convenience (and perhaps even a practical necessity in survey data collection since in general it is hard to collect sufficient samples for short time periods), it may bias modeling results by concealing factors related to departure time which can affect driver's route choice.

As an alternative, variations in the value of reliability could be modeled by dividing the study time interval into smaller periods, in each of which a determined value of travel time and its reliability are assumed. As the temporal coordinate is divided into sufficiently small periods, the value of travel time and its reliability should be expressible as a continuous function of time. Yan et al. (2002) have discussed the effects of the time of day in their route choice model, but did not estimate the time-dependent parameters and value of reliability.

As alluded to above, a principal difficulty in studying the effect of time-dependency on VOT and VOR is data collection. Typically, research on static VOT and VOR employs direct methods, either based on revealed preference (RP) surveys (Lam and Small, 2001) or stated preference (SP) surveys (Abdel-Aty et al., 1995; Bates et al., 2001), or a combination of the two (Small et al., 2005), to investigate these values. Unfortunately, neither the RP nor SP survey approach is particularly well-suited for time-dependent parameter estimation. In the case of SP data, there should be sufficient samples lying in each study period. Because travelers commonly depart at a specific time based on habitual behavior, data collected by SP surveys may be subject to biases resulting from hypothetical questions related to scenarios not actually experienced by the respondent. And, although the RP data uncover the actual response to a typical situation, it is virtually impossible to find RP situations where there is sufficient perceived variation to allow statistically reliable estimates, especially for the time-dependent case.

In this paper, we adopt a time-dependent approach to formulate motorists' route choice behavior. Our model is based on the more realistic assumption that travelers' tastes toward travel time and its reliability are distinctly related to departure time. As in previous efforts (Small et al., 2005; Liu et al., 2004), we adopt a mixed logit model formulation to depict motorists' route choice behavior, as determined by three randomly distributed parameters: travel time, its reliability, and monetary cost.

The probabilistic-based heuristic, genetic algorithm (GA), is introduced as the solution method, to estimate the model parameters by minimizing the differences between observed and estimated data. As an alternative to survey data, inductive loop detector data are utilized as an expeditious source for observed data (Yan et al., 2002; Liu et al., 2004). Compared to traditional survey data, loop detector data provide a simple way to reveal travelers' actual responses to different traffic conditions, on the aggregated level. With such loop detector data, traveler surveys employing hypothetical scenarios are unnecessary, since the data accurately reflect the actual environment in the network at any particular time. Moreover, aggregating the loop detector data over any length of time interval is straightforward, making time-dependent estimation extremely flexible. The dynamic parameters estimated from the mixed logit model are used to measure the time-dependent VOT, VOR, and also the degree of risk aversion (DORA) by simulation. In order to enhance computational efficiency and to lower variances in the Monte Carlo simulation used in the mixed logit coefficient estimation, systematic sequences are embedded in simulation procedure.

We applied the estimation procedure to the study site of a recent value-pricing project, California State Route 91 (SR91), the same as used by Lam and Small (2001), Small et al. (2005), Yan et al. (2002) and Liu et al. (2004). SR91 presents a simple network exhibiting commuters' alternatives between two parallel routes connecting a unique origin and destination pair. The time-dependent values of travel time and values of reliability for the morning commuting period are estimated and their implications on the toll pricing policy are discussed.

The remainder of the paper is organized as follows. In Section 2, we review a background of route choice behavior and present our dynamic route choice model. Section 3 discusses how we collect and deal with the necessary data from the 30-s raw loop detector data. Section 4 presents the details of the estimation and solution procedure, including formulation, heuristic and data preparation. Estimation results and discussion are included in Section 5. Finally, we summarize our research in the last section.

## 2. Econometric background

### 2.1. Time-dependent route choice

Consider a commuter faced with the choice of different routes connecting the origin and destination of a trip. Suppose further that this commuter has prior perfect information of the average (mean) travel time and variation (standard deviation) in each given peak-hour interval for each route. Based on the model of Jackson and Jucker (1982), the time-dependent route choice of a commuter is determined by

$$\min_{p \in Q_{rs}} E_t(T_p) + \lambda_k V_t(T_p) \quad (1)$$

where

- $E_t(T_p)$  the expected travel time of path  $p$  at time  $t$
- $V_t(T_p)$  the variance of travel time of path  $p$  at time  $t$
- $\lambda_k$  degree of variability aversion to motorist  $k$
- $Q_{rs}$  the set of routes connecting origin  $r$  and destination  $s$

Under this model, the traveler has two approaches to avoid/reduce the variability of travel time for the trip, either by altering departure time or by choosing a route with more reliable travel time. For instance, as reported in Lam and Small, 2001, females and commuters with high income generally more strongly prefer avoiding travel-time variation than do other travelers. These commuters may choose some ‘express lane’, such as the toll facilities cropping up in California, Florida and Texas providing high peak-hour level of service, to guarantee the travel time. Alternatively, many other travelers who equally abominate the unreliability of travel time, but who are unwilling to pay to use such facilities, will choose to depart earlier/later to avoid congestion, generating the phenomenon that the peak-hour appears earlier and lasts longer than before.

### 2.2. The time-dependent VOT and VOR

Following our previous study (Liu et al., 2004) on VOT and VOR, we assume that the trip-cost experienced by any particular motorist is captured mainly by three parts: travel time, travel-time variability and out-of-pocket monetary cost. Moreover, we assume that the various weights in the utility formulation are both time dependent and specific to the individual traveler; i.e., the traveler has different preference to these three costs depending on time circumstances. Our time-dependent disutility function is formulated as:

$$U_{np}(t) = \beta'_n(t)x_{np}(t) + \varepsilon_{np} \quad (2)$$

where

- $U_{np}(t)$  the total disutility of path  $p$  at time  $t$  for traveler  $n$
- $x_{np}(t)$   $[T_p(t), R_p(t), C_p(t)]'_n$  = the cost vector of path  $p$  at time  $t$  for traveler  $n$
- $T_p(t)$  the travel time of path  $p$  at time  $t$
- $R_p(t)$  the variability of path  $p$  at time  $t$
- $C_p(t)$  the out-of-pocket monetary cost of path  $p$  at time  $t$
- $\beta_n(t)$   $[\beta_p^T(t), \beta_p^R(t), \beta_p^C(t)]'$  = the aversion parameters vector of traveler  $n$
- $\varepsilon_{np}$  unobserved extreme random value for traveler  $n$  using path  $p$

In this disutility function, the random term  $\varepsilon_{np}$ , which generally is unknown and assumed to be identically and independently distribution across all travelers and routes, captures the person-varying differences between true disutility value  $U_{np}(t)$  and deterministic disutility calculated by the given linear function  $V_{np}(t) = \beta'_n(t)x_{np}(t)$ .

