

## STRATEGIES FOR SELECTING ADDITIONAL TRAFFIC COUNTS FOR IMPROVING O-D TRIP TABLE ESTIMATION

ANTHONY CHEN<sup>1</sup>, SURACHET PRAVINVONGVUTH<sup>2</sup>, PIYA CHOOTINAN<sup>3</sup>, MING LEE<sup>4</sup> AND WILL RECKER<sup>5</sup>

Received 31 January 2007; received in revised form 24 June 2007; accepted 25 June 2007

Traditional traffic counting location (TCL) problem is to determine the number and locations of counting stations that would best cover the network for the purpose of estimating origin-destination (O-D) trip tables. It is well noted that the quality of the estimated O-D trip table depends on the estimation methods, an appropriate set of links with traffic counts, and the quality of the traffic counts. In this paper, we develop strategies in the screen-line-based TCL model for selecting additional traffic counts for improving O-D trip table estimation. Using these selected traffic counts, the O-D trip table is estimated using a modified path flow estimator that is capable of handling traffic count inconsistency internally. To illustrate the impact of the additional number of traffic counts on O-D estimation, we set up a unique experiment in a real world setting to visually observe the evolution of O-D estimation as the number of traffic counting locations increases. By comparing the O-D trip tables in a GIS, we visualize the actual impacts of counting locations on the estimation results. Various spatial properties of O-D trip tables estimated from traffic counts of different locations are identified as results of the study.

KEYWORDS: Traffic counting location, origin-destination estimation, route choice, path flow estimator, integer program

### 1. INTRODUCTION

Traffic counts are often collected to monitor traffic circulation. They measure the number of vehicles passing through a point (or a measurement station) during a specified time period. They are usually conducted to monitor and describe traffic characteristics (Garber and Hoel, 1999) such as average annual daily traffic (AADT), average daily traffic (ADT), peak hour volume (PHV), vehicle miles travel (VMT), etc. In addition, these counts can be efficiently used to estimate an O-D demand trip table, which depicts the spatial distribution of trips among the traffic analysis zones in a transportation network. The O-D trip table is the prime source of input for many transportation studies such as future travel demand forecasting and transportation management and control.

Conventionally, an O-D trip table is estimated from a large scale survey which is costly, time-consuming, and labor-intensive. Hence, in the past three decades, many researchers (Van Zuylen and Willumsen, 1980; Maher, 1983; Bell, 1984; Cascetta, 1984; Spiess, 1987; Fisk, 1988; Yang et al., 1992; Ashok and Ben-Akiva, 1993; Sherali et al., 1994; Bell and Shield, 1995; Yang, 1995; Bell et al., 1997; Hazelton, 2000; Maher et al., 2001; Chen et al., 2005; Chootinan et al., 2005a) focused on how to use these traffic counts to estimate the O-D demand trip table. This process can be viewed as the inverse problem of the traffic assignment problem (Bell and Iida, 1997). The process estimates the O-D trip table such that, when assigned back to the network, the trip table can

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<sup>1</sup> Department of Civil and Environmental Engineering, Utah State University, Logan, UT 84322-4110, USA. Corresponding author (Email: achen@cc.usu.edu).

<sup>2</sup> Cambridge Systematics, Inc., Oakland, CA 94607, USA.

<sup>3</sup> Bureau of Planning, Department of Highways, Bangkok 10400, Thailand.

<sup>4</sup> Department of Civil and Environmental Engineering, University of Alaska, Fairbanks, AK 99775-5900, USA.

<sup>5</sup> Department of Civil Engineering, University of California, Irvine, CA 92697-3600, USA.

reproduce the observed counts. In addition, the estimated O-Ds from traffic counts can be updated frequently, and are relatively inexpensive compared with the conventional survey methods. Therefore, O-D trip table estimation from traffic counts is regarded as a convenient and practical way to obtain up-to-date information about travel demand patterns in a region.

The traditional traffic counting location (TCL) problem can be considered as a pre-process of the O-D estimation problem. It has been overlooked, but it is a practically important problem (Yang and Zhou, 1998). TCL problem refers to the problem of selecting locations to obtain traffic counts in order to estimate the O-D trip table. There are various methods to conduct traffic counts ranging from manual, semi-automatic, to fully automatic counts. The main disadvantages of the manual count method are labor-intensive, limited by human errors, and only for short counting periods. However, this method can be used anywhere without any instruments. The semi-automatic method introduces some electronic tools such as counters, telephone transmitters, software package, etc., to reduce human errors and make the process faster. The automatic method avoids human-involvement by using some instruments such as pneumatic road tubes, and magnetic or electric contact devices. These instruments detect the passing vehicle and transmit the information to a recorder at the road side (see some examples of real world instruments from Garber and Hoel (1999)).

Counting costs vary for different techniques and regions. Examples of manual counting costs in California, USA are \$200 for a 2-hour count per intersection, and \$140 for a 24-hour count per counting station. Compared to the conventional survey method, the estimated O-D from traffic counts is much cheaper, and easier to update. Research on the O-D estimation from traffic counts indicates that the quality of the estimated O-D trip table depends on both the number and locations of traffic counting stations (Yang et al., 1991; Yang and Zhou, 1998; Chootinan et al., 2005a; Gan et al., 2005; Ehlert et al., 2006). Intuitively, the traffic counting stations are located at critical points on the network such as congested intersections and freeway entrances. Clearly, this subjective selection cannot guarantee the quality of information obtained. Lam and Lo (1990) and Yim and Lam (1998) proposed some heuristic procedures for identifying the order in which the link should be selected. Yang et al. (1991) examined the reliability of the estimated O-D trip table with respect to the number and locations of counting stations in the network and proposed the "O-D covering rule" for counting locations. This rule states that, for O-D estimation error to be bounded, traffic counting points must be located on the network so that the trips between any O-D pair are observed for at least one link of their path. Ehlert et al. (2006) adopted this rule to develop "second best" solution to locate additional counting stations with budget consideration. Bianco et al. (2001) developed an iterative two-stage procedure that first derives the complete traffic flow vector in a network and then produces a reliable O-D trip table estimate. The procedure is based on the flow measurements provided by a minimal cost set of traffic sensors that are placed on the network by solving the sensor location problem that requires knowledge of traffic turning coefficients at each node.

Yang and Zhou (1998) conducted a comprehensive investigation on the traffic counting locations for effective estimation of O-D trip tables from traffic counts. Based on the theory of maximal possible relative error (MPRE) in O-D trip table estimation (Yang et al., 1991), they derived four rules to locate traffic counting points: the O-D covering rule, the maximal flow fraction rule, the maximal flow-intercepting rule and the link independence rule. The problem of locating counting points on a network is formulated as an integer mathematical program, where the O-D covering rule and the

link independence rule are incorporated as constraints, and the total net traffic flows observed are taken as the objective function to be maximized. Two different cases have been investigated: the case where path flow information is available and the other case where only the initial flow distribution and the turning coefficients at each node are required. They also presented an integer linear programming method to determine the minimum number of counting points required to observe a prescribed fraction of the total traffic flow through the network. Recently, Yang et al. (2001) proposed screen-line-based TCL models to optimally select traffic counting stations in road network based on the O-D separation rule without the need for explicit references to existing O-D flows, path flows, turning proportions at each node, and behavioral assumptions of link/route choice proportions. Trips between a particular O-D pair are considered observed (or separated) if and only if no path can bypass the selected traffic counting locations. Integer programming models were formulated and a genetic algorithm (GA) heuristic procedure was developed to solve two screen-line-based TCL problems: one is to locate a given number of counting stations to separate as many O-D pairs as possible and the other is to determine the minimal number and locations of counting stations required to separate all O-D pairs. Yang et al. (2005) also provided a column generation approach to solve these screen-line-based TCL models. On the other hand, Chootinan et al. (2005b) extended the two single-objective TCL problems to a bi-objective binary integer program for determining optimal screen lines for the purpose of O-D trip table estimation. A distance-based GA solution procedure was developed to solve the multi-objective screen-line-based TCL problem.

