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On optimal freeway ramp control policies for congested traffic corridors

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Abstract

This paper examines the conditions for which ramp metering can be beneficial to the overall system in terms of travel time savings for a simple traffic corridor that consists of a freeway and a set of parallel arterials connected by entrance ramps. The focus is on analyzing state and control relationships to arrive at general analytical results regarding optimal metering policies, rather than on either developing specific control algorithms or solving a specific application. The analysis is concerned with the general behavior of the system under ramp control and traffic diversion. The analysis assumes that time-varying traffic demands that originate from various locations are destined for a single location and that the freeway is uniformly congested throughout the control period. Under these assumptions, some general results are obtained regarding the effectiveness of ramp metering for various traffic diversion propensities and differentials between freeway and arterial traffic conditions. It is shown that the optimal ramp control policies are determined by the magnitudes of two co-state vectors that depend on traffic conditions on the freeway and its alternative, and the drivers diversion propensity. The results of two limiting cases imply that when the freeway is uniformly congested ramp control is counter-productive unless diversion occurs, and where drivers have a high propensity to divert, the optimal policy is dependent on the travel speed on the freeway alternative and on the wave speed of backward propagating waves on the freeway. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Corridor control; Ramp metering

1. Background

Ramp metering has gained general acceptance as an essential part of modern traffic congestion management systems in either preventing or reducing freeway congestion. Ramp metering is effective if it can be used to ensure that the traffic volume delivered downstream of the ramp does

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Nomenclature

Summarized below is the notation used in the analysis:

i:	road section number
k:	time step number
K:	time step horizon
t:	denotes vector or matrix transpose when used as a superscript
T:	size of each time increment
TT:	total system travel time
c_1:	wave speed of backward-propagating waves on the freeway
c_2:	wave speed of backward-propagating waves on the freeway alternative
d_i^k:	demand at ramp i during time interval k
l_f:	number of lanes of the freeway
l_a:	the number of lanes of the freeway alternative
P_k:	control gradient
$\hat{q}_0^k$:	inflow rate from the upper boundary of the freeway alternative during time interval k
q_0^k:	inflow rate from the upper boundary of the freeway during time interval k
$\hat{q}_{i-1/2}^k$:	traffic influx to section i of the freeway alternative during time interval k
$q_{i-1/2}^k$:	traffic influx to section i of the freeway during time interval k
r_i^k:	ramp metering rate of ramp i during time interval k
T_a:	total time spent on freeway alternative
T_f:	total time spent on freeway
T_q:	total time spent in ramp queues
V_1:	free flow speed on the freeway
V_2:	free flow speed on the freeway alternative
β:	ramp queue penalty function
Δ:	length of each road section
δd_i^k:	portion of the demand d_i^k diverted from ramp i during time interval k
ρ_i^k:	density of section i of the freeway during time interval k
$\rho_{\text{jam}}, \rho_c$:	jam and critical densities, respectively, of a section of the freeway
ξ_i^k:	density of section i of the freeway alternative during time interval k
ξ_{jam}, ξ_c:	jam and critical densities, respectively, of a section of the freeway alternative
ζ_i^k:	number of vehicles waiting at ramp i during time interval k
ζ_m:	maximum allowable queue length for ramps
ψ:	ratio of freeway backward wave speed to free flow freeway alternative speed
I_d:	an identity matrix of proper dimensions
$A_{yz}^k, y, z = \rho, \xi, \zeta$:	various state Jacobians
$B_{\rho r}^k, B_{\xi r}^k, B_{\zeta r}^k$:	various control Jacobians
$u_k = r^k$:	matrix of control variables
$1^t = [1, 1, \ldots, 1]$:	unit vector of appropriate dimension

$\rho^k = \left[\rho_1^k, \ldots, \rho_n^k\right]^t$:	transpose of freeway section parameter density matrix
$\xi^k = \left[\xi_1^k, \ldots, \xi_n^k\right]^t$:	transpose of freeway alternative section parameter density matrix
$\lambda_k = \left[\lambda_\rho^k, \lambda_\xi^k, \lambda_\zeta^k\right]$:	co-state vector
$\mu_k = \left[\mu_\rho^k, \mu_\xi^k, \mu_\zeta^k\right]$:	modified co-state vector
$\zeta^k = \left[\zeta_1^k, \ldots, \zeta_n^k\right]^t$:	transpose of section parameter ramp queue matrix
$r^k = \left[r_1^k, \ldots, r_n^k\right]^t$:	transpose of ramp metering matrix
$x_k = \left[\left(\rho^k\right)^t, \left(\xi^k\right)^t, \left(\zeta^k\right)^t\right]^t$:	transpose of state variable matrix
$w_k = \left[\hat{q}_0^k, q_0^k, d_i^k, \ldots, d_n^k\right]^t$:	transpose of disturbance matrix
$\dot{h}(x) = \dfrac{\mathrm{d}h}{\mathrm{d}x}(x)$	
$F_u = \dfrac{\partial F}{\partial u}$	

not create a bottleneck situation in which upstream demand exceeds the downstream capacity. Under conditions of heavy, prolonged demand on the freeway facility, the principal mechanism by which ramp metering can improve freeway operation is by encouraging diversion of freeway-bound traffic to alternate routes. However, since the freeway is part of a larger traffic system, the result of such diversion may simply displace congestion effects from the freeway to the surface streets unless the alternate routes have sufficient capacity available to carry the diverted traffic (Newman, 1969). And, while this may be beneficial for freeway operations, it may likewise be detrimental to the operation of the overall traffic system.

Awareness of the potential for ramp metering to have negative impacts on the overall traffic system has led to efforts to study how ramp metering affects system performance, and to develop integrated control strategies that attempt to improve the overall performance of a corridor by combining ramp metering with intersection signal control and route guidance (e.g. Chang, Ho and Wei, 1993; Stephanedes and Chang, 1993; Yang and Yagar, 1994; Lam and Huang, 1995; Papargorgiou, 1995). Using simulation, Nsour et al. (1992) studied the effects of metering and traffic diversions on a system's performance for a 7-mile long corridor comprising a freeway, two parallel arterials and seven connecting arterials. They found that while restrictive ramp metering significantly improved freeway flow it adversely affected the overall system performance because overflowing queues behind meters blocked street traffic, creating a severe disturbance on feeder streets. Less restrictive ramp metering was reported to be not sufficient to bring the congested freeway to its normal condition during the simulated time period. A more recent field study by Haj-Salem and Papageorgiou (1995) draws a somewhat more favorable conclusion regarding the effectiveness of ramp metering in a corridor setting. Using a local ramp metering algorithm called ALINEA (Papageorgiou et al., 1991) to control a motorway in Paris, they estimated such performance measures as Total Time Spent (TTS) from field-measured data for a period of about six weeks. It was reported that TTS had an average improvement of 8.1% for the freeway, 6.9% for parallel arterials, and −17.8% for radial streets. Although there was a significant performance degradation on radial streets, the benefit 'gain' on freeways and parallel arterials offset the benefit 'loss' on radial streets in this case, and an overall system improvement of about 5.9% was reported. Both of the aforementioned studies employed only a limited number of control tactics. A study by Pooran et al. (1994) tested the effectiveness of a variety of control tactics and their combinations on a simulated corridor. They considered four levels of ramp metering and 16 arterial control tactics for various traffic conditions. The number of case combinations was found to be overwhelming and, consequently, only a few major cases were tested against the "no control" option. Daganzo and Lin (1994), in a study notable as one of the very few that has uncovered general qualitative results on the relationship between metering effectiveness and traffic conditions, showed that ramp metering did not reduce total travel time spent by the system users for a morning rush hour along a traffic corridor leading to the central business district (CBD).