A notable difference between disutility function above and others in the literature is that the time variable  $t$  is included in the parameter vector  $\beta_n(t)$ , such that the VOT and VOR are time dependent and defined by

$$VOT_n(t) = \frac{\partial U_{np}(t)/\partial T_p(t)}{\partial U_{np}(t)/\partial C_p(t)} = \frac{\beta_n^T(t)}{\beta_n^C(t)} \tag{3}$$

$$VOR_n(t) = \frac{\partial U_{np}(t)/\partial R_p(t)}{\partial U_{np}(t)/\partial C_p(t)} = \frac{\beta_n^R(t)}{\beta_n^C(t)} \tag{4}$$

And the degree of risk aversion (DORA) is defined by

$$DORA_n(t) = \frac{VOR_n(t)}{VOT_n(t)} = \frac{\beta_n^R(t)}{\beta_n^T(t)} \tag{5}$$

By introducing the time variable  $t$  into the formulation of VOT, VOR and DORA, Eqs. (3)–(5) exhibit temporal variation in a commuter’s tastes. A simple intuitive example is that the route choice of a commuter starting a work trip later than usual may be influenced by a higher level of aversion to the travel time and travel-time uncertainty than that associated with his/her habitual starting time.

To accommodate taste variation across individuals, preference heterogeneity is introduced by assuming that the coefficients  $\beta_n$  are realizations of random variables  $\beta$ . This specification, known as “mixed” logit or “random coefficients” logit or logit kernel, is identical to standard logit except that  $\beta_n$  varies over decision makers rather than being fixed (McFadden and Train, 2000; Bhat, 2001; Small et al., 2005; Train, 2003). The main idea of the mixed logit model is the assumption that the coefficients in the deterministic utility function satisfy certain distributions. This assumption generalizes the standard multinomial logit (MNL) model and allows the coefficients of variables in our time-dependent route choice model to vary over different drivers and time. Throughout this paper, we assume that the coefficients to travel time, its reliability and out-of-pocket cost satisfy normal distributions.

Although normal distribution is a commonly accepted distribution used in the mixed logit model, its unboundedness property implies unreasonable behaviors towards travel time and reliability because every real (positive or negative) number may have non-zero probability as a draw in the estimation procedure. Many previous studies (Small et al., 2005; Train and Sonnier, 2004; Hess et al., 2005) discussed the disadvantages of using normal distribution and provided alternative distributions. For example, Small et al. (2005) adopted lognormal distribution to reveal motorists’ preferences; Hess et al. (2005) discussed the advantages and disadvantages of using bounded distributions, and they suggested triangular and  $S_B$  distributions in bounded parameters’ estimation. In this paper, we adopt a simple truncation strategy in normal distribution to avoid parameters’ sign-changes, although it generates other disadvantages need further discussions.

Under the mixed logit model formulation, the probability that traveler  $n$  will depart at time  $t$  and choose route  $p$ , conditioned on  $\beta(t)$  is given by

$$L_{np}(\beta(t); t) = \frac{e^{\beta^t(t)x_{np}(t)}}{\sum_{\forall q \in Q^s} e^{\beta^t(t)x_{nq}(t)}} \tag{6}$$

The unconditional probability is the integral of  $L_{np}(\beta(t); t)$  over the distribution of all possible values of  $\beta(t)$ , i.e.,

$$P_{np}(t) = \int \frac{e^{\beta^t(t)x_{np}(t)}}{\sum_{\forall q \in Q^s} e^{\beta^t(t)x_{nq}(t)}} \cdot f(\beta(t))d\beta(t) \tag{7}$$

where  $f(\beta(t))$  represents the density function that the coefficient variable vector  $\beta(t)$  satisfies.

In practice, this mixed logit model does not lend itself to an analytical probability value because of difficulties arising from incorporation of the density function in the integral. Empirical approaches applying the mixed logit model in studies of discrete choice behavior generally are based on simulations; see, e.g., those found in Bhat (2001) and Train (2003).

### 2.3. Simulated mixed logit estimation

The Monte Carlo or quasi-Monte Carlo (QMC) simulation makes it possible to integrate out mixed logit probability by discretizing the density function of the coefficient variable  $\beta(t)$ . Let  $\Theta(t) = [b^T(t), W^T(t);$

$b^R(t), W^R(t); b^C(t), W^C(t)]'$ , where  $b(t)$  and  $W(t)$  are mean and variance to respective dynamic coefficients, be the characteristic vector describing the density function of  $\beta(t)$ . Then the unbiased Monte Carlo simulation to the mixed logit model is generated by averaging the MNL values over a set of samples which are drawn from the conditional density function  $f(\beta(t)|\Theta(t))$ . Since the coefficient vector  $\beta_n(t)$  reflects the aversion degree of traveler  $n$  at time period  $t$ , it is reasonable to make the assumption that the characteristics vector  $\Theta(t)$  is fixed over sufficiently short time period  $t$ . That is, we suppose that travelers cannot distinguish the small differences within sufficiently small time intervals.