In this paper, we develop strategies in the screen-line-based TCL model for selecting additional traffic counts to determine optimal screen lines for the purpose of improving O-D trip table estimation. Suppose the study area has already collected some traffic counts for monitoring purpose, the screen-line-based TCL problem is to determine an additional set of traffic counts for the purpose of improving the O-D trip table estimation. In the next section, we describe the characteristics of the screen-line-based TCL problem and present integer-programming formulations of interest. In Section 3, we describe the modified path flow estimator adopted in this paper for estimating path flows (hence O-D flows) using the traffic counts determined by the screen-line-based TCL models. In Section 4, we provide numerical results of a unique experiment set up in a real world setting to explore the impact of traffic counting locations on O-D trip table estimation. General conclusions and future research are summarized in Section 5.

## 2. THE SCREEN-LINE-BASED TRAFFIC COUNTING LOCATION PROBLEM

In this section, we first describe the characteristics of the screen-line-based TCL problem for the purpose of O-D trip table estimation. Before formulating the strategies for selecting additional traffic counts for improving the O-D trip table estimates, we briefly review the two integer formulations proposed by Yang et al. (2001).

### 2.1 Characteristics of screen-line-based TCL problem

Consider a directed road network  $G(N, A)$  where  $N$  is the set of nodes and  $A$  is the set of directed links in the network. Let  $W$  be the set of O-D pairs with nonzero traffic demand and  $i$  and  $j$  be the origin and destination of O-D pair  $w \in W$ , further let  $|W|$  be

the total number of O-D pairs and  $|A|$  be the total number of links in the network. We define that the network is connected if there exists at least one directed simple path (a path that contains no repeated arcs and no repeated nodes) between each O-D pair  $w \in W$  starting at origin  $i$  and ending at destination  $j$ .

Now, we introduce a binary integer variable:  $x_a = (0,1)$ ,  $x_a = 1$  if a traffic counting station is located on link  $a$ , and 0 otherwise,  $\mathbf{x}$  denotes the corresponding binary integer variable vector with element  $x_a$ . Let  $t_a$  be a virtual travel time on link  $a \in A$ . For the sake of our model formulation, we suppose  $t_a$  is a function of  $x_a$  and is simply defined as

$$t_a(x_a) = x_a, \forall a \in A. \tag{1}$$

Since each link has a nonnegative value of travel time, we can use an appropriate shortest path algorithm to find the shortest path and its corresponding travel time from each origin  $i$  to each destination  $j$  within a finite number of iterations. Let  $u_w$  be the shortest travel time between O-D pair  $w \in W$  determined by an appropriate shortest path algorithm such as Dijkstra method (Ahuja et al., 1993; Bertsekas, 1998). Clearly,  $u_w$  is a function of the binary integer variable vector  $\mathbf{x} = (\dots, x_a, \dots)$ , and we can easily understand that if  $u_w(\mathbf{x}) > 0$  then the shortest path between O-D pair  $w \in W$  includes at least one counting link. In view of the definition of link travel time function (1), it is straightforward to see that if  $u_w(\mathbf{x}) > 0$  then origin  $i$  and destination  $j$  of O-D pair  $w \in W$  is separated by at least one screen line. Otherwise, there exists at least one shorter path with zero travel time from  $i$  to  $j$  that does not go through any counting link or cross any screen line. To illustrate the concept of virtual travel time, consider the network depicted in Figure 1. This network consists of 9 nodes, 12 links, and 4 O-D pairs. Node 1 and node 4 are origins and node 6 and node 9 are destinations.

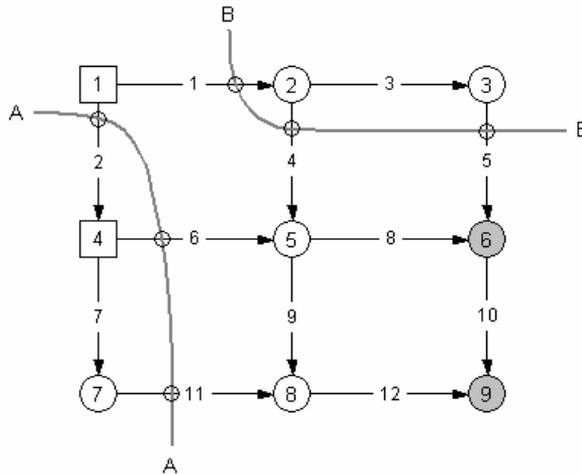


FIGURE 1: Two sets of sensors forming two screen lines

Let us consider two sets of counting stations,  $[2,6,11]$  and  $[1,4,5]$ , respectively forming screen lines A and B. Screen line A is able to intercept flows originating from origin

node 4 completely since there is no alternate path to nodes 6 and 9 that can bypass this set of counting stations. Clearly, it can be observed that costs of all possible paths from node 4 to node 6 and from node 4 to node 9 are greater than zero (e.g., path 6-8 for O-D pair (4,6), and path 7-11-12 or 6-8-10 for O-D pair (4,9)). However, this set of counting stations can partially intercept flows originating from node 1. Part of the flows using the upper portion of network such as flows on path 1-3-5, 1-3-5-10, or 1-4-9-12, is not intercepted at all. One can simply verify that the costs (virtual travel time) of those paths are actually zero. In other words, by using these shorter paths (with zero virtual travel time), part of flows between these O-D pairs is not necessarily passing through the traffic counts forming screen line A. By applying the same analogy to screen line B, it is easy to verify that the costs of shortest paths of all O-D pairs are essentially zero, for examples, path 2-6-8 for O-D pair (1,6), path 2-7-11-12 for O-D pair (1,9), path 6-8 for O-D pair (4,6), and 6-8-10 or 7-11-12 for O-D pair (4,9). This means that screen line B cannot completely intercept flows from all O-D pairs in this network.

Although both screen lines contain the same amount of counting stations, from the interpretation given above, their capabilities of intercepting flows are different and largely dependent on the location of counting stations. For demonstration purpose, we assume that these sets of counting stations can be manipulated and combined to form new screen lines ( $A_1 - [1,6,11]$  and  $B_1 - [2-4-5]$ ) as depicted in Figure 2.

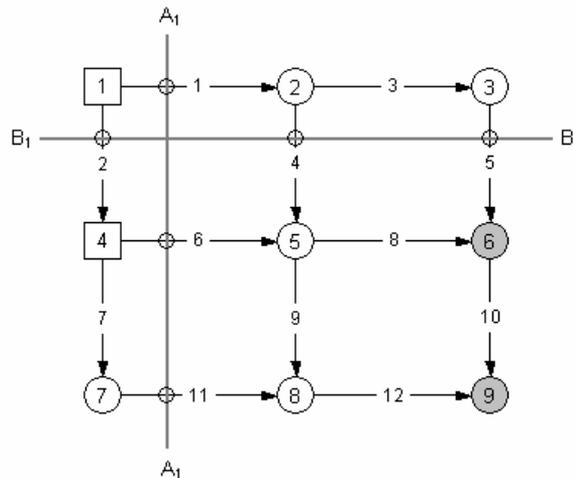


FIGURE 2: New screen lines generated from screen lines A and B

Again, using the concept of virtual travel time, it is found that, with the counting stations on links 1, 6, and 11 (screen line  $A_1$ ), there is no path with virtual cost less than one connecting any O-D pair. In other words, traveling from any origin to any destination has to traverse at least one link with counting stations. On the other hand, the locations of counting station set  $B_1$  can observe the total traffic flow emanating only from origin 1 (O-D pairs (1,6) and (1,9)), but not from origin 4 (O-D pairs (4,6) and (4,9)). By computing the virtual travel time of all possible paths connecting O-D pairs (4,6) and (4,9), they (cost of path 6-8, 7-11-12, or 6-9-12) are essentially zero.