Despite the results of the specific studies noted above, as well as others, general definitive quantitative relationships between the potential effectiveness of ramp metering in a corridor setting and some measure of traffic conditions have remained elusive. An approach that has the potential to offer some enlightenment on the general principles underlying effective ramp control policies is optimal control theory. This theory has been applied formally to the ramp control problem (Papageorgiou, 1983; Papageorgiou et al., 1990,1991; Stephanedes and Chang, 1993), and its rudiments have formed the basis of a number of ramp control algorithms (Payne et al., 1973; Goldstein and Kumar, 1982; Zhang et al., 1994). The advantage of optimal control formulations lies in formal declaration of state and control relationships, rather than relying on simulation and/or simple heuristics as a basis for defining the system. Although the optimal control formulations noted above have this characteristic, their applications have precluded analytical generalizations, mostly because of the complexity of either their formulation or the application itself; most often, formulation being coupled directly to solution heuristics with little or no meaningful analysis of the general structural properties of the system possible.

In a previous effort (Zhang et al., 1996), the authors exploited the advantages offered by optimal control theory to obtain some general theoretical results on the efficacy of ramp metering for loosely-defined regimes of traffic flow under some rather restrictive assumptions. That analysis considered a system comprised of a freeway section and its entry/exit ramps only, and formulated the ramp control problem as a dynamic optimal process to minimize the total time spent in the system. Under the assumption that the controlled freeway had to serve all of its demand (i.e., no ramp diversion), and the traffic flow process follows the rules prescribed by the LWR theory (Lighthill and Whitham, 1955; Richards, 1956), it was shown that when traffic is either uniformly congested or uniformly uncongested, ramp metering leads to inferior solutions to the problem; its effectiveness in reducing total time spent in the system restricted to traffic conditions that have the potential to switch between congested and non congested situations.

This paper extends the results of Zhang et al. (1996) to a simple road corridor, such as that leading to a CBD, comprising a freeway, a set of parallel arterials that function as a freeway alternative and the associated connecting streets. As with the previous paper, the focus here is on analyzing the state and control relationships to arrive at general analytical results regarding optimal metering policies, rather than on either developing specific control algorithms or solving a specific application. Our approach is to simplify the problem in a manner that will allow the development of general conclusions in terms of the fundamental parameters of the system. A few assumptions are therefore made to make the problem more tractable and the results clearer. Specifically, we assume that:

- time-varying traffic demands that originate from various locations are destined for a single location—the CBD (hereafter referred to as "many-to-one" demand, see Fig. 1),
- the freeway is *uniformly congested*[1] throughout the control period (because of a downstream bottleneck),[2] but the set of parallel arterial streets is not, and
- traffic diverted from entering the freeway at the ramps does not rejoin the freeway.[3]

Under these assumptions, we obtain some general results regarding the effectiveness of ramp metering for various traffic diversion propensities and differentials between freeway and arterial traffic conditions.

The remainder of this paper is organized as follows: we first describe the dynamic equations that govern the behavior of the corridor traffic system, then formulate the problem as a dynamic optimization process. Next we present the results of our analysis and discuss the implications of these results.

2. Traffic dynamics

The traffic dynamics of the simple corridor[4] considered herein (Fig. 1) consists of three parts—freeway traffic dynamics, freeway alternative traffic dynamics and queue dynamics on the ramps that link the freeway alternative to the freeway. We assume that the traffic behavior of both

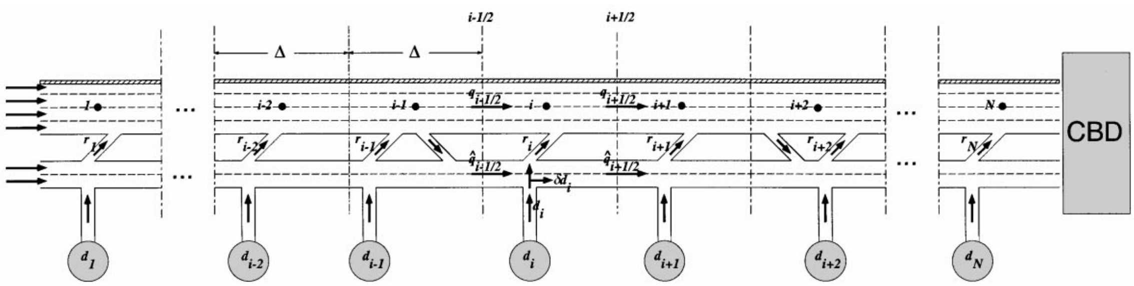

Fig. 1. Linear traffic corridor and discretization scheme.

[1] Traffic is said to be uniformly congested if traffic density is greater than critical density for all road segments in the entire control period.

[2] The case in which transition occurs from congested to congestion-free conditions was addressed by Zhang et al. (1996).

[3] This assumption is not overly restrictive since, under conditions of uniform congestion on the freeway, drivers would not gain advantage by rejoining the freeway at a downstream location. See discussions in Danganzo and Lin (1994). A route choice survey conducted by Stephanedes et al. (1989) also provides some empirical evidence supporting this assumption.