The standard Monte Carlo simulation generates the samples randomly from the density function, while the quasi-Monte Carlo (QMC) simulation procedure draws the samples systematically from certain predetermined sequences; for example, the Halton sequence (Halton, 1960) introduced to the mixed logit model in Bhat (2001). Because of the properties of variance reduction with smaller sample size and faster speed compared to traditional random samples, as shown in Train (2003), we employ the QMC simulation in this paper, and adopt the randomized and scrambled Halton sequences on the systematic draws from densities in the simulated mixed logit model, as proposed in Bhat (2003), to reduce the correlations between parameters. Other systematic sequences, although not being used in this paper, are also available in quasi-random simulation, for instance,  $(t, m, s)$ -nets and Latin hypercube sampling are two of the alternatives to Halton sequences, provided in recent studies (Sandor and Train, 2004; Hess et al., 2006).

The coefficients' density functions in the mixed logit model describe the travelers' preferences on route choices over different traffic conditions. The probability for an individual motorist's route choice, under the condition of a certain value of travel time, travel-time variability and out-of-pocket monetary cost, is captured by the MNL part in mixed logit model. Therefore, the heterogeneous choices of different travelers can be captured by the standard deviations of all these traveler-choices MNL probabilities. In our studies, the parameter estimations have the basic assumption that the travelers make their departure time and route choices based on empirical travel time and travel-time variability values at specific time intervals. Because of this assumption, the dynamic variations of the route choices probabilities on specific routes over different days reflect the heterogeneities of travelers' time-dependent route choices. Therefore, the estimated parameters in our model should not only capture the mean probabilities, but also reflect their heterogeneities over different travelers' route choices at specific time.

### 3. Freeway travel-time variability estimation

#### 3.1. Deriving point-of-departure travel time from loop detector data

To reveal the traffic condition and therefore estimate the VOT and VOR, time-dependent travel time needs to be estimated. In this section, we offer one such methodology for calculating an index (by time-of-day and day-of-week) that captures the variability in travel time for trips between any pair of on/off ramps on a freeway network, based on loop data.

Consider a freeway segment (or combination of contiguous freeway segments),  $S$ , containing  $n$  loop stations,  $l_i, i = 1, 2, \dots, n$  (Fig. 1).

Suppose a trip entering at O starts at time  $t = t_0$  with eventual destination at D. An estimate of the travel time for this trip on any particular observation day can be obtained from loop data as follows:

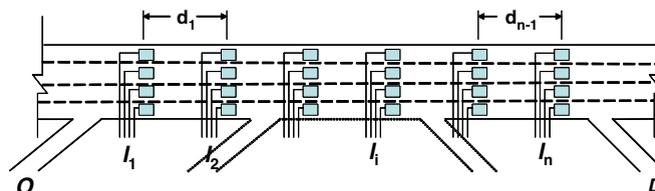


Fig. 1. Freeway segment.

1. Estimate prevailing speed,  $S_{12}(t_0)$ , between loops  $l_1$  and  $l_2$  at time  $t = t_0$  as:

$$S_{12}(t_0) = \frac{120 \cdot Q_1(t_0)}{D_{12}(t_0)} \tag{8}$$

where  $Q_1(t_0) = 30$ -s volume count at loop station 1 for the 30-s period preceding  $t_0$ ;  $Q_1(t_0)$  is multiplied by 120 in order to transform 30-s volume into hourly rate;

$$D_{12}(t_0) = \frac{5280 \cdot \theta_1(t_0)}{100(L_{veh} + d_{det})} \tag{9}$$

where

- $D_{12}(t_0)$  density in vehicles per mile (vpm) for the 30-s period preceding  $t_0$
- $L_{veh}$  average vehicle length, in feet
- $d_{det}$  length of the detector, in feet
- $\theta_1(t_0)$  occupancy in percent for the 30-s period preceding  $t_0$

2. Estimate travel time to loop station 2 for a trip starting at  $t = t_0$  as:

$$t_{12}(t_0) = \frac{d_1}{S_{12}(t_0)} \tag{10}$$

3. Estimate arrival time at loop station  $l_2$  for a trip entering at O at time  $t = t_0$  as

$$t_2^*(t_0) = t_0 + t_{12}(t_0) \tag{11}$$

4. Repeat steps 1–3 for each loop station in succession, e.g., for loop station  $i$  at  $t_i^*$ :

$$S_{i-1,i}(t_i^*) = \frac{120 \cdot Q_i(t_i^*)}{D_{i-1,i}(t_i^*)} \tag{12}$$

$$t_{i-1,i}(t_0) = \frac{d_{i-1}}{S_{i-1,i}(t_i^*)} \tag{13}$$

$$t_i^*(t_0) = t_{i-1}^*(t_0) + t_{i-1,i}(t_0) \tag{14}$$

5. Compute estimate of travel time from O to D for trip beginning at  $t = t_0$  as

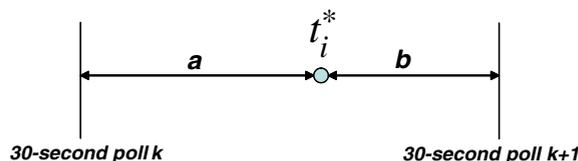
$$t_{OD}(t_0) = \sum_{i=1}^n t_{i-1,i} \tag{15}$$

6. Call this estimate of travel time from O to D for trip beginning at  $t = t_0$  on any particular day  $d$ , a sample point, say the  $d$ th such observation  $t_{OD}^d(t_0)$  among a total sample of  $d^*$  observations.

### 3.2. Some practical considerations

To implement the above procedure, the followings are some practical considerations:

1. In general, the calculated  $t_i^*$  will not be coincident with the ending of a particular 30-s polling count. In this case, the calculations can be taken as a weighted average of conditions, i.e.,



$$Q_i(t_i^*) = \frac{a \cdot Q_i(30\text{-s pollk}) + b \cdot Q_i(30\text{-s pollk} + 1)}{30} \quad (16)$$

2. Missing data. Simply skip over the loop, as long as there are not too many contiguous non-reporting loops, this should not pose a serious problem.

### 3.3. Candidate indices of travel-time variability

Using the  $t_{OD}^d(t_0)$ , any number of statistical measures of variation (e.g., variance, difference between the 80th percentile travel time and the mean travel time, ratio of variance to mean, percent of observations that exceed the mean by some specific threshold, etc.), can be constructed to capture the variability of travel time between any two points on a freeway path by time-of-day (referenced to the start of travel on the freeway). The nature of these measures is that they are positive, monotonically increasing functions of variability.