From this small network, it is easy to verify that only three counting stations are required to intercept all O-D pairs in this network. However, they have to be set up at

some specific locations. This implies that there may be multiple combinations of locations (or solutions), which can achieve the same goal (intercept all O-D pairs). Figure 3 shows some possible locations to set up the three counting stations for this simple grid network (a set of more than three counting stations such as [1-4-8-12] is also feasible, but not the optimal for the minimization problem). From Figure 3, it can be seen that some combinations of counting stations, [1,6,7] and [3,8,12], are able to form a screen line, which divides a network into two parts and some combinations such as [1,6,12] represented by the shaded tiny triangles are not. Note that we should distinguish the screen line considered here and the traditional cut in network theory. A cut is a partition of the node set  $N$  into two parts,  $S$  and  $\bar{S} = N - S$ . Each cut defines a set of links consisting of those links that have one endpoint (either starting or ending point) in  $S$  and another endpoint in  $\bar{S}$ . A source-terminal cut is defined with respect to distinguished nodes  $i$  and  $j$  and is a cut  $[S, \bar{S}]$  satisfying the property that  $i \in S$  and  $j \in \bar{S}$  (Ahuja et al., 1993; Bertsekas, 1998). In contrast, a screen line here is established with respect to the availability of a path that does not cross that line. A screen line so determined may not necessarily divide the network in to two disjoint parts.

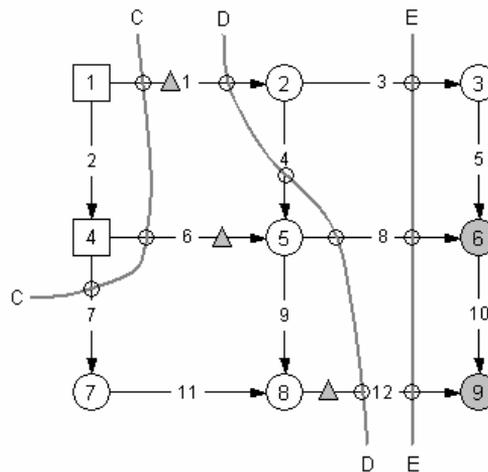


FIGURE 3: Multiple solutions for set covering problem (excluding screen line D)

## 2.2 Two existing screen-line-based TCL models

This section reviews two screen-line-based TCL models recently proposed by Yang et al. (2001, 2005). They defined the “O-D separation rule” as an O-D pair is separated when all routes connecting the origin to the destination pass through at least one traffic counting station. Using the O-D separation rule, the two screen-line-based TCL models can be stated as follows: (1) how to determine the minimum number of counting stations to separate all O-D pairs, and (2) how to choose the optimal locations of a given number of counting stations to separate as many origin-destination pairs as possible. Both models assume that there is no existing traffic counting station in the network. Only network topology and the delimitation of O-D zones are assumed to be given. There is

no need to explicitly reference to an existing O-D trip table, turning proportions at each node, and link/route choice proportions.

*P1: Determine the optimal number and locations of traffic counting stations to separate all O-D pairs in a network.*

$$\text{Minimize } Z_1 = \sum_{a=1}^{|\mathbf{A}|} x_a \quad (2a)$$

subject to

$$\sum_{a=1}^{|\mathbf{A}|} \delta_{ra}^w x_a \geq 1, \quad \forall r \in R_w, w \in W, \quad (2b)$$

$$x_a \in (0,1), \quad \forall a \in A, \quad (2c)$$

where  $R_w$  is the set of paths between an O-D pair  $w$ , and  $\delta_{ra}^w$  is a path-link indicator denoting 1 if link  $a$  is on path  $r$  between O-D pair  $w$ , and 0 otherwise.

The objective function (2a) of *P1* is to minimize the number of traffic counting stations required to separate all O-D pairs in the network. Equation (2b) ensures that all O-D pairs are separated by at least one screen line (or all routes connecting all origins to all destinations must pass through at least one traffic counting station). Equation (2c) constrains the solution to be a binary integer. If the number of available traffic counting stations is less than the number required to separate all O-D pairs as required in *P1*, we can formulate *P2* as follows.

*P2: Determine the locations for a given number of traffic counting stations to maximize the number of O-D pairs being separated.*

$$\text{Maximize } Z_2 = \sum_{w=1}^{|\mathbf{W}|} y_w \quad (3a)$$

subject to

$$y_w \leq \sum_{a=1}^{|\mathbf{A}|} \delta_{ra}^w x_a, \quad \forall r \in R_w, w \in W, \quad (3b)$$

$$\sum_{a=1}^{|\mathbf{A}|} x_a \leq L, \quad (3c)$$

$$x_a \in (0,1), \quad \forall a \in A, \quad (3d)$$

$$y_w \in (0,1), \quad \forall w \in W, \quad (3e)$$

where  $y_w$  is a binary integer variable denoting 1 if O-D pair  $w$  is separated by the set of traffic counts and 0 otherwise, and  $L$  is the number of available traffic counting stations.

Unlike *P1*, the objective function (3a) of *P2* is to determine the locations of traffic counting stations to maximize the number of O-D pairs being separated. Whether O-D pair  $w$  is separated or not is determined by equation (3b). If O-D pair  $w$  is not separated according to the “*O-D separation rule*”, it forces  $y_w$  to be zero. Equation (3c) constrains the total number of traffic counting stations to be located less than or equal to the number of available stations ( $L$ ). Equations (3d) and (3e) constrain the solution to be a binary integer.

### 2.3 Two extensions of the screen-line-based TCL model

As mentioned above, *P1* and *P2* assume that there is no existing traffic counting station in the network. In many situations, there may already exist some traffic counting stations (though may not be optimally located). To account for existing traffic counting stations, *P3* and *P4* extend *P1* and *P2* respectively. Ehlert et al. (2006) referred to these extensions as “second best” solutions. However, it should be noted that their study adopt the “*O-D covering rule*”, which does not require all paths between an O-D pair to be intercepted. It relies on the link choice proportions, which are dependent on the route choice model and demand level used to generate the link choice proportions for the TCL problem. In contrast, the screen-line-based TLC models proposed in this paper are not dependent on any route choice behavioral assumptions, explicit reference to existing O-D trip table, or turning proportions at each node. Only network topology and the delimitation of O-D zones are assumed in the models.

*P3*: Given some existing traffic counting stations, determine the minimal number and locations of additional traffic counting stations required to separate all O-D pairs in a network.

$$\text{Minimize } Z_3 = \sum_{a \in A - A_e} x_a \quad (4a)$$

subject to

$$\sum_{a=1}^{|\mathbf{A}|} \delta_{ra}^w x_a \geq 1, \quad \forall r \in R_w, w \in W, \quad (4b)$$

$$x_a = 1, \quad \forall a \in A_e, \quad (4c)$$

$$x_a \in (0,1), \quad \forall a \in A - A_e, \quad (4d)$$

where  $A_e$  is the set of (existing) counted links in the network (i.e.,  $x_a = 1$  if  $a \in A_e$ ). The objective function (4a) of *P3* is to minimize the number of additional traffic counting stations required to separate all O-D pairs in the network. Equation (4b) is the same as equation (2b), which is to ensure that all O-D pairs are separated by at least one screen line. Equation (4c) constrains links in the existing counted set to be 1, while equation (4d) constrains those that are not in the existing counted set to be a binary integer.

*P4*: Given some existing traffic counting stations, determine the locations for a given number of additional traffic counting stations to maximize the number of O-D pairs being separated in a network.

$$\text{Maximize } Z_4 = \sum_{w=1}^{|\mathbf{W}|} y_w \quad (5a)$$

subject to

$$y_w \leq \sum_{a=1}^{|\mathbf{A}|} \delta_{ra}^w x_a, \quad \forall r \in R_w, w \in W, \quad (5b)$$

$$\sum_{a \in A_e} x_a \leq L, \quad (5c)$$

$$x_a = 1, \quad \forall a \in A_e, \quad (5d)$$

$$x_a \in (0,1), \quad \forall a \in A - A_e, \quad (5e)$$

$$y_w \in (0,1), \quad \forall w \in W. \quad (5f)$$

Similar to *P2*, the objective function (5a) of *P4* is to determine the locations of additional traffic counting stations to maximize the number of O-D pairs being separated. Equation (5b) determines whether the set of traffic counting stations (existing plus new) separates O-D pair  $w$  or not. Equation (5c) constrains the total number of additional traffic counting stations to be located less than or equal to the number of available counting stations ( $L$ ). Equation (5d) constrains links in the existing counted set to be 1; equation (5e) constrains those that are not in the existing counted set to be a binary integer; and equation (5f) constrains the solution to be a binary integer.