[4] The configuration of the corridor considered here is similar to that treated in Newell (1977).

freeway and freeway alternative traffic follows the rules prescribed by the well-known Lighthill-Witham-Richard (LWR) theory (Lighthill and Whitham, 1955; Richards, 1956), i.e.

$$\partial q/\partial x + \partial \rho/\partial t = 0$$
$$q = \rho \cdot u(\rho) = f(\rho)$$

where x denotes space and t time; q, u and ρ are flow rate per lane, space mean speed, and traffic density (in vehicles per lane per distance unit), respectively; and where $u(\rho)$ is defined by some empirical speed-density relationship that gives rise to an implicit flow-density relationship . Although there are many forms that the flow-density relationship may take, we assume in this exposition the triangular flow-density relationship proposed by Newell (1991):

$$q = V\rho, \qquad \text{if } \rho \leqslant \rho_c$$
$$q = c(\rho_{jam} - \rho), \qquad \text{if } \rho \geqslant \rho_c$$

where V is the free flow travel speed, c is the wave speed traveling in a direction against the traffic stream, ρ_c is the critical density, and ρ_{jam} is the jam density. For purposes of obtaining the general theoretical qualitative results that are the focus of this paper, we further assume that the freeway and freeway alternative have the same jam and critical densities (Fig. 2), but different capacities. The effect of this simplifying assumption is to ignore the details of traffic control on the freeway alternative (e.g. signal control on arterial streets) and, rather, assume that the traffic flow conditions on the freeway alternative can be represented adequately by uninterrupted flow with these equivalent properties. This assumption is not overly restrictive and allows us to interpret results subject to only a few parameters.

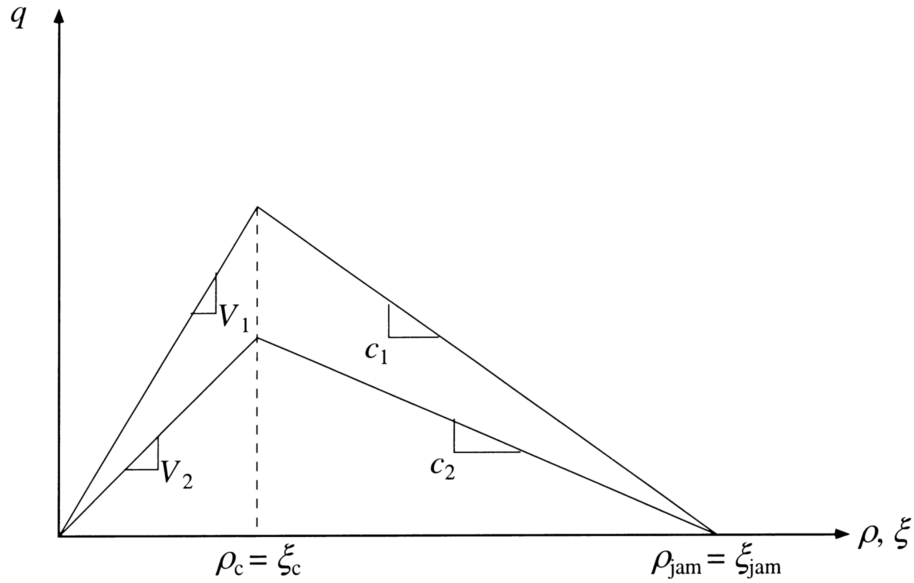

Fig. 2. The triangular fundamental diagrams.

2.1. Freeway traffic dynamics

For the purpose of exposition, we assume that the freeway segment is comprised of an arbitrary number of sections of equal length, Δ, and equal number of lanes l_f, and that each section has at most one entry ramp. (No similar assumption regarding the number of exit ramps is necessary under the problem statement.) To facilitate analysis, we convert the LWR formulation into difference equations, adding sources to represent entry ramps. The discretization scheme adopted is shown in Fig. 1, where: ρ_i^k, q_i^k and u_i^k are respectively the section density, flow, and space mean speed of section i during time step k;r_i^k is the entry rate from any ramp located in section i during time step k, and where the notation $i-1/2$ and $i+1/2$ is used to designate the upstream and downstream boundaries of section i, respectively.

The nature of the discontinuities associated with the LWR traffic flow law requires the use of difference techniques that guarantee entropy solutions (Ansorge, 1990). We here adopt the numerical scheme due to Murman (1974) that is known to produce entropy satisfying solutions to the LWR formulation. Then, in difference form, the equations governing the traffic dynamics of the freeway can be represented by:

$$\rho_i^{k+1} = \rho_i^k + (T/\Delta)\left(q_{i-1/2}^k - q_{i+1/2}^k + r_i^k/l_f\right) \tag{1.1}$$

$$q_{i+1/2}^k = \frac{1}{2}\left[f_1\left(\rho_{i+1}^k\right) + f_1\left(\rho_i^k\right) - |\mu_1|_{i+1/2}^k\left(\rho_{i+1}^k - \rho_i^k\right)\right] \tag{1.2}$$

$$\begin{aligned} |\mu_1|_{i+1/2}^k &= \left|\frac{f_1(\rho_{i+1}^k) - f_1(\rho_i^k)}{\rho_{i+1}^k - \rho_i^k}\right|, \qquad \text{if } \rho_{i+1}^k \neq \rho_i^k \\ |\mu_1|_{i+1/2}^k &= |f_1'(\rho_i^k)|, \qquad \text{if } \rho_{i+1}^k = \rho_i^k \end{aligned} \tag{1.3}$$

$$\begin{aligned} f_1(\rho) &= V_1\rho, \qquad \text{if } \rho \leqslant \rho_c \\ f_1(\rho) &= c_1(\rho_{jam} - \rho), \qquad \text{if } \rho_{jam} \geqslant \rho \geqslant \rho_c \end{aligned} \tag{1.4}$$

in which: T is the duration of each time increment, $q_{i-1/2}^k$ and $q_{i+1/2}^k$ are respectively the traffic influx and outflux of section i of the freeway during time interval k, and the remaining terms are as defined by the freeway flow density relationship in Fig. 2. The detailed derivations of these forms can be found in Leo and Pretty (1992) and are omitted in this development.

2.2. Freeway alternative traffic dynamics

As stated above, we have made the simplifying assumption that the alternative route(s) to the destination can be represented sufficiently by specifying a single freeway alternative conforming to the LWR equations with appropriate flow-density characteristics. Under this assumption, and using notation similar to that used in developing Eqs. (1) above, the traffic dynamics for the freeway alternative can be represented in discrete form by:

$$\xi^{k+1} = \xi_i^k + (T/\Delta)\left(\hat{q}_{i-1/2}^k - \hat{q}_{i+1/2}^k + \delta d_i^k/l_a\right) \tag{2.1}$$

$$\hat{q}^k_{i+1/2} = \frac{1}{2}\left[f_2(\xi^k_{i+1}) + f_2(\xi^k_i) - |\mu_2|^k_{i+1/2}(\xi^k_{i+1} - \xi^k_i)\right] \tag{2.2}$$

$$|\mu_2|^k_{i+1/2} = |\frac{f_2(\xi^k_{i+1}) - f_2(\xi^k_i)}{\rho^k_{i+1} - \rho^k_i}|, \qquad \text{if } \xi^k_{i+1} \neq \xi^k_i$$
$$|\mu_2|^k_{i+1/2} = |f'_2(\xi^k_i)|, \qquad \text{if } \xi^k_{i+1} = \xi^k_i \tag{2.3}$$

$$f_2(\xi) = V_2\xi, \qquad \text{if } \xi \leqslant \xi_c$$
$$f_2(\xi) = c_2(\xi_{\text{jam}} - \xi), \qquad \text{if } \xi_{\text{jam}} \leqslant \xi \leqslant \xi_c \tag{2.4}$$

in which ξ^k_i is the density of section i of the freeway alternative during time interval k, δd^k_i is the portion of the demand d^k_i diverted from ramp i during time interval k, $\hat{q}^k_{i-1/2}$ and $\hat{q}^k_{i+1/2}$ are respectively the traffic influx and outflux of section i of the freeway alternative during time interval k, l_a is the effective number of lanes of the freeway alternative, and the remaining terms are as defined by the freeway alternative flow density relationship shown in Fig. 2. (We note that, by assumption, $\xi_{jam} = \rho_{jam}$ and $\xi_c = \rho_c$; to facilitate clarity, this substitution has not been made here.)