### 3.4. Data smoothing

The data given by loop detectors are discrete, and the mean or quantile values of these data are sawtooth-like, even though we aggregated them in small intervals. Since a smooth curve is reasonable to describe the changes in a continuous time interval, we adopted a non-parametric method to smooth the mean and quantiles.

The basic idea of the smoothing is using locally weighted regressions. All data in a small band will add certain weights into a predetermined linear function. And the estimate can be obtained from the local linear quantile regression optimization process. The details on data smoothing can be found in Small et al. (2005).

## 4. Estimation and solution procedure

### 4.1. Estimation procedure

The estimated parameters  $\Theta(t)$  characterize the density function  $f(\beta(t)|\Theta(t))$ ; the time-dependent probability formulation is only dependent on parameter vector  $\Theta(t)$ , such that the summation of mean square error (MSE) between the estimated mean  $\tilde{\mu}_p(t)$  and the observed mean  $\hat{\mu}_p(t)$ , and MSE between estimated standard deviation  $\tilde{\sigma}_p(t)$  over  $L_{np}(\beta(t); t)$  and the observed standard deviation  $\hat{\sigma}_p(t)$  is minimal, over the specific time period  $t_1 \leq t < t_2$ , i.e.,

$$\min_{\Theta(t)}(\text{MSE}) = \int_{t_1}^{t_2} \left[ \sum_{p \in P_{rs}} (\hat{\mu}_p(t) - \tilde{\mu}_p(t))^2 + \lambda \sum_{p \in P_{rs}} (\hat{\sigma}_p(t) - \tilde{\sigma}_p(t))^2 \right] dt \quad (17)$$

Upon dividing the time period  $t$  into  $N$  smaller time intervals, the integral in the above objective may be approximated by discretization as

$$\min_{\Theta(t)}(\text{MSE}) = \sum_{i=1}^N \left[ \sum_{p \in P_{rs}} (\hat{\mu}_p(t_i) - \tilde{\mu}_p(t_i))^2 + \lambda \sum_{p \in P_{rs}} (\hat{\sigma}_p(t_i) - \tilde{\sigma}_p(t_i))^2 \right] \quad (18)$$

For convenience, we let the weight parameter  $\lambda = 1$  in this study.

For each observed data set in specific time period  $t_i$ , we calculate traffic volumes on a certain path  $p$ , and the observed probability of route choice at time interval  $t$  is computed by  $P_{rs}^p(t) = v_{rs}^p(t)/v_{rs}(t)$ , where  $P_{rs}^p(t)$  is the probability of traveler choosing route  $p$  at time interval  $t$ ,  $v_{rs}^p(t)$  is the traffic volumes, by aggregating the 30-s loop detector volume count into one study interval, on route  $p$  connecting origin  $r$  and destination  $s$  at time period  $t$ , and  $v_{rs}(t)$  presents the total trip demands from origin  $r$  to destination  $s$  at time period  $t$ . With these values of  $P_{rs}^p(t)$  across multiple days, the observed mean  $\hat{\mu}_p(t)$  and observed standard deviations  $\hat{\sigma}_p(t)$  can be calculated. It should be noted that, in general, capturing travelers' route choices from loop detector data is

difficult because computing the probability of travelers' route choices is complicated. In this study, we can calculate  $P_{rs}^p(t)$  due to the network simplicity, as is shown in the later sections.

Finally, applying the quasi-Monte Carlo simulation, the estimated probabilities are computed by

$$\check{P}_{rs}^p(t) = \frac{1}{K} \cdot \sum_{k=1}^K L_p^k(\beta_k(t); t) \quad (19)$$

where the values of  $\beta_k(t)$  are calculated by  $k = 1, \dots, K$  randomized and scrambled Halton draws from the density function  $f(\beta(t)|\Theta(t))$ . The estimated mean  $\check{\mu}_p(t) = \check{P}_{rs}^p(t)$  and the estimated standard deviation  $\check{\sigma}_p(t)$  are captured by the standard deviation over the  $K$  samples,  $L_p^k(\beta_k(t); t)$ .

#### 4.2. Genetic algorithm

An analytical solution to the minimization problem (18), and therefore to complete the parameter estimation in this study, is impractical. Traditionally, this estimation can be conducted by simulated maximum likelihood estimation (SMLE), as Bhat (2001), or genetic algorithm (GA), as Liu et al. (2004). With the loop detector data aggregation combined with all travelers' responses, instead of individual activities describing in SP or RP data, it is difficult to apply SMLE, which is based generally on individual discrete data. Moreover, the calculation of gradients with the aggregated loop counter data can be obtained only with extraordinary computational cost. The complexity of mixed logit model with aggregated loop data can also produce non-convexity which could drive gradient-based methods to converge to local optima with poor stability. Because of these considerations, we herein employed the popular heuristic, genetic algorithm, to solve the minimization problem (18). In this study, we followed the simple genetic algorithm (SGA) described by Goldberg (1988) containing following steps:

1. *Population representation*: a number of chromosomes, which are uniformly distributed in a given search range, are generated to search the optimal solution of objective. For convenience, we use binary codes as genes in chromosomes representing the solution features.
2. *Evaluation*: evaluating the chromosome performances by calculating the objective function and fitness function. In SGA, a linear transformation which offsets the objective is used as the fitness function.
3. *Selection*: a procedure determining the number of times that a particular chromosome is chosen for generating its offspring. The probability of selecting a chromosome is determined by its performance or corresponding fitness function value.
4. *Crossover*: the basic procedure reproducing new chromosomes in SGA. Crossover produces offspring by combining different parts of both parents' genetic materials, such that their offspring has parts of their parents' features.
5. *Mutation*: as in natural evolution, a random process by which genes alter or change. This process guarantees that GA has the possibility of finding out a global optimum in given feasible region. In SGA, mutation occurs with low probability.
6. *Reinsertion*: a procedure maintaining the population size in each generation. If fewer new chromosomes are generated after crossover and mutation than the size of the original population, new chromosomes must be reinserted.
7. *Termination*: stopping iteration. Since the SGA is a probabilistic method, it cannot use the convergence criteria as traditional methods. We terminate SGA after a predetermined number of generations.