#### 2.4 Incorporating land use information to improving O-D trip table estimates

Note that the above screen-line-based TCL models (*P1* to *P4*) view all O-D pairs equally since only network topology and the delimitation of O-D zones are used in determining the optimal number and locations of traffic counts. If an existing O-D trip table is available, it can be incorporated into the screen-line-based TCL models by appending a weighting factor to each O-D pair to rank the importance of the O-D pairs to be separated by the traffic counts. Ehlert et al. (2006) suggested using a nonlinear scale based on the concept of information theory to influence the selection of traffic counts. However, such an O-D trip table may not always exist. In this paper, we suggest to use local land use maps to determine if the trip production and attraction of an estimated O-D trip table correspond to the actual trip making propensity suggested by the land use designation. Such land use zoning designation information is typically available in the city's general plan as illustrated later in the numerical results section. Using the published trip generation rates by the Institute of Transportation Engineers (ITE, 1997), such land use information can be converted to estimates of trip production and attraction for each traffic analysis zones (TAZs) and incorporated in the screen-line-based TCL models to influence the selection of traffic counts. It can also be used to verify the quality of the O-D trip table estimates as shown later in the numerical results section.

#### 2.5 Solution procedure

The screen-line-based TCL models (*P1* to *P4*) presented above are combinatorial problems, which are known to be *NP-hard* (Megiddo, et al., 1983; Yang et al., 2003). In addition, solving these models requires determining the path-link indicator ( $\delta_{ra}^w$ ) for every O-D pair. One way to obtain  $\delta_{ra}^w$  is path enumeration. However, this approach is prohibited for large-scale networks since the number of paths between each O-D pair grows exponentially with respect to network size. Following Yang et al. (2001) and Chootinan et al. (2005b), we adopt a genetic algorithm (GA) embedded with a shortest path algorithm to solve the screen-line-based TCL models. As discussed in Section 2.1, the embedded shortest path obviates the need to enumerate paths since the value of the shortest path between each O-D pair can be checked to determine whether the O-D pair is separated by the traffic counts or not. An alternative approach for generating efficient paths for the traffic counting location problem using the "*O-D covering rule*" is given by Meng et al. (2005).

A GA-based approach differs from conventional search methods in that it searches among a population of points. The transition scheme of GA is probabilistic, whereas

traditional methods use gradient information. Because of these features, GAs are considered as an effective solution approach for solving combinatorial problems. In general, a GA involves the following steps: chromosome representation, creating initial population, evaluation and selection, crossover, mutation and next generation. A GA begins with an initial population of randomly generated members. The fitness or performance of each member in the first generation is evaluated, and members are reproduced in proportion to their relative fitness. The bias toward higher fitness members ensures that the high fitness characteristics are passed along to future generations and that succeeding generations are more fit. In this way, the solution quality improves with the generation. The GA-based procedure used in this study is briefly detailed below. For the detailed descriptions of GA approach, the reader may refer to Goldberg (1989) and Gen and Cheng (2000).

### 2.5.1 Chromosome representation

For  $P1$  and  $P2$ , the location variables,  $\mathbf{x} = (\dots, x_a, \dots)$ , are represented by a string of binary integers with a length equal to the number of network links,  $|A|$ . The value of each gene indicates the existence of a counting station on link  $a$  (i.e., 1 if a count is located on link  $a$ , and 0 otherwise). For  $P3$  and  $P4$ , the location variables are also represented by a string of binary integers with a length of  $|A - A_e|$  (i.e., number of network links minus the number of existing links with traffic counts).

### 2.5.2 Fitness evaluation

The objective value of  $P1$  and  $P3$  is simply the sum of the values of all genes in the chromosome (i.e., number of traffic counts required to separate all O-D pairs), while the objective value of  $P2$  and  $P4$  is determined by the number of O-D pairs that has a shortest path value greater than zero (i.e.,  $u_w(\mathbf{x}) > 0$ ).

### 2.5.3 Reproduction

The reproduction is a process of selecting the chromosomes from the population pool for mating purpose. It directs the genetic search toward the promising area of the search space. The reproduction operator used in this study is based on the roulette wheel selection and elitist approaches. The elitist method is employed to preserve healthy chromosomes from the current population set to be the survivals for the next generation. The roulette wheel selection, on the other hand, is a method used to reproduce new chromosomes proportional to the fitness of each chromosome in the current generation.

### 2.5.4 Crossover and mutation

Crossover and mutation provide a way to stochastically manipulate the existing chromosomes in order to generate new offsprings. The crossover plays a major role in exchanging genetic materials between a pair of chromosomes previously selected. The crossover can be as simple as a one-point crossover or slightly complicated as a multi-point crossover. In this study, we use the uniform crossover in which the exchanges of genetic material occur at the points corresponding to the crossover mask. The crossover

mask has the same length as chromosome and it consists of 0s and 1s, which indicate the parent chromosomes supplying genetic units to new offsprings.

After the crossover is applied to a certain number of chromosome pairs according to crossover probability ( $P_c$ ), the mutation will be applied next. The major role of mutation is to introduce a new genetic material to the pool of chromosomes to provide the genetic search an ability to jump out of a local optimum. There is also the probability associated with this operator, probability of mutation ( $P_m$ ), which is in general set at a very small number. However, this setting is problem dependent.

### 3. MODIFIED PATH FLOW ESTIMATOR

In this study, the path flow estimator (PFE), originally developed by Bell and Shield (1995) and further enhanced by Ceylan and Bell (2004), Chootinan et al. (2004, 2005a), Chen et al. (2005), and Chen and Chootinan (2007), is used to estimate path flows from traffic counts. The basic idea is to find a set of path flows that can reproduce the observed link counts. The resulting path flows can be used to derive flows on other spatial levels, such as turning movement flows, unobserved link flows, O-D flows, production flows, attraction flows, and total demand. The attractiveness of PFE lies on the fact that it is a single level mathematical program in which the interdependency between O-D demand and route choice behavior (congestion effect) is taken into account without the need to employ the bi-level mathematical program (one level estimates the O-D trip table while the other represents the behavioral responses of network users). Network users are assumed to follow the stochastic user equilibrium (SUE) assumption, which allows the selection of non-equal travel time paths due to the imperfect knowledge of network travel times and yields the unique path flow estimates. Although PFE does not require traffic counts to be collected on all network links when inferring unmeasured traffic conditions, it requires all available counts to be consistent. This requirement is difficult to fulfill in most of real applications due to the errors inherited in data collection and processing. The original PFE handles this issue by specifying appropriate error bounds on the traffic counts. This method enhances the flexibility of PFE by allowing the user to incorporate local knowledge about the network conditions into the estimation process. However, specifying appropriate error bounds for all measured links in a real network application is laborious. In addition, improper specification of the error bounds could lead to biased estimates of the O-D demand (Chootinan et al., 2005a).

In this study, we adopt the modified PFE developed by Chen and Chootinan (2007) to internally handle inconsistent traffic counts within the PFE model. Due to measurement errors inherited in traffic counts, there may not exist a path flow solution that can reproduce all traffic counts exactly; however, if measurement errors are allowed in the estimation, a path flow solution may be found to match all traffic with different degrees of deviation between the estimated and observed link flows. This path flow pattern is usually associated with some estimation errors given by:

$$\Psi_a = \left| v_a - \sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_{ra}^w \right|, \quad \forall a \in M, \quad (6)$$

where  $M$  is the set of links with traffic counts,  $v_a$  is the observed flow (or traffic count) on link  $a$ ,  $f_r^w$  is the estimated flow on path  $r$  between O-D pair  $w$ , and  $\psi_a$  is the error associated with the selected path flow pattern fails to satisfy the observed flow on link  $a$ .