2.3. *Ramp traffic dynamics*

Traffic dynamics for ramps is influenced by two factors—drivers' diversion behavior, and queuing. When queues grow on metered ramps, drivers are more likely to bypass these ramps in favor of an alternate route, usually a surface arterial, that is perceived to be faster; this, of course, explicitly is the intended effect of the metering. Such diversion behavior is inherently complex and there is little experimental data or theory to support definitively any specific diversion model; a comprehensive review of models that have been used and their limitations is provided in Stephanedes et al. (1989). It appears reasonable, however, to assume that, in the absence of real-time traffic information, a driver's propensity to divert from a metered ramp is a function of the driver's estimate of the additional travel time due to queuing on the ramp (which, under conditions of a constant metering rate is proportional to the observed queue length), and some expectation of the travel time on the alternate route that is based primarily on experience (rather than on current real-time traffic information). A similar behavioral assumption, used by Stephanedes and Kwon (1993) as an element of their adaptive demand-diversion prediction model, has been shown to correspond with observed field data. Further, we assume an ideal system where, in the absence of any queue on the ramp, traffic originally destined for the ramp will use the ramp; and, any traffic arriving at the ramp upon the queue on the ramp reaching its maximum storage capacity, will divert to the freeway alternative.[5] As a mathematical simplification (and in the absence of any supported rationale for assuming otherwise), we assume that the variation between these two extremes of the portion of diverted vehicles from a ramp is linear; i.e. we assume that the portion of the demand d^k_i diverted from ramp i in time interval k, δd^k_i, is given by:

[5] Under the assumed condition of constant metering rate, and in the absence of upstream queue detectors, the metering rate would, *de facto*, be set to achieve this ideal. We note that, in any event, this assumption is in no way restrictive, since the value of ζ_m can always be adjusted to achieve this desired result.

$$\delta d_i^k = h(\zeta_i^k) d_i^k \quad (3.1)$$

in which

$$h(\zeta_i^k) = \zeta_i^k / \zeta_m, 0 \leqslant \zeta_i^k \leqslant \zeta_m$$

and where ζ_i^k and ζ_m are the number of vehicles waiting at ramp i during time interval k and the maximum allowable queue length for ramps, respectively. (There is at least weak empirical evidence that the transition of the probability of such diversion to its maximum value of unity may be approximated by a somewhat linear function of perceived travel time savings; see Ullman et al., 1994)

The second input to ramp traffic dynamics is queuing, which can be described by the following difference equation:

$$\zeta_i^{k+1} = \zeta_i^k + T(d_i^k - r_i^k - \delta d_i^k) \quad (3.2)$$

Finally, we impose the condition that the ramp metering rates of the controlled ramp cannot exceed their virtual demand, or be negative,

$$0 \leqslant r_i^k \leqslant d_i^k - \delta d_i^k + \zeta_i^k / T \quad (3.3)$$

Eqs. (1)–(3) complete the governing equations of corridor traffic dynamics. Next, we select the control objective and formulate the control problem for this corridor.

3. Formulation of the optimal ramp control problem

Although there are a number of performance measures by which optimal control for this problem may be specified (e.g. travel speed, travel delay, and throughput), we here consider total system travel time, TT, plus some penalty for having vehicles still queued on ramps at the end of the control period, as the performance measure. The total system travel time is the summation of the total time spent in each subsystem, i.e.

$$TT = T_f + T_a + T_q \quad (4)$$

where:

$$T_f = T \sum_{k=1}^{K} \Delta \cdot l_f \cdot 1^t \cdot \rho^k \quad (5.1)$$

is the total time spent on the freeway,

$$T_a = T \sum_{k=1}^{K} \Delta \cdot l_a \cdot 1^t \cdot \zeta^k \quad (5.2)$$

is the total time spent on the freeway alternative, and

$$T_q = T \sum_{k=1}^{K} 1^t \cdot \zeta^k \quad (5.3)$$

is the total time spent in ramp queues, and where $\rho^k = [\rho_1^k, \dots, \rho_n^k]^t$, $\xi^k = [\zeta_1^k, \dots, \xi_n^k]^t$, and $\zeta^k = [\zeta_i^k, \dots, \zeta_n^k]^t$ are the transposes of the respective matrices of section parameters. Collecting terms, we write

$$TT = T\sum_{k=1}^{K} L^k(x_k, u_k)$$

in which $L^k(x_k, u_k) = 1^t.(\Delta \cdot l_f \cdot \rho^k + \Delta \cdot l_a \cdot \xi^k + \zeta^k)$, and $x_k = \left[(\rho^k)^t, (\xi^k)^t, (\zeta^k)^t\right]^t$ and $u_k = [r_1^k, \dots, r_n^k]^t$ are the transposes of the state and ramp control vectors, respectively.

We select as the residual ramp queue penalty function the simple linear relation $\tilde{\beta} \cdot \zeta^k$, $\tilde{\beta} \geqslant 0$. Under these conditions, it is easily seen that the objective function corresponding to the performance measure stated above can be written as

$$\text{minimize} \qquad J = \phi(x_k) + \sum_{k=1}^{K-1} L^k(x_k, u_k) \tag{6.1}$$

where

$$\phi(x_k) = 1^t.(\Delta.l_f.\rho^k + \Delta.l_a.\xi^k + \beta.\zeta^k), \beta = 1 + \tilde{\beta} \geqslant 1$$

Using the recursive difference expressions defined by Eqs. (1.1), (2.1) and (3.2), and defining a disturbance vector, W_k, comprising the upstream boundary inflow rates of both the freeway and its alternative, together with the vector of ramp demands, i.e.

$$w_k = [\hat{q}_0^k, q_0^k, d_1^k, \dots, d_n^k]^t$$

where $\hat{q}_0^k$ and q_0^k respectively are the inflow rates from the upper boundary of freeway and its alternative during time interval k, the corridor traffic dynamics can be written in a compact vector form:

$$x_{k+1} = F^k(x_k, u_k, w_k) \tag{6.2}$$

where F^k is a vector-valued function whose elements comprise the RHS of Eqs. (1.1), (2.1) and (3.2) and where u_k satisfies the constraint

$$0 \leqslant u_k \leqslant c(x_k, w_k) = h(w_k) + \zeta^k / T \tag{6.3}$$

Eqs. (6) complete the transformation of the corridor control problem into a standard optimal control problem. Moreover, Eqs. (6.1) and (6.2) are linear and the convexity of the constraint given by Eq. (6.3) ensures that the optimal policy obtained for this problem is global.