#### 4.3. Study site specification

Route choice data from California State Route 91 (SR91), lying in Orange County, California, has been used in a number of recent research on the pricing policies; see, e.g., Lam and Small (2001), Small et al. (2005) and Liu et al. (2004). The site is a 10-mile section connecting the residential areas in Riverside and San Bernardino Counties with the job centers in Orange and Los Angeles Counties. The study site of SR91 has six parallel lanes in each direction, in which two are toll lanes and four are free lanes, named as 91X

and 91F lanes, respectively. Commuters who wish to use the 91X for express trips must pay tolls that vary hourly according to a predetermined schedule. Data used in our study were for the westbound lanes for weekday mornings in 2001, and the fixed toll schedules were \$3.20 for 5–6 a.m., \$3.30 for 6–7 a.m., \$3.60 for 7–8 a.m., \$3.30 for 8–9 a.m. and \$2.65 for 9–10 a.m. Because there are no additional exits on the 91X lanes, it is reasonable to regard this SR91 section as a network connecting a unique origin–destination pair with two parallel routes; the simplicity of network greatly facilitates data collection and model construction, and significantly reduces the difficulty of capturing travelers' route choices.

We assume that the trips on 91X lanes have fixed 8-min travel time, according to an observed 75 mph free-flow speed; the speed of traffic on 91F lanes is calculated from the data exhibited by the loop detectors on each free lane. The loop detector data is collected from a data interface to the local Caltrans District's Front End Processor, and represents the raw data reported from the individual loop detectors in the field.

#### 4.4. Data preparation

The study period in this paper is the a.m. peak-hour, from 5:00 a.m. to 10:00 a.m. Because we have assumed that the 91X lanes exhibit reliable free-flow travel time, the main task is in determining the temporal variation in travel time for the 91F lanes. We accomplish this by applying the travel-time calculation steps, described in Section 3, to 16 selected weekdays' free lanes loop detector data from June 1st to July 26th, 2001. The raw data are 30-s loop detector data over 16 weekdays and are aggregated into 5-min intervals. Upon calculation of the travel time on free lanes, the differences between the travel time on free lanes and the free-flow travel time on this 10-mile freeway section exhibit the travel-time savings at distinct time intervals.

The travel-time variability can be captured by a number of measures, including, among others, the standard deviation of the travel time on congested free lanes, or the difference between 80th percentile and the median travel time, or the difference between 90th percentile and the median travel time. Based on experimental comparison of consistency among these different measurements of travel-time variability, we employ the 80–50th percentile to capture the variability. The median travel time and its 80–50th percentile variability after local smoothening in our study are shown in Fig. 2.

The median travel-time savings shown in Fig. 2 indicate that the maximal travel-time differences between free lanes and toll lanes occurs at about 6 a.m., with a value of about 5 min. After 6 a.m., travel-time savings start to drop until 9:30 a.m., which indicating the end of morning peak-hours and 91F becomes less congested.

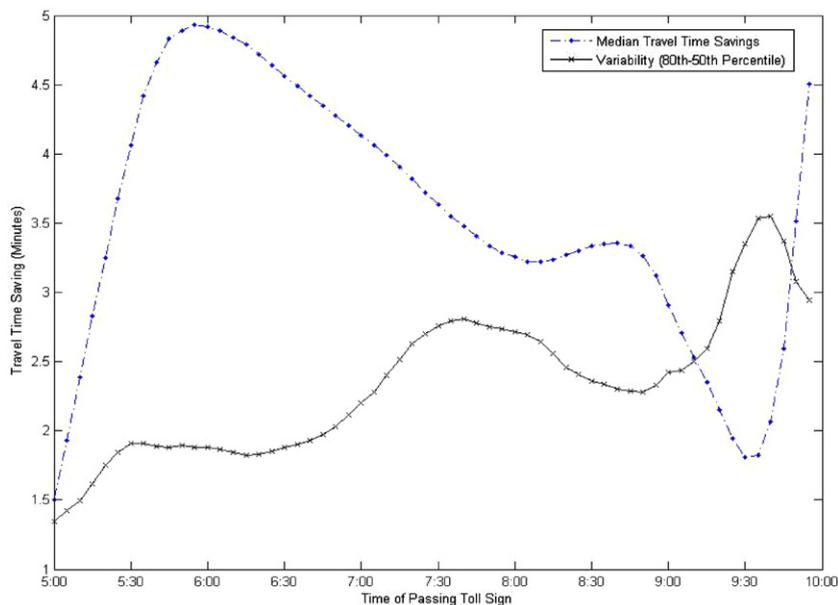


Fig. 2. Travel-time savings and variability in different time intervals.

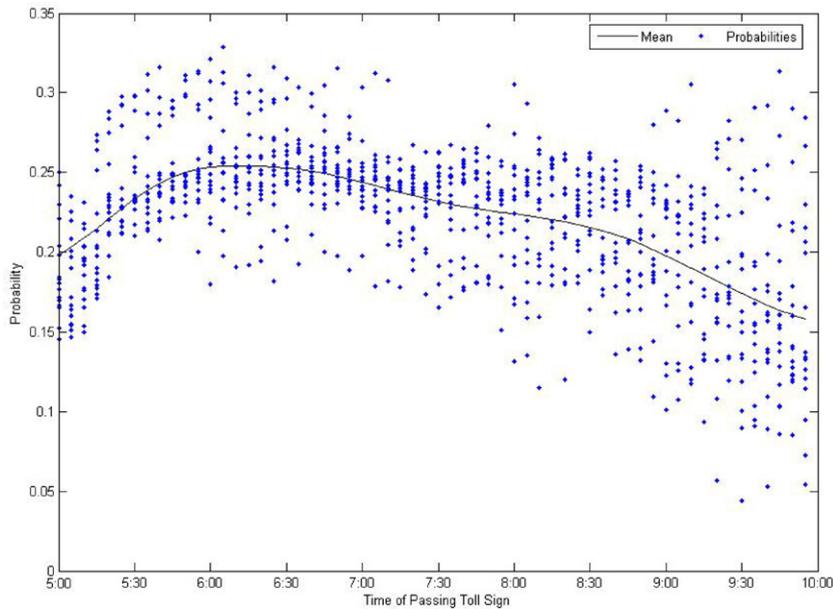


Fig. 3. Probabilities of motorists choosing 91X lanes over 16 weekdays.