Intuitively, the best approximate path flow pattern is the solution that keeps such deviation as small as possible. For this study, we use the  $L_1$ -norm, which is to minimize the average absolute error. As discussed by Chvatal (1983), minimizing the  $L_1$ -norm leads to the most robust approximate solution (i.e., solution insensitive to outliers). Hence, the  $L_1$ -PFE formulation is to minimize the mean absolute error (MAE) while searching for a SUE path flow pattern that produces a link flow pattern with the minimum MAE as follows.

$$\begin{aligned} \text{Minimize } Z_{L_1} = & \sum_{a \in A} \int_0^{f_a} t_a(w) dw + \frac{1}{\theta} \sum_{w \in W} \sum_{r \in R_w} f_r^w (\ln f_r^w - 1) \\ & + \frac{1}{\theta} \sum_{a \in M} \psi_a (\ln \psi_a - 1) + \sum_{a \in M} \rho_a \psi_a \end{aligned} \quad (7a)$$

subject to

$$\sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_{ra}^w \geq v_a - \psi_a, \quad \forall a \in M, \quad (7b)$$

$$\sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_{ra}^w \leq v_a + \psi_a, \quad \forall a \in M, \quad (7c)$$

$$f_a \leq C_a, \quad \forall a \in U, \quad (7d)$$

$$f_r^w \geq 0, \quad \forall r \in R_w, w \in W, \quad (7e)$$

$$\psi_a \geq 0, \quad \forall a \in M, \quad (7f)$$

where

$$f_a = \sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_{ra}^w, \quad \forall a \in A, \quad (7g)$$

$$q_w = \sum_{r \in R_w} f_r^w, \quad \forall w \in W, \quad (7h)$$

$U$  is the set of unmeasured links,  $A$  is the set of all network links ( $A = M \cup U$ ),  $\theta$  is the dispersion parameter,  $t_a(\cdot)$  is the link cost function of link  $a$ ,  $C_a$  is the capacity of link  $a$ ,  $f_a$  is the estimated flow on link  $a$ , and  $q_w$  is the estimated flow between O-D pair  $w$ .

The objective function (7a) of the  $L_1$ -PFE model is to minimize path entropies and travel costs for both physical and virtual paths. The entropy of the virtual paths is treated in the same manner as those of the physical paths while the travel cost of the virtual paths is treated as a penalty term ( $\rho_a$ ). Ideally, this cost penalty must be raised to the level at which the average absolute deviation (i.e., MAE) is minimized. Equations (7b) and (7c) define the lower and upper limits of the estimated link flows. These two constraints restrict the estimated link flows (derived from the physical path flow estimates) to be within the boundaries defined by link observations and  $\psi_a$ . Equation (7d) constrains the estimated flows on the unobserved links to be less than or equal to their respective capacities. Equations (7e) and (7f) constrain the estimated path flows and the estimated link errors to be non-negative, respectively. Equations (7g) and (7h) are

definitional constraints that sum up the path flows at the link level and at the O-D level, respectively.

The solution procedure for the  $L_1$ -PFE model is based on the partial linearization method. The method consists of two major steps (i) a direction finding step and (ii) a line search step. In the direction finding step, certain part of the objective function is linearized. The solution to the linearized problem defines a feasible direction and can be solved by the iterative balancing technique. The line search step determines how far the current solution should move in the feasible direction. These two steps are iterated until convergence is reached. A column generation procedure is also implemented to avoid path enumeration for a general transportation network. For details of the solution procedure, the readers are referred to Patriksson (1994) for the partial linearization method and to Bell and Iida (1997) for the iterative balancing solution method.

#### 4. NUMERICAL RESULTS

We start the experiment by building a planning network based on the actual network of the City of St. Helena, located in the famous wine-producing region of Napa Valley in California, approximately 65 miles north of San Francisco. We select the city as the study area for the experiment because of the availability of traffic counts in the city and the knowledge of local traffic patterns established from various field work performed in the city. St. Helena is a full service city with a population of 6,019 (as of January 1, 2002) within an area of 4 square miles. The city's development pattern is relatively compact. Commercial development and wineries concentrate along Highway 29 (Main Street) corridor and residential development radiate out from Main Street. As a result, the Main Street is the busiest street in town with an average of over 1,000 vehicles during the evening peak hour. Most of the traffic studies done in the city involve the evaluation of traffic impacts on the Main Street. The city does not maintain a travel demand forecasting model. The traffic count data and land use zoning map found in the city's general plan (City of St. Helena, 1993) are the only resources available for this study.

For the experiment, we use Census Blocks as Traffic Analysis Zones (TAZ) for the planning network. The planning network is essentially an exact replication of the actual street network. The network contains 113 TAZs, 802 links, and 344 nodes (see Figure 4). We coded the speed and capacity of each link with information derived from the design class and field measurement of the actual street. We first use the  $L_1$ -PFE model to estimate an O-D table based on 106 of the most up-to-date link traffic counts (collected for a recent study of a specific plan proposal in the city) along the Main Street corridor. The 106 link counts are based on the evening peak hour, when traffic congestion is the most problematic. Through trips passing St. Helena via Highway 29 are estimated with field observation. The through trips are then subtracted from the traffic counts on the highway. The subtraction is made such that the estimated O-D tables represent internal-to-internal and internal-to-external trips. We then treat these 106 traffic counts as the base and use  $P4$  to determine additional counting locations for observation of O-D pattern evolution. The locations of the additional counts are determined such that the number of O-D pairs separated is maximized. The O-D pair is considered observed (or separated) when there is no path between that O-D pair can bypass any link with traffic count. Figure 5 shows the relationship between number of traffic counts and number of O-D pairs separated (at an increment of 10 locations). Though not reported in Figure 5, it requires 248 additional counts (or  $354 = 106 + 248$  traffic counts in total) to completely separate all 12,656 O-D pairs by solving  $P3$ .