4. Optimal control policies

The optimal control policy for a linear optimal control problem with box constraints is:

$$\begin{aligned} u_k^* &= 0, \qquad \text{if } p_k > 0 \\ u_k^* &= c(x_k, w_k), \qquad \text{if } p_k < 0 \end{aligned} \tag{7}$$

where p_k is the gradient of the objective function with respect to controls, i.e.

$$p_k = L_u^k + (\lambda_{k+1})^t F_u^k \tag{8.1}$$

Note that $\lambda_k = [\lambda_\rho^k, \lambda_\xi^k, \lambda_\zeta^k] \in R^{3n}$, with $\lambda_\rho^k \in R^n$, $\lambda_\xi^k \in R^n$, $\lambda_\zeta^k \in R^n$, is called a co-state vector and is determined by the following recursive formula:

$$\lambda_k = L_x^k + \lambda_{k+1} F_x^k, \lambda_K = \phi_x(x_K) \tag{8.2}$$

We now derive explicit expressions for the co-state (row) vector and control gradient. The first step in this derivation is to obtain formulas for Jacobian matrices F_x^k and F_u^k, where F_x^k is the Jacobian of corridor traffic dynamics with respect to state variables (i.e. freeway and freeway alternative traffic densities and ramp queue lengths) and is F_u^k the Jacobian of corridor traffic dynamics with respect to controls (i.e. ramp metering rates). To simplify notation, we define $A^k \equiv F_x^k$, $B^k \equiv F_u^k$, and write

$$[A^k|B^k] = \left[\begin{array}{ccc|c} A_{\rho\rho}^k & A_{\rho\xi}^k & A_{\rho\zeta}^k & B_{\rho r}^k \\ A_{\xi\rho}^k & A_{\xi\xi}^k & A_{\xi\zeta}^k & B_{\xi r}^k \\ A_{\zeta\rho}^k & A_{\zeta\xi}^k & A_{\zeta\zeta}^k & B_{\zeta r}^k \end{array}\right] \tag{9}$$

The first row of $[A^k|B^k]$ is derived from freeway dynamics, the second row from freeway alternative traffic dynamics and the third row from ramp traffic dynamics. To give explicit expressions for $[A^k|B^k]$, we first define the following parameters: $\alpha_1 = (T/\Delta)c_1, \alpha_1 = (T/\Delta)V_1, \alpha_2 = (T/\Delta)c_2$, $\alpha_2 = (T/\Delta)V_2$.

The state Jacobians that form the sub-matrices of $[A^k|B^k]$ for the freeway, freeway alternative and ramps that comprise an N-section road corridor are, respectively:

$$A_{\rho\rho}^k = \begin{bmatrix} 1-\alpha_1 & \alpha_1 & 0 & \dots & 0 \\ 0 & 1-\alpha_1 & \alpha_1 & & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \alpha_1 \\ 0 & 0 & 0 & \dots & 1-\alpha_1 \end{bmatrix}, A_{\rho\xi} = 0, A_{\rho\zeta} = 0$$

$$A_{\xi\xi}^k = \begin{bmatrix} 1-\alpha_2' & 0 & \dots & 0 & 0 \\ \alpha_2' & 1-\alpha_2' & & 0 & 0 \\ 0 & \alpha_2' & \ddots & 0 & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_2' & 1-\alpha_2' \end{bmatrix} \tag{10.1}$$

$$A^k_{\xi\zeta} = \frac{T}{l_a\Delta}\begin{bmatrix} d^k_1\dot{h}(\zeta^k_1) & 0 & 0 & \dots & 0 \\ 0 & d^k_2\dot{h}(\zeta^k_2) & 0 & & 0 \\ 0 & 0 & d^k_3\dot{h}(\zeta^k_3) & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & d^k_N\dot{h}(\zeta^k_N) \end{bmatrix}, A_{\xi\rho} = 0 \tag{10.2}$$

$$A^k_{\zeta\zeta} = \begin{bmatrix} 1 - T\dot{h}(\zeta^k_1)d^k_1 & 0 & 0 & \dots & 0 \\ 0 & 1 - T\dot{h}(\zeta^k_2)d^k_2 & 0 & & 0 \\ 0 & 0 & 1 - T\dot{h}(\zeta^k_3)d & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 - T\dot{h}(\zeta^k_N)d \end{bmatrix}, A^k_{\zeta\xi} = 0, A^k_{\zeta\rho} = 0. \tag{10.3}$$

The control Jacobians for the three subsystems (freeway, freeway alternative and ramps) are, respectively:

$$B^k_{\rho r} = \frac{T}{\Delta l_f} I_d \tag{11.1}$$

$$B^k_{\xi r} = 0 \tag{11.2}$$

$$B^k_{\xi r} = -TI_d \tag{11.3}$$

Substituting Eqs. (10) and (11) into Eq. (9), we have

$$[A^k|B^k] = \left[\begin{array}{ccc|c} A^k_{\rho\rho} & 0 & 0 & B^k_{\rho r} \\ 0 & A^k_{\xi\xi} & A^k_{\xi\zeta} & 0 \\ 0 & 0 & A^k_{\zeta\zeta} & B^k_{\zeta r} \end{array}\right] \tag{12}$$

The structure of $[A^k|B^k]$ shows that the dynamics of freeway and freeway alternative, freeway and ramp queues are completely decoupled from each other; that is, the traffic density on the freeway alternative and ramp queue length do not *directly* influence freeway traffic density and vice versa. Conversely, a change in queue length does directly affect the traffic density on the freeway alternative because of drivers' diversions. Changes in any state variable (i.e. queue length, freeway alternative and freeway traffic densities), however, *indirectly* affect the others through a common link—control action (ramp metering).