Concurrently, the values of the travel-time variability show a fairly steady increase to about 2.8 min at 7:40 a.m., followed by a slight decrease until reaching another peak as the time approached 9:30 a.m., perhaps affected by delays associated with incremental non-work trips and congestion loosening. These results are similar to those in Liu et al. (2004).

From the volume counts provided by the loop detectors, the aggregated probability for usage of toll lanes during any time interval can be computed by the ratio of volume on toll lanes to the total volume that entered the network during that period. Fig. 3 shows the observed aggregated 5-min probabilities of using toll lanes for 16 weekdays. Each dot in this figure represents the value of observed toll-lane-chosen probability at that time on a particular day; the line appearing in Fig. 3 gives the local-smoothed mean value over these 16-day probabilities. These mean values of probabilities indicate that, in general, the peak percentage of motorists using the toll lanes occurs at about 6 a.m., which coincides with the time having highest travel-time savings, and then slowly decreases. Note that the differences of toll lane usage probabilities are statistically significant since the probabilities of using toll lane can neither be a constant, nor be linearly estimated. Also evident in the figure, the standard deviations from 5 a.m. to 8 a.m. (not shown) are much smaller than the values outside of this interval, indicating that motorists have stable tendencies to use toll lanes from 5 a.m. to 8 a.m.

Qualitatively, Figs. 2 and 3 indicate that the observed usage of toll lanes has positive correlation with the travel-time savings, an observation that is consistent with intuitive reasoning that commuters prefer to pay the toll for higher benefit—shorter travel time.

Following data preparation, we obtained the observed travel-time savings and travel-time variability over these 16 sample days; the toll schedule for 2001 was obtained online ([www.91expresslanes.com](http://www.91expresslanes.com)). These data were used to specify  $x_p(t)$  for calculating the simulated probabilities  $L_{np}(\beta(t);t)$  and  $\check{P}^p(t)$ . The observed time-dependent route choice probabilities' mean  $\hat{\mu}_p(t)$  and standard deviation  $\hat{\sigma}_p(t)$  are given by the data shown in Fig. 3.

## 5. Estimation results and analysis

### 5.1. GA parameters

In the estimation of  $\Theta(t)$ , which describes the distributions of the time-dependent parameters  $\beta(t)$ , we used the values in Table 1 to specify the parameters in the GA to determine the estimates with best fitness value to the objective identified in Section 4.

Table 1  
Control parameters for the GA

Population size	40
Maximal number of generations	50
Substring length per parameter	10
Crossover probability	0.6
Mutation probability	0.033
Elitism flag	1

We divided the study time period (5–10 a.m.) into 10 intervals, each of which contains half an hour. Since the estimated parameters set  $\Theta(t)$  for each time period contains six parameters:  $b^T(t)$ ,  $W^T(t)$ ,  $b^R(t)$ ,  $W^R(t)$ ,  $b^C(t)$ ,  $W^C(t)$ , half an hour offers the least number of six observed aggregated 5-min data for the parameter estimation. We then estimated the parameter values of  $\Theta(t)$  during each interval, to uncover the variations of travelers' VOT and VOR tendencies dependent on time.

## 5.2. Computational results

The properties of the simulated mixed logit model and the GA determine the consistency of results. In each GA run, the estimates are different, since GA is a probabilistic-based algorithm and the estimated probabilities in GA are approximated by quasi-Monte Carlo simulation. In order to generate estimated parameters within a certain confidence level, we performed 30 GA runs based on these time-dependent route choice data. Statistical details of estimated parameters in different time periods resulting from these 30 runs are shown in Table 2.

Randomized and scrambled Halton sequences, instead of random draws, are generated for parameter estimation in each GA run, in order to achieve better coverage and faster computational speed. With respect to the number of the samples, Bhat (2001) and Train (2003) have revealed that 125 Halton draws were somewhat better than 2000 random draws. To avoid the correlation between parameters in our estimation, we quadruple the sample size of 125 in every randomized and scrambled Halton sequence.

The estimated statistical distributions of the parameters obtained from the quasi-Monte Carlo simulations make it possible to calculate the extent of the heterogeneity of the time-dependent VOT, VOR and DORA. Similar to the procedure used in parameter estimation, we drew 500 randomized and scrambled Halton samples from the normal distributions of  $\beta(t)$ , according to the parameters under assumptions that  $\beta^T(t) \in N(b^T(t), W^T(t))$ ,  $\beta^R(t) \in N(b^R(t), W^R(t))$  and  $\beta^C(t) \in N(b^C(t), W^C(t))$ . The calculation of randomized and scrambled Halton sequences is referred to Bhat (2003). The dynamic VOT, VOR and DORA are captured