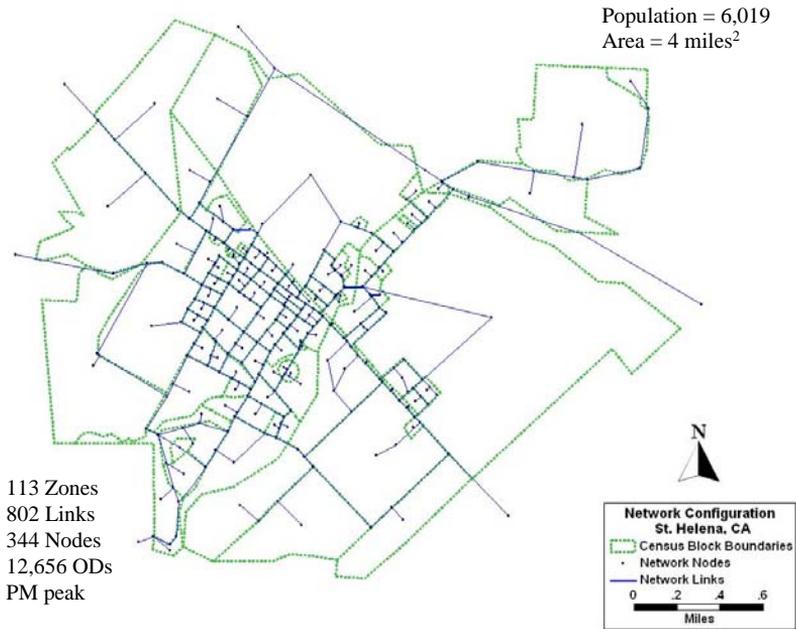


FIGURE 4: Planning network configuration

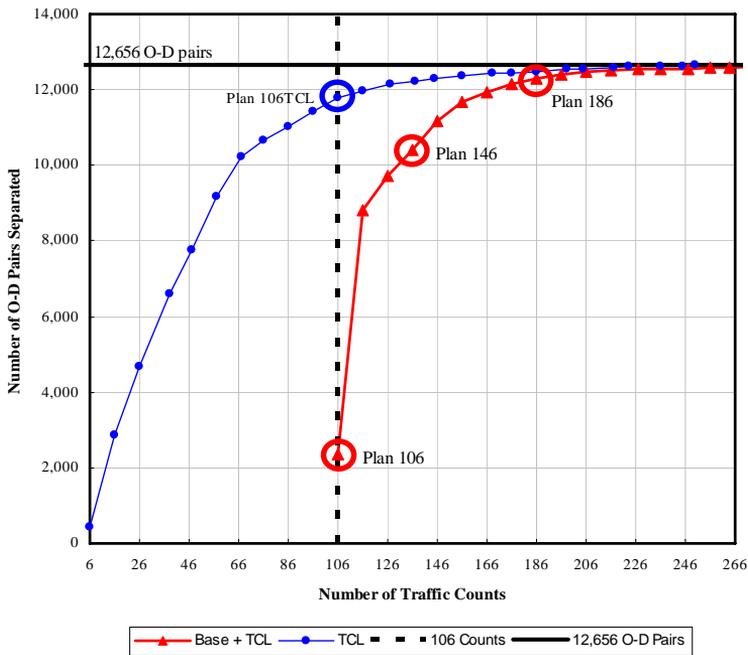


FIGURE 5: Relationship between number of traffic counts and number of O-D pairs separated

Based on the curve in Figure 5, we estimate a series of four additional O-D trip tables with 10, 20, 40, and 80 additional counting locations using the  $L_1$ -PFE model. The set with a smaller number of links is not necessarily a subset of the subsequent sets. That is, links in the additional 10 link set may or may not overlap those in the additional 20 link set. We also use the existing screen-line-based TCL model ( $P_2$ ) to determine another independent set of 106 optimal counting locations (only 19 of the link locations overlap with those in base case) so we can observe the emergent O-D pattern when traffic-counting locations are entirely determined by the screen-line-based TCL solution. The comparison of the series of O-D trip tables to the 106 base case can also illustrate the implications when majority of the traffic counts are derived from major corridors. For illustrative purposes, we plot on a map (Figure 6) the locations of the base case (Plan 106), the 40 (Plan 146), the 80 (Plan 186) addition links, and the independent set of 106 links that are entirely based on  $P_2$  (Plan 106TCL). The arrangement of these four figures creates distinct visual patterns that illustrate how O-D evolves as the number of counting location increases.

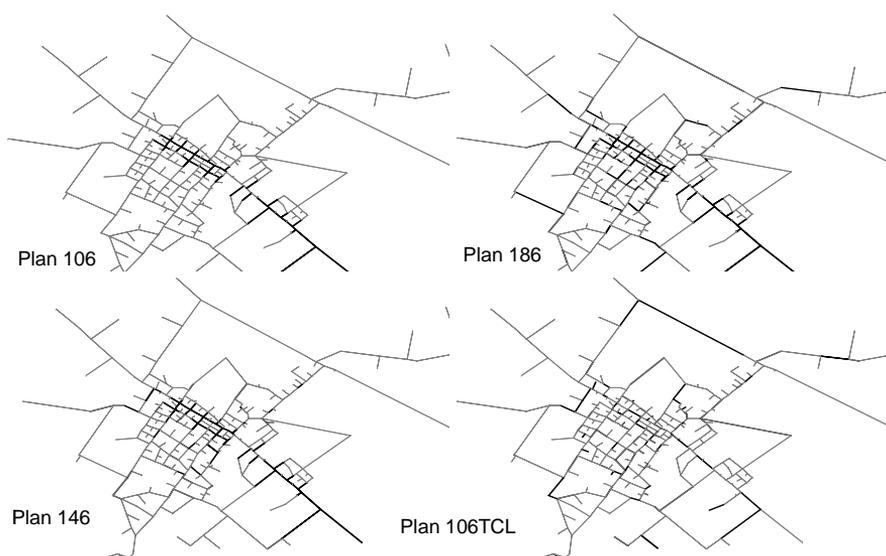


FIGURE 6: Relationship between number of traffic counts and number of O-D pairs separated

The numerical results of the estimation are summarized in Table 1. Table 1 shows that, as the number of counting locations increases, we can separate more O-D pairs, capture more flows, estimate higher total demand, and have more O-D pairs with significant flows. The values of the root mean square error (RMSE) show that the  $L_1$ -PFE model can reproduce link flows that match the observed flows well. Note that flow capturing is essentially the sum of traffic counts on all links. When a vehicle is counted on a particular link of a network, we say that it is “captured” once. When the same vehicle moves to another link and is counted there again, we say that it is captured twice. The number of flow capturing gives an indication of how well links with high traffic flows are observed in the counting locations. It can be seen in Table 1 that Plan 106, when compared with Plan 106TCL, contained more links with higher traffic volumes. Plan 106TCL, on the other hand, is designed to separate more O-D pairs. Hence, it can

separate 93%, which is much higher than Plan 106 that can separate only 18%. It is important to note that the results presented in Table 1 can only be assessed in relative terms since these plans do not have sufficient number of traffic counts to separate all 12,656 O-D pairs. That is, the results (i.e., number of O-D pairs separated, amount of captured flows, total demand, and number of O-D pairs with significant flows, etc.) are increasing because the number of additional traffic counts is less than the minimum additional number required to separate all O-D pairs (248 by solving  $P3$ ). However, if more counts are included beyond 248 (i.e., more than the minimum additional required number to separate all O-D pairs) and the counts are of good quality, the measurements of the estimated O-D trip table should stabilize.

TABLE 1: Summary of estimated O-D trip tables

Location plan	Base	Base + TCL				TCL
	106	116	126	146	186	106
	Measurements of location plan					
No. of O-D pairs separated	2,340	8,825	9,706	11,157	12,287	11,780
Percent of O-D pairs separated	18%	70%	77%	88%	97%	93%
	Measurements of estimated O-D trip table					
Flow capturing (times)	39,077	40,063	40,753	43,073	47,922	14,891
No. of O-D pairs with flow > 5 vph	165	180	186	190	215	193
Total estimated demand (vph)	5,118	5,493	5,633	5,954	6,645	5,755
Link RMSE	18.71	17.82	17.04	15.89	17.43	31.92

Due to the lack of prior O-D information, we can not directly assess the quality of the estimated O-D trip tables. To overcome this limitation, we devise a unique scheme that compares the PFE estimates of trip productions and attractions with estimates derived from the city's land use map. With reference to the city's general plan (City of St. Helena, 1993), quantities of the land use zoning designation are converted to estimates of trip production and attraction for each TAZ (see Figure 7) using the trip generation rates published by the Institute of Transportation Engineers (ITE, 1997). The numbers of production and attraction estimated from each corresponding PFE-estimated O-D trip table can be visualized in Figure 8 along with the estimated link flow patterns. We discover that the trip production and attraction patterns estimated by PFE, when compared with the ITE estimates, represent a reasonable estimation of trip-making propensities in the city. The TAZs in the central business district have higher productions and attractions. In addition, the productions and attractions for most of the medium to low density residential zones in the outskirts of the town also appear to be reasonable.

Figures 9 and 10 show the desire lines of the estimated O-D trip tables. Figure 9 shows all desire lines (i.e., from all origins to all destinations) that has flow greater than 20 vehicles per hour. It can be seen that as the number of traffic counting locations increases from 106 to 186 more flows can be captured in the estimation. Flows going in and out of the residential zones in the outer skirt of town are finally captured in Plan 186. On the other hand, with just 106 counting locations, Plan 106TCL can spread flow capturing to a wider coverage than Plan 106. Figure 10 displays the desire lines of a selected set of origins and destinations with the intention of comparing the O-D flow patterns in more details. Figure 10 shows that in Plan 106 the two circled residential zones do not have any flows greater than 5 vehicles. Then, one of them is estimated with flow in Plan 146. Eventually, both zones have flows in Plan 186. Despite a wider

coverage of flow capturing, the flow pattern of Plan 106TCL is not adequately consistent with the actual travel pattern in the City of St. Helena.