Substituting Eq. (12) into Eq. (8), we obtain the following recursive formulas for the co-state vectors:

$$\lambda_\rho^k = \Delta\cdot l_f\cdot 1^t + \lambda_\rho^{k+1}\cdot A_{\rho\rho}^k, \lambda_\rho^K = \Delta\cdot l_f\cdot 1^t, \tag{13.1}$$

$$\lambda_\xi^k = \Delta\cdot l_a\cdot 1^t + \lambda_\xi^{k+1}\cdot A_{\xi\xi}^k, \lambda_\xi^K = \Delta\cdot l_a\cdot 1^t, \tag{13.2}$$

$$\lambda_\zeta^k = 1^t + \lambda_\zeta^{k+1}\cdot A_{\zeta\zeta}^k + \lambda_\xi^{k+1}\cdot A_{\xi\zeta}^k, \lambda_\zeta^K = \beta\cdot 1^t, \tag{13.3}$$

and control gradient:

$$p_k = T\left(\frac{\lambda_\rho^{k+1}}{\Delta\cdot l_f} - \lambda_\zeta^{k+1}\right) \tag{14}$$

5. Some general implications for optimal control policies

The evolution of the co-state vectors, governed by Eq. (13), present some interesting general results regarding ramp metering strategies within the context of this problem. Before proceeding further, we simplify Eqs. (13) by introducing a set of new co-state vectors: $\mu_\rho^k = \frac{\lambda_\rho^k}{\Delta l_f}$, $\mu_\xi^k = \frac{\lambda_\xi^k}{\Delta l_a}$ and $\mu_\zeta^k = \lambda_\zeta^k$. We then rewrite Eqs. (13) using the new co-state vectors:

$$\mu_\rho^k = 1^t + \mu_\rho^{k+1}\cdot A_{\rho\rho}^k, \mu_\rho^K = 1^t \tag{15.1}$$

$$\mu_\xi^k = 1^t + \mu_\xi^{k+1}\cdot A_{\xi\xi}^k, \mu_\xi^K = 1^t \tag{15.2}$$

$$\mu_\zeta^k = 1^t + \mu_\zeta^{k+1}\cdot A_{\zeta\zeta}^k + \mu_\xi^{k+1}\cdot A_{\xi\zeta}^k/\Delta\cdot l_a, \lambda_\zeta^K = \beta\cdot 1^t \tag{15.3}$$

and the control gradient:

$$p_k = T\cdot(\mu_\rho^{k+1} - \mu_\zeta^{k+1}) \tag{16}$$

Now, the sign of p_k is solely determined by two transformed co-state vectors μ_ρ^{k+1} and μ_ζ^{k+1}. Eq. 15.1 shows that change in μ_ρ^k is determined by $A_{\rho\rho}^k$. Eq. (11.1) indicates that $A_{\rho\rho}^k$ is of Jordan form, whose eigenvalues are $1-\alpha_1$, which in turn is determined by c_1, the wave speed for congested freeway traffic. Similarly, μ_ξ^k is determined by $A_{\xi\xi}^k$, whose structure is similar to $A_{\rho\rho}^k$ but has lower, rather than upper, non-zero off-diagonal elements. The eigenvalues of $A_{\xi\xi}^k$ are $1-\alpha_2'$, which is dependent on V_2, the travel (and wave) speed of freeway alternative traffic for congestion-free conditions. To ensure numerical stability for Eqs. (1) and Eq. (2), the following condition is required (Zhang et al., 1996):

$$0_{\max}\{\alpha_1, \alpha_1', \alpha_2, \alpha_2'\} \leqslant 1 \tag{17}$$

Eq. (17) guarantees that the spectra of $A_{\rho\rho}^k$ and $A_{\xi\xi}^k$ lie within the unit hypersphere; therefore, μ_ρ^k and μ_ξ^k are bounded. Note that μ_ξ^k does not contribute to the control gradient p_k directly. It affects p_k indirectly through μ_ζ^k. We observe that the interdependency of the co-state vectors is opposite to that of the state vectors revealed by Eq. (12). And since, under practical considerations, $T.d_i^k \leqslant \zeta_m$ (the number of vehicles that join a ramp queue in one time interval is not greater than the maximum allowable queue), it can be shown that μ_ζ^k is also bounded.

Since we have a linear problem with a convex constraint set, the global optimal solution of this problem depends on the sign of the control gradient p_k. The sign of the control gradient, as given by Eq. (16) is determined by only two co-state vectors—μ_ρ^{k+1} and μ_ζ^{k+1}. There is a natural interpretation of the co-state vectors—they measure the sensitivity of performance with respect to the change of states; that is, λ_ρ^{k+1} and λ_ζ^{k+1} relate how much the system travel time changes for a unit change in the freeway density or ramp queue, respectively. Because $\mu_\rho(k+1)$ and μ_ζ^{k+1} are proportional to λ_ρ^{k+1} and λ_ζ^{k+1}, respectively, they have the same interpretation. What the optimality condition tells us is therefore quite intuitive—if increasing freeway density increases system travel time more than increasing queue length (i.e.$\mu_\rho^{k+1} > \mu_\zeta^{k+1}, p_k > 0$), the system time can be decreased by reducing freeway density through ramp metering; however, if increasing queue length increases system travel time more than increasing freeway density (i.e. $\mu_\rho^{k+1} < \mu_\zeta^{k+1}$, $p_k < 0$), then system travel time can be decreased by allowing more traffic to enter the freeway (i.e., decreasing metering headways), thereby decreasing waiting time in queues.

Although the conclusion that the optimal control strategy for metering depends on whether or not travel time saved on the freeway can offset travel time delayed at controlled ramps, the operative answer to this question lies in the magnitudes of the two co-state vectors, which in turn (as is demonstrated in the following sections) depend on traffic conditions on the freeway and its alternative, and the drivers' diversion propensity. To demonstrate this, we examine two limiting cases of drivers' diversion propensity. The first case covers totally insensitive drivers, i.e., drivers who do not divert from ramps regardless how long the ramp queues are; the second case deals with extremely sensitive drivers, i.e. drivers who divert if there is a ramp queue of any length.

5.1. Case 1: totally insensitive drivers

From Eq. (3.1), drivers' diversion sensitivity to ramp queues is given by $\dot{h}(\zeta_i) = 1/\zeta_m$. The case of totally insensitive drivers therefore corresponds to $\dot{h}(\zeta_i) \to 0$ (or, equivalently, $\zeta_m \to \infty$), which leads to $A_{\xi\zeta}^k = 0$ and $A_{\zeta\zeta}^k = I_d$. Because both $A_{\rho\rho}^k$ and $A_{\xi\xi}^k$ are independent of k, we drop the superscript k in $A_{\rho\rho}^k$ and $A_{\xi\xi}^k$, and obtain the following formulae:

$$\mu_\rho^k = 1^t \sum_{j=K}^{k} (A_{\rho\rho})^{K-j} \tag{18.1}$$

$$\mu_\zeta^k = 1^t \sum_{j=K}^{k} (I_d)^{K-j} \tag{18.2}$$

Note that, because there is no diversion from ramps, μ_ξ^k does not enter Eq. (18.2). Because of $A_{\rho\rho}$ the form of and because its spectra are within the unit hypersphere, μ_ρ^k increases at a slower rate than does λ_ζ^k, such that the p_k is negative throughout the control period. This implies that when the freeway is uniformly congested and no diversion occurs, ramp control is counter-productive—a result that is consistent with the findings of Zhang et al. (1996) under conditions that are a degenerative case of the diversion behavior assumed in this analysis.