Table 2  
Estimated time-dependent parameters

Parameter	Range	Estimated value (median/[5 percentile, 95 percentile])				
		5:00–5:30 a.m.	5:30–6:00 a.m.	6:00–6:30 a.m.	6:30–7:00 a.m.	7:00–7:30 a.m.
$b^T$	[10,40]	16.51/[15.81, 20.40]	21.38/[12.83, 32.19]	28.42/[17.06, 36.38]	27.30/[19.79, 38.50]	30.63/[25.15, 39.66]
$W^T$	[0,10]	0.78/[0.07, 1.92]	0.73/[0.06, 1.60]	0.78/[0.10, 1.68]	1.08/[0.31, 1.59]	1.17/[0.20, 1.80]
$b^V$	[10,40]	39.49/[32.41, 39.99]	30.03/[20.85, 39.88]	27.07/[12.46, 38.63]	27.54/[17.50, 37.74]	22.95/[15.04, 37.43]
$W^V$	[0,10]	2.73/[0.14, 7.04]	2.67/[0.67, 4.72]	2.95/[0.36, 5.18]	1.48/[0.33, 3.30]	0.96/[0.28, 2.74]
$b^C$	[1,2]	1.00/[1.00, 1.01]	1.13/[1.01, 1.47]	1.29/[1.02, 1.50]	1.24/[1.05, 1.50]	1.15/[1.01, 1.41]
$W^C$	[0,1]	0.06/[0.03, 0.07]	0.03/[0.01, 0.05]	0.03/[0.00, 0.05]	0.02/[0.00, 0.04]	0.02/[0.00, 0.04]
		7:30–8:00 a.m.	8:00–8:30 a.m.	8:30–9:00 a.m.	9:00–9:30 a.m.	9:30–10:00 a.m.
$b^T$	[10,40]	27.45/[12.88, 35.40]	30.85/[17.28, 36.84]	28.37/[14.78, 38.92]	27.33/[19.30, 34.57]	10.12/[10.00, 10.65]
$W^T$	[0,10]	1.16/[0.19, 2.50]	0.77/[0.10, 3.54]	1.33/[0.14, 3.48]	1.93/[0.15, 5.87]	1.50/[0.10, 2.54]
$b^V$	[10,40]	27.74/[15.51, 38.46]	24.33/[12.22, 33.58]	24.97/[15.67, 33.85]	24.63/[14.80, 35.91]	34.66/[23.85, 39.79]
$W^V$	[0,10]	1.69/[0.23, 3.01]	3.07/[0.27, 4.82]	2.04/[0.41, 3.82]	4.26/[0.27, 6.39]	6.26/[2.64, 7.49]
$b^C$	[1,2]	1.08/[1.00, 1.28]	1.13/[1.04, 1.37]	1.20/[1.01, 1.40]	1.39/[1.14, 1.67]	1.53/[1.31, 1.65]
$W^C$	[0,1]	0.03/[0.01, 0.04]	0.03/[0.00, 0.06]	0.04/[0.01, 0.06]	0.10/[0.04, 0.12]	0.11/[0.04, 0.15]

Table 3  
Estimated time-dependent value of time and reliability

	Estimated value (median/[5 percentile, 95 percentile])				
	5:00–5:30 a.m.	5:30–6:00 a.m.	6:00–6:30 a.m.	6:30–7:00 a.m.	7:00–7:30 a.m.
<b>Value of time (\$/h)</b>					
Median	16.50/[15.80, 20.36]	18.53/[12.11, 22.03]	22.02/[15.81, 26.64]	22.97/[18.77, 26.41]	27.66/[22.05, 30.18]
Heterogeneity (75–25th)	1.93/[1.01, 2.88]	1.12 / [0.52, 2.05]	1.20/[0.77, 1.87]	1.30/[0.58, 1.95]	1.48/[1.01, 2.07]
<b>Value of reliability (\$/h)</b>					
Median	39.24/[32.38, 39.99]	25.66/[18.16, 37.24]	23.60/[10.59, 33.86]	23.30/[15.54, 28.12]	20.25/[14.32, 27.92]
Heterogeneity (75–25th)	4.84/[3.13, 9.42]	3.53/[1.44, 5.44]	3.72/[0.98, 5.60]	1.73/[0.67, 3.56]	1.30/[0.69, 3.12]
<b>Degree of risk aversion</b>					
Median	2.39/[1.59, 2.52]	1.31/[0.85, 3.11]	1.04/[0.40, 2.14]	1.04/[0.61, 1.43]	0.71/[0.50, 1.28]
Heterogeneity (75–25th)	0.33/[0.10, 0.60]	0.22/[0.06, 0.42]	0.18/[0.04, 0.34]	0.10/[0.04, 0.17]	0.06/[0.03, 0.14]
<hr/>					
<b>Value of time (\$/h)</b>					
Median	24.66/[12.60, 31.44]	24.23/[16.50, 30.36]	23.18/[13.63, 30.77]	19.58/[16.28, 21.98]	6.82/[6.18, 7.70]
Heterogeneity (75–25th)	1.87/[0.79, 3.15]	2.00/[0.53, 4.33]	2.16/[0.69, 4.60]	2.47/[1.60, 6.50]	1.47/[0.75, 2.32]
<b>Value of reliability (\$/h)</b>					
Median	23.61/[14.28, 34.67]	21.00/[11.23, 28.38]	22.53/[13.33, 29.21]	17.49/[12.45, 21.34]	22.68/[18.32, 24.30]
Heterogeneity (75–25th)	2.44/[0.69, 3.96]	3.78/[0.81, 5.77]	2.67/[1.00, 4.22]	4.21/[2.08, 7.16]	5.83/[3.67, 7.21]
<b>Degree of risk aversion</b>					
Median	0.96/[0.46, 2.87]	0.85/[0.37, 1.68]	0.98/[0.43, 1.67]	0.91/[0.62, 1.14]	3.36/[2.38, 3.91]
Heterogeneity (75–25th)	0.13/[0.05, 0.51]	0.19/[0.07, 0.35]	0.13/[0.05, 0.41]	0.24/[0.08, 0.50]	1.01/[0.52, 1.49]

by median values of time-dependent parameter  $\beta(t)$ , by Eqs. (3)–(5). We use the parameter set,  $\{b^T(t), W^T(t), b^R(t), W^R(t), b^C(t), W^C(t)\}$  given by each run of GA, as the input parameters in the QMC simulations. For each QMC simulation, the program generated 500 randomized and scrambled Halton draws from the distributions with given parameters for the relevant time period, and VOT, VOR and DORA are calculated, together with their heterogeneity as measured by the inter-quartile differences. Finally, the median of these values are computed based on 30 sets of parameters given by 30 GA runs. Their confidence intervals are calculated as well. All these data are contained in Table 3.