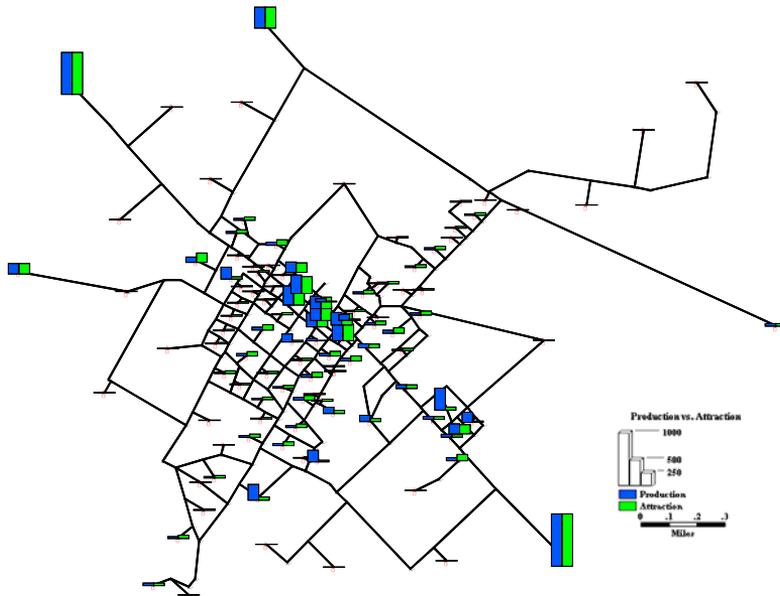


FIGURE 7: Trip production and trip attraction from ITE

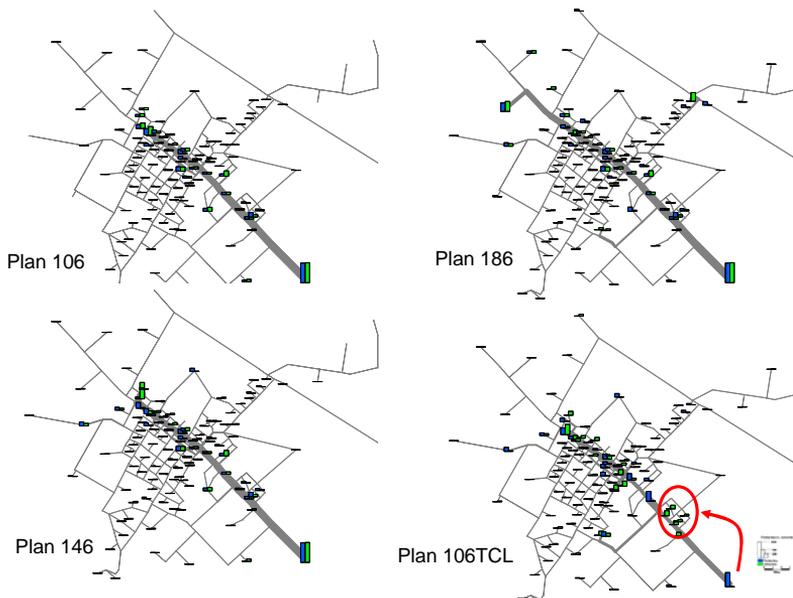


FIGURE 8: Estimated link flow and estimated trip production and trip attraction

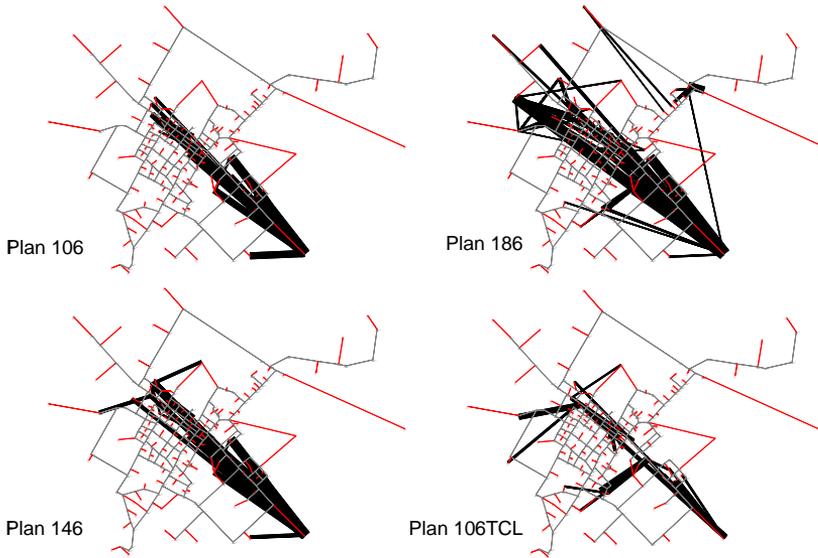


FIGURE 9: Desire line analysis of all zones (O-D flows > 20 vph)

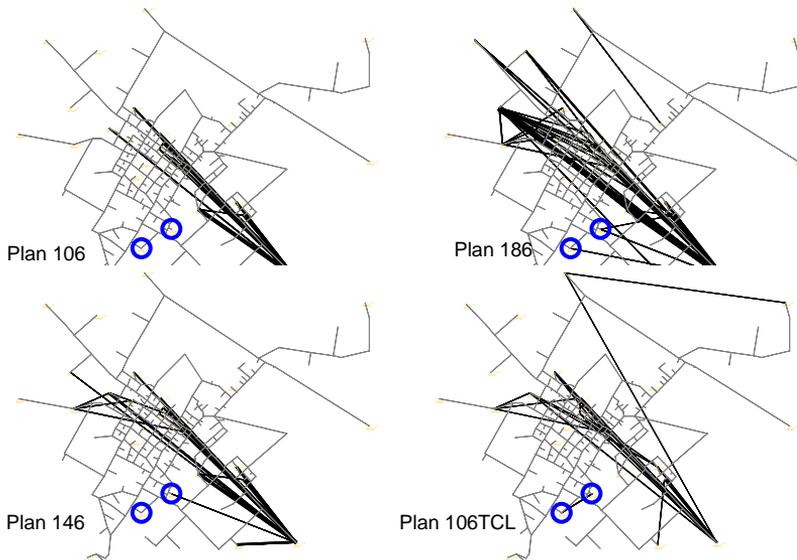


FIGURE 10: Desire line analysis of selected zones (O-D flows > 5 vph)

## 5. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we develop strategies for selecting additional traffic counts in the screen-line-based TCL model formulated as integer programming formulations. To solve these combinatorial problems which are NP-hard, a genetic algorithm embedded with a shortest path algorithm is developed. The advantage is that it obviates the need to enumerate paths when solving the integer programs and is suitable for large-scale networks. We illustrate the impact of traffic counts on O-D estimation by setting up a

unique experiment in a real world setting under a GIS environment to visually observe the evolution of O-D estimation as the number of traffic counting locations increases. The visualization indicates that, as the number of counting locations increases, the resultant O-D can be more reasonable in that more zones are observed and estimated with trip interchanges. The study also shows that when links on major corridors are used in the estimation, the heavy traffic on the corridor can help “stabilize” the estimation such that important trip producers and attractors are observed. The same principle may also be applied to corridor gateways. By including traffic counts on the gateway links, the chance for a correct estimation of the internal-to-external trip interchange may be improved.

The study suggests that a potentially better strategy for selecting traffic counting locations for practical O-D estimation could be based on two principles: 1) make sure critical links such as major corridors and gateway links are sufficiently covered, and 2) links not covered in the major corridors should be strategically included to improve the overall reasonableness of the resultant O-D. Since high quality, up-to-date traffic counts on major roadways are readily available from modern traffic surveillance systems with nominal cost, the practical problem of selecting traffic counting locations essentially reduces to the strategies of how to select additional counts to supplement the major roadway data. We suggest that solutions from a screen-line-based TCL model can provide some useful insights. In addition, land use designation and population and employment density maps may also provide useful information as to where zones with great potential for trip interchange are located.