5.2. Case 2: extremely sensitive drivers

The flow equations and diversion model [Eqs. (1)–(3)] assume that all of the demand d_i^k will join the freeway if there is no metering. If there is metering, then r_i^k flow will join the freeway and δd_i^k will use the freeway alternative. The latter depends on the sensitivity of the diversion behavior. The smallest unit of ramp queue increase in any one period is $T.d_i^k$; the most sensitive diversion behavior therefore corresponds to $\dot{h}(\zeta_i) = 1/(T.d_i^k)$, which leads to $A_{\xi\zeta}^k = \frac{1}{l_a.\Delta} I_d$ and $A_{\zeta\zeta}^k = 0$. The new formulae for calculating co-state vectors for this case are:

$$\mu_\rho^k = 1^t \sum_{j=K}^{k} (A_{\rho\rho})^{K-j} \tag{19.1}$$

$$\mu_\xi^k = 1^t \sum_{j=K}^{k} (A_{\xi\xi})^{K-j} \tag{19.2}$$

$$\mu_\zeta^k = 1^t + \mu_\xi^{k+1}, \lambda_\zeta^K = \beta.1^t \tag{19.3}$$

In this case, unlike in the previous, μ_ζ^k is solely determined by μ_ξ^{k+1}, the transformed co-state vector for freeway alternative traffic dynamics. The control gradient is thus determined by the spectra of $A_{\rho\rho}$ and $A_{\xi\xi}$. As previously noted, the spectra of $A_{\rho\rho}$ is controlled by c_1 (the speed of backward propagating waves on the congested freeway), and the spectra of $A_{\xi\xi}$ is controlled by V_2 (the uncongested speed on the freeway alternative); the relative values of these two traffic parameters completely determine if ramp metering is beneficial to the overall system performance under extremely sensitive diversion behavior. That is, the optimal policy in this case, while dependent on the travel speed on the freeway alternative, is not directly dependent on the associated travel speed on the congested freeway; rather, it is dependent on the wave speed of backward propagating waves.

We first define a parameter $\psi \equiv c_1/V_2$ and consider four illustrative examples involving a range of ψ values of for the four-section road corridor shown in Fig. 3 with $T = 30\,\mathrm{s}$: $\psi \equiv c_1/V_2 = 0.1, 0.5, 1$ and 5.0. We note that the parameter ψ is related to the free speeds on the freeway and its alternative through the fundamental diagram as:

$$\frac{V_1}{V_2} = \psi\left(\frac{\rho_{jam}}{\rho_c} - 1\right) \tag{20}$$

As such, specification of ψ is itself not sufficient to determine the relative speeds along the freeway and its alternative; only for values of $\psi < 1/(\rho_{jam}/\rho_c - 1)$ can it be stated unequivocally that the congested travel speed on the freeway is less than the free speed on the alternative. If, for example,

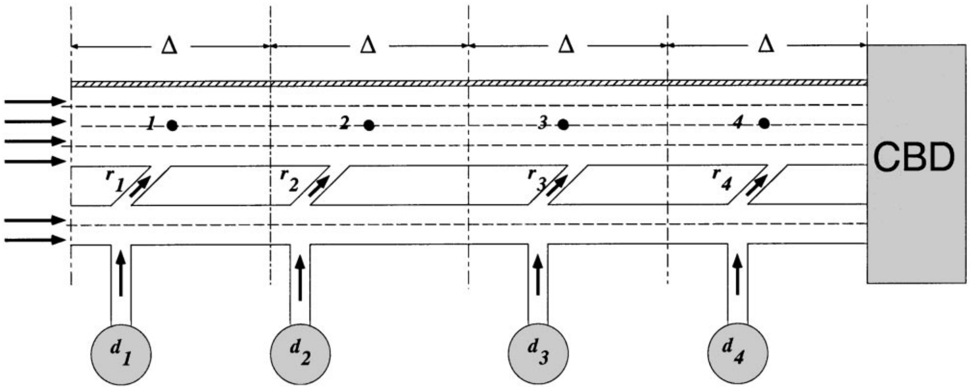

Fig. 3. Four-section road corridor used in numerical example.

$\rho_{jam}/\rho_c = 3$, then $V_1/V_2 = 2\psi$ and the freeway alternative is certain to have a travel time advantage over the freeway for values of $\psi < 0.5$. (For practical purposes, scenarios in which the freeway alternative has a travel time advantage over the freeway are likely to correspond to values of $\psi < 1$; the example for which $\psi = 5$ represents a scenario in which the vast majority of densities possible under the congested portion of the assumed piece-wise linear flow-density relationship freeway are associated with travel speeds greater than that of the freeway alternative.)

The numerical results obtained for the four scenarios tested are shown in Fig. 4. The results show that, under the conditions specified in this example, metering for at least some time period for certain ramps is beneficial to the system time savings for cases that are likely to represent scenarios in which freeway alternative travel is advantageous over freeway travel ($\psi = 0.1, 0.5$) and metering is not beneficial under conditions in which even congested freeway travel is likely to have a large advantage over freeway alternative travel. In all cases for which metering is beneficial, the optimal policy has the following characteristics:

1. ramp metering should not be employed near the end of the control period, thereby ensuring that no vehicles are waiting on the ramps after the control period is over (a consequence of the penalty function);
2. downstream ramps should be metered more persistently than upstream ramps; and
3. the extent of ramp metering varies inversely with the relative travel time advantage that the freeway alternative has over the freeway. Because of the properties of the assumed driver diversion propensity function, these results hold for any set of ramp demands, d_i^k, that does not violate the assumptions that the freeway is always congested and the freeway alternative is always uncongested.