### 5.3. Implications of time-dependent VOT and VOR

The values shown in Table 3 yield a variety of useful information. First, by combining the time-dependent VOT, VOR and respective peak values of time savings and variability in that period, the maximal benefits of using toll lanes are approximated by the summation of benefits of time savings and variability. The comparisons between benefits and costs for the 95 percentile values are displayed in Table 4. Since the maximal benefit values from choosing to use the express lanes are lower than out-of-pocket costs for the full range of departure times considered, the probabilities of using 91X should be less than 1/3, which is also demonstrated by the loop detector data in Fig. 3.

The median VOT and VOR reveal some interesting phenomena. For the earlier time periods, the median VOR was found to be significantly higher than the median VOT (Fig. 4). This indicates that motorists departing during the earlier periods weigh more benefits from the stability of travel time than from the time savings; commuters choosing to use express lanes were expecting receiving highest benefit from reliable travel time. Alternatively, with the passage of time toward the end of a.m. peak, the median VOT becomes greater than the median VOR. While the data do not permit imputation of motive for the phenomena, the results are consistent with the observation that, in general, we expect that workers with an early start time are more likely to

Table 4  
Comparison between benefits and costs of using toll lanes

	Departure time				
	5:00–5:30 a.m.	5:30–6:00am	6:00–6:30 a.m.	6:30–7:00 a.m.	7:00–7:30 a.m.
VOT (\$/h)	20.36	22.03	26.64	26.41	30.18
VOR (\$/h)	39.99	37.24	33.86	28.12	27.92
Time savings (min)	3.68	4.94	4.92	4.56	4.13
Variability (min)	1.85	1.91	1.88	2.20	2.70
Benefit (\$)	2.48	3.00	3.25	3.04	3.33
Toll (\$)	3.2	3.2	3.3	3.3	3.6
	7:30–8:00 a.m.	8:00–8:30 a.m.	8:30–9:00 a.m.	9:00–9:30 a.m.	9:30–10:00 a.m.
VOT (\$/h)	31.44	30.36	30.77	21.98	7.70
VOR (\$/h)	34.67	28.38	29.21	21.34	24.30
Time savings (min)	3.63	3.30	3.35	2.70	4.5
Variability (min)	2.81	2.71	2.36	3.32	3.55
Benefit (\$)	3.52	2.95	2.87	2.17	2.02
Toll (\$)	3.6	3.3	3.3	2.65	2.65

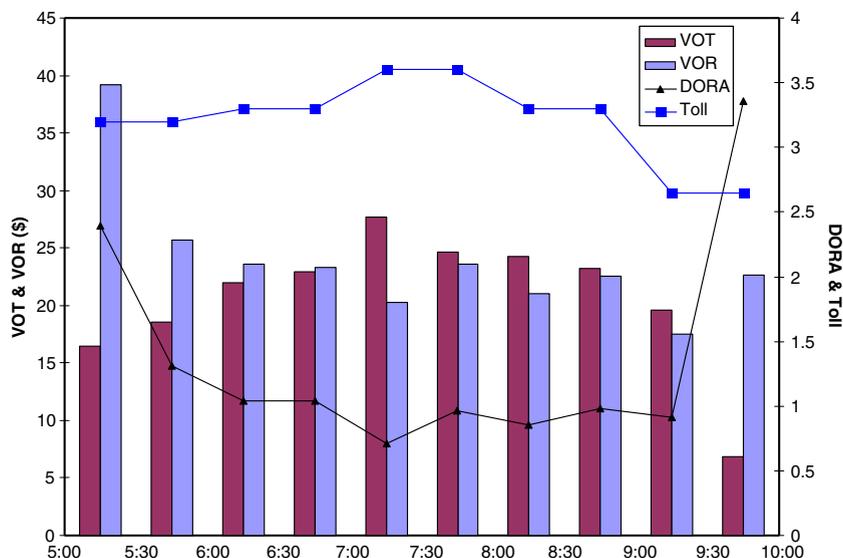


Fig. 4. Time-dependent estimates and tolls.

be employed in positions for which tardiness results in loss of pay, while those with later (nominal) start times are more likely to be salaried, professional, employees. In addition, after the a.m. peak hours, travelers choosing toll lanes are those with high risk aversions, as shown by the tail part of the DORA curve in Fig. 4.

Observing travelers' heterogeneity in Table 3, heterogeneity with respect to neither travel time nor its variability is higher than 26% of its median value, and the values of heterogeneity at the beginning periods are around 10% of their median values. This observation indicates that the preference structure to choosing toll lanes among motorists is similar for the same time periods. This result is quite different from previous research observation, where commuters were determined to exhibit a wide distribution of preferences for speed and reliability (Liu et al., 2004). We attribute this difference to the series of shorter study periods with identical toll prices making up the current study.

The median values of DORA calculated by Eq. (5) at different periods, shown in Table 3 and Fig. 4, reveal the extent to which motorists have aversion to routes with unreliable travel time. These values decrease from a

high of 2.39 at the beginning to a low of 0.71 in the middle of the study interval and then keep in a small range that is smaller than 1 before jumping to 3.36. This result indicates that travelers (taking work-trips) with high DORA are willing to depart earlier in order to escape the high variability of travel time during peak volumes and more likely to choose the express lanes during such early departures to maintain reliable travel times, consistent with Figs. 2 and 3.

## 6. Conclusions

This paper considers the variations of VOT and VOR as a function of departure time, and offers an alternative approach to the traditional traveler survey for the parameter estimation. Route choice behavior has been formulated as a mixed logit model that represents the commuters' different preferences by probabilistic distributions of parameters. Travel time, its variability and monetary cost are included in the travelers' disutility function. The difficulty of systematic calculation of the mixed logit is mitigated by employing quasi-Monte Carlo simulation to approximate the estimated probabilities; randomized and scrambled Halton sequences are also embedded into QMC simulation to speed up the computations.

Based on computational results drawn from the California SR91 site, we estimate the time-dependent parameters to the travel time, its variability and monetary cost in the mixed logit model, and calculate the time-dependent VOT, VOR and DORA, whose variations in different periods reveal the relationship between commuters' route choices and their departure times, as well as the variation of their preferences with departure time. The heterogeneity values further show that travelers have certain consistency with respect to paying the toll within relatively small time intervals. These results may have significance for researchers and transportation planners who are concerned with toll pricing policies.

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