Currently, tools for O-D estimation are often products of academic research that do not necessarily have sufficient visualization capability. For those commercial products, the limitation is usually in the lack of customized visualization tools. Our experience indicates that visualization with reference to land use maps or prior OD tables (if available) can be a powerful means for assessing the quality of the estimation O-D tables. There is a need to develop a software application that integrates an O-D estimator with customized visualization in GIS. Such an application can expand the capability of the O-D estimator and potentially increase the quality of the estimated O-D tables by an iterative process that uses visualization to identify problems of the current estimation and re-estimates once the problems are amended.

#### ACKNOWLEDGMENT

The work described in this paper was partially supported by a research project from the California Partners for Advanced Transit and Highways (PATH) Program (TO 5502). The contents of this paper reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein and do not necessarily reflect the views of our sponsors.

#### REFERENCES

- Ahuja, R.K., Magnanti, T.L. and Orlin, J.B. (1993) *Network Flows: Theory, Algorithms and Applications*, Prentice Hall, Englewood Cliffs, New Jersey.
- Ashok, K. and Ben-Akiva, M.E. (1993) Dynamic origin-destination matrix estimation and prediction for real-time management systems. Proceedings of the 12th International Symposium on Transportation and Traffic Theory, Berkeley, California.

- Bell, M.G.H., Shield, C.M., Busch, F., and Kruse, G. (1997) A stochastic user equilibrium path flow estimator. *Transportation Research Part C*, 5, 197-210.
- Bell, M.G.H. (1984) The estimation of junction turning volumes from traffic counts: the role of prior information. *Traffic Engineering and Control*, 25, 279-283.
- Bell, M.G.H. and Iida, Y. (1997) *Transportation Network Analysis*, John Wiley and Sons, Inc, New York.
- Bell, M.G.H. and Shield, C.M. (1995) A log-linear model for path flow estimation. *Proceedings of the 4th International Conference on the Applications of Advanced Technologies in Transportation Engineering*, Carpi, Italy, pp. 695-699.
- Bertsekas, D.P. (1998) *Network Optimization: Continuous and Discrete Models*, Athena Scientific, Belmont, Massachusetts.
- Bianco, L., Confessore, G. and Reverberi, P. (2001) A network based model for traffic sensor location with implications on O/D matrix estimates. *Transportation Science*, 35, 50-60.
- Cascetta, E. (1984) Estimation of origin-destination matrices from traffic counts and survey data: a generalized least squares estimator. *Transportation Research Part B*, 18, 289-299.
- Ceylan, H. and Bell, M.G.H. (2004) Sensitivity analysis on stochastic equilibrium transportation networks using genetic algorithm. *Journal of Advanced Transportation*, 38, 291-321.
- Chvatal, V. (1983) *Linear Programming*, W.H. Freeman and Company, New York.
- Chen, A. and Chootinan, P. (2007) Handling inconsistency of traffic counts in path flow estimator. *Proceedings of the 6th TRISTAN Conference*, Phuket, Thailand.
- Chen, A., Chootinan, P., and Recker, W.W. (2005) Examining the quality of synthetic origin-destination trip table estimated by path flow estimator. *Journal of Transportation Engineering*, 131, 506-513.
- Chootinan, P., Chen, A. and Recker, W. (2004) Handling the inconsistency issue of traffic counts in path flow estimator. In Hai Yang and Hong K. Lo (eds.) *Proceedings of the 9th Hong Kong Society of Transportation Studies Conference: Transportmetrica: Advanced Methods for Transportation Studies*, Hong Kong, P.R. China, pp. 272-281.
- Chootinan, P., Chen, A., and Recker, W. (2005a) Improved path flow estimator for estimating origin-destination trip tables. *Transportation Research Record*, 1923, 9-17.
- Chootinan, P., Chen, A., and Yang, H. (2005b) A bi-objective traffic counting location problem for origin-destination trip table estimation. *Transportmetrica*, 1, 65-80.
- City of St. Helena. (1993) *General Plan*, St. Helena, California.
- Ehlert, A., Bell, M.G.H. and Grosso, S. (2006) The optimisation of traffic count locations in road networks. *Transportation Research Part B*, 40, 460-479.
- Fisk, C. (1988) On combining maximum entropy trip matrix estimation with user equilibrium. *Transportation Research Part B*, 22, 69-73.
- Garber, N.J. and Hoel, L.A. (1999) *Traffic and Highway Engineering (Second Edition)*, PWS Publishing Company, Boston, Massachusetts.
- Gan, L., Yang, H. and Wong, S.C. (2005) Traffic counting location and error bound in origin-destination matrix estimation problems. *Journal of Transportation Engineering*, 131, 524-534.
- Gen, M., and Cheng, R. (2000) *Genetic Algorithms and Engineering Optimization*, John Wiley and Sons, Inc., New York.
- Goldberg, D. (1989) *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading, MA.

- Hazelton, M.L. (2000) Estimation of origin-destination matrices from link flows on uncongested networks. *Transportation Research Part B*, 34, 549-566.
- ITE (1997) *Trip Generation*, Sixth Edition, Institute of Transportation Engineers, Washington D.C.
- Lam, W.H.K. and Lo, H.P. (1990) Accuracy of O-D estimates from traffic counts. *Traffic Engineering and Control*, 31, 358-367.
- Maher, M. (1983) Inferences on trip matrices from observations on link volumes: a Bayesian statistical approach. *Transportation Research Part B*, 17, 435-447.
- Maher, M., Zhang, X., and Van Vliet, D. (2001) A bi-level programming approach for trip matrix estimation and traffic control problems with stochastic user equilibrium link flows. *Transportation Research Part B*, 35, 23-40.
- Megiddo, N., Zemel, E. and Hakimi, S. (1983) The maximum coverage location problem. *SIAM Journal on Algebraic and Discrete Methods*, 4, 253-261.
- Meng, Q., Lee, D.H. and Cheu, R.L. (2005) Counting the different efficient paths for transportation networks and its applications. *Journal of Advanced Transportation*, 39, 193-220.
- Patriksson, M. (1994) *The Traffic Assignment Problem-Models and Methods*, VSP, Utrecht, The Netherlands.
- Sherali, H.D., Sivanandan, R., and Hobeika, A.G. (1994) A linear programming approach for synthesizing origin-destination trip tables from link traffic volumes. *Transportation Research Part B*, 28, 213-234.
- Spieß, H. (1987) A maximum likelihood model for estimating origin-destination matrices. *Transportation Research Part B*, 21, 395-412.
- Van Zuylen, H.J. and Willumsen, L.G. (1980) The most likely trip estimated from traffic counts. *Transportation Research Part B*, 14, 281-293.
- Yang, C., Chootinan, P. and Chen, A. (2003) Traffic counting location planning using genetic algorithms. *Journal of Eastern Asia Society for Transportation Studies*, 5, 898-913.
- Yang, H. (1995) Heuristic algorithms for the bi-level origin-destination matrix estimation algorithm. *Transportation Research Part B*, 29, 231-242.
- Yang, H. and Zhou, J. (1998) Optimal traffic counting locations for origin-destination matrix estimation. *Transportation Research Part B*, 32, 109-126.
- Yang, H., Iida, Y. and Sasaki, T. (1991) An analysis of the reliability of an origin-destination trip matrix estimated from traffic counts. *Transportation Research Part B*, 25, 351-363.
- Yang, H., Iida, Y., and Sasaki, T. (1992) Estimation of origin-destination matrices from traffic counts on congested networks. *Transportation Research Part B*, 26, 417-434.
- Yang, H., Gan, L. and Tang, W.H. (2001) Determining cordons and screen lines for origin-destination trip studies. *Proceedings of the 3rd Eastern Asia Society for Transportation Studies*, Hanoi, Vietnam, pp. 85-99.
- Yang, H., Yang, C., and Gan, L. (2005) Models and algorithms for the screen-line-based traffic counting location problem. *Computers and Operations Research*, 33, 836-858.
- Yim, P.K.N. and Lam, W.H.K. (1998) Evaluation of count location selection methods for estimation of O-D matrices. *Journal of Transportation Engineering*, 124, 376-383.