Observations (1) and (3) above are intuitive. Observation (2) is logical on two fronts. First, in the corridor setting selected, traffic congestion should dissipate from downstream to upstream; metering downstream ramps therefore will help traffic to recover more quickly than metering upstream traffic. Second, a driver who enters from a downstream ramp will affect the travel times of more drivers than a driver who enters from an upstream ramp because vehicles ahead can

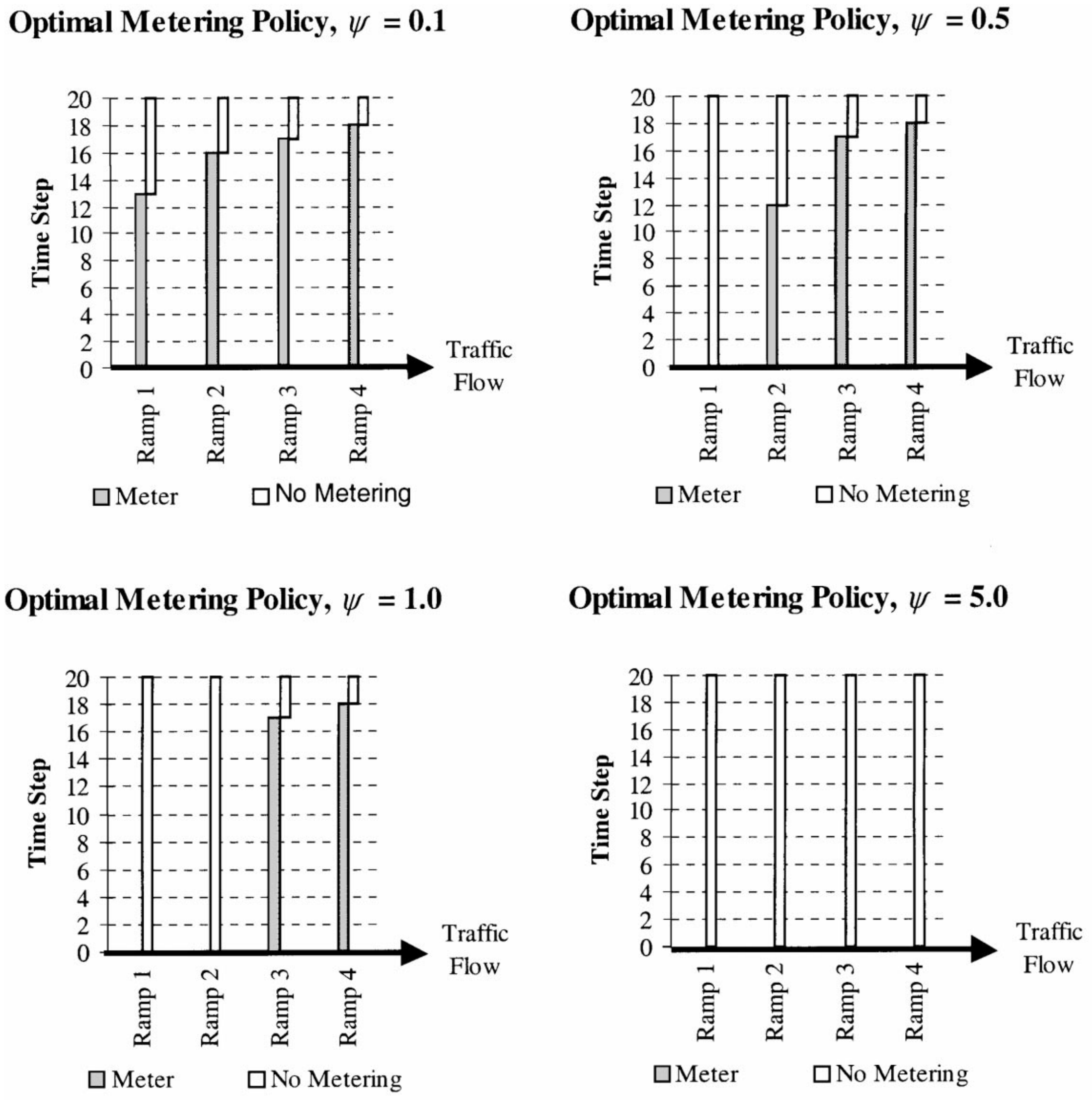

Fig. 4. Optimal policies under sensitive diversion behavior.

influence vehicles behind, and not vice versa. That is to say, a downstream driver adds a higher marginal cost to the system than does an upstream driver.

6. Discussion

The primary objective of this paper has been to present some general properties underlying the effectiveness of ramp metering along a congested freeway corridor, taking into account queuing, traffic diversion and freeway alternative traffic dynamics. It is shown that optimal ramp control policies depend both on traffic diversion propensities and on differentials between freeway and surface street traffic conditions. The results indicate that, unless drivers have at least some propensity to divert from entering the freeway based on the queue at the entry ramp, it is never beneficial to the total system performance (as measured by total travel time) to meter the entry ramps. Even in the limiting case in which drivers are extremely sensitive to the presence of ramp queues, metering a congested freeway may not be beneficial to overall system performance—rather, the decision to meter depends on the wave speed ratio c_1/V_2. These results, of course, are subject to both the context and the simplifying assumptions used in the formulation of the problem; for example, many-to-many flow networks and corridors in which the freeway is not uniformly congested are deliberately excluded from the analysis. While it is improper to draw any firm conclusions regarding the general efficacy of ramp metering from the analysis, it is reasonable to presume that the general tone of the findings may have applicability in less restrictive settings. As such, it is hoped that the results presented in this paper could be used as guidelines for developing system-wide ramp metering strategies.

As noted, there are significant limitations to the scope of applicability of the results presented herein—limitations that can be lessened substantially through the relaxation of restrictive assumptions along two general directions, one of which related to the behavioral aspects of the system, and the other to the physical setting of the system. The simplest example in the first direction is to consider nonlinear traffic diversion behavior and flow-density relationships. Although this extension is straightforward, results would be less definitive than those presented here because of the possible existence of multiple optima. A somewhat more difficult and more interesting extension to the current study is to examine the dynamic interaction between controls and drivers' *long-term* adaptation to these controls, and the effects of such behavioral change on system controllability and performance. This adaptation process is arguably highly important to modern traffic management and is perhaps the least understood link in the control—system-response—control loop. The difficulty of such a study lies not only in the complexity of the problem, but also in the lack of experimental observations to develop models and to test certain hypotheses. Hopefully, the latter aspect will be addressed to a certain degree by some of the Intelligent Transportation System demonstration projects in the U.S. and elsewhere.

In the second direction, a natural extension to the current study is to consider the details of signal settings on the surface streets that comprise the freeway alternative. Incorporation of signal settings into the formulation is not difficult. There is, however, a trade-off between the level of detail considered and the generality of the results obtained. It is unlikely that the results of such an extension would depend on as few parameters as we have presented here. The corridor considered in this analysis consists of only one destination, which covers many cases of CBD-leading corridors. Still, many-to-many corridors abound and should be considered in further research. Realistic dynamic traffic flow models that can separate flow by their destinations have to be developed for this purpose, which is not an easy task. The Cell Transmission Model developed recently by Daganzo (1994) may be a useful development in this regard. We note, however, that the results of many-to-one corridors provide a lower bound of metering benefits for those many-to-many corridors by forcing all of the flow to a single destination. A third extention of the current analysis is to study the effects of ramp metering under hyper-congestion, where portions of the road could reach jam density for a period of time. This case is more involved than that considered here and is left for future work.

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