# Stochastic pre-planned household activity pattern problem with uncertain activity participation (SHAPP)

Li Ping Gan and Will Recker
Department of Civil and Environmental Engineering
Institute of Transportation Studies
University of California, Irvine
Irvine, CA 92697 USA

**Abstract**

The so-called activity-based approach to analysis of human interaction with the social and physical environments dates back to the original time-space geography works of Hägerstrand and his colleagues at the Lund School in 1970. Despite their obvious theoretical attractiveness, activity-based approaches to understanding and predicting travel behavior have suffered from the absence of an analytical framework that unifies the complex interactions among the resource allocation decisions made by households in conducting their daily affairs outside the home, while preserving the utility-maximizing principles presumed to guide such decisions. In this paper, we develop a computationally-tractable system, based on an extension and modification of some rather well-known network-based formulations in operations research, to model human dynamics in uncertain environments. The research builds on the mathematical programming formulation of the Household Activity Pattern Problem (HAPP) by embedding stochastic elements in the planned household activity schedule decision process that capture the uncertainty of the need for rescheduling.

## 1. Introduction

Recognition that conventional travel demand approaches that examine each trip in isolation at best provide only limited information regarding the particular trip (since they generally ignore both the history that precedes the trip as well as the future that follows) and virtually no information on the impact of decisions regarding the particular trip on other travel decisions (both prior and subsequent), has led to research to develop and operationalize activity-based travel demand analyses. The original work providing the foundation of the activity-based approach can be dated back to Hägerstrand (1970) and his colleagues. Introducing time-space geography as a paradigm for understanding human movement, the seminal work of Hägerstrand (1970) offered the potential to better integrate the spatial and temporal components of human interaction decisions that underpin the concepts of human movement within the built environment and its relationship to accessibility. His concept is characterized as a "constraint-based approach," given the defining role that spatial and temporal constraints in the formulation. Within this three-dimensional space, so-called time-space prisms define the limits of what is accessible, or "reachable" in the urban environment. A number of studies following the concept of Hägerstrand have been undertaken to develop activity-based models, which are purported to be able to evaluate spatial and temporal effects on human interaction. A line of previous research efforts has been carried out that focuses on the following themes: analysis of activity demand, scheduling of activities, investigation of constraints on activity and travel choices, spatial-temporal dynamics of activity-travel decisions and how they relate to one's role in the household, and overall effect of household structure (e.g. lifestyle, lifecycle, role) on individual activity/travel.

Notable among the various features of the existing models loosely based on Hägerstrand's original concept is that, to date, research has been restricted to a deterministic environment almost without exception; lacking in all of these existing models is a systematic incorporation of uncertain factors associated with the agenda of activities, either desired or needed to be performed, as a complement to established deterministic elements. That such uncertainty can play a significant, and sometimes dominating, role in travel decisions has been well documented in the literature (Abdel-Aty et al., 1996; Bell et al., 1999; Chen et al., 2002). Recker et al. (2001) indicate that an extension of existing approaches in a manner that embeds associated "soft" constraints, inherent uncertainties and physical constraints in a single, comprehensive, operational system would greatly increase capability to address the household activity scheduling problem under uncertain environment. However, the challenges involved to describe and model the human decision process for consumption of time-space to accomplish the need or desire for out-of-home activities within a household context incorporating the uncertain factors are formidable.

It is generally recognized that, rather than being a static system, the activity scheduling problem involves a dynamic process that incorporates the degree of uncertainty arising from factors associated with the uncertain characteristics previously mentioned. Given an initial activity/travel schedule, at any moment in the day, there may be the need to reschedule those remaining, not-yet-completed, activities due to unexpected events that occur in real time, including: traffic congestion, the need for more time to accommodate a certain activity, illness, etc. This is in contrast to the basic assumption of existing activity-based models that almost universally are posed as a static problem, and designed to produce the optimal activity/travel decisions relative to the prescribed objective of completing its activity agenda at the outset of the day. It should be acknowledged that observed activity schedules are the result of an unobserved decision "process" involving the planning and execution of activities over time within a household context (Doherty and Axhausen, 1999; Doherty and Miller, 2000). However, this process has been unspecified in the literature due to its complexity. The problem of how household members adjust their planned activity/travel schedules when faced with unexpected events has gained more attention recently (Auld et al, 2011; Clark and Doherty, 2009; Doherty et al, 2002; Joh et al 2005a, Mohammadian et al, 2005). There have been a few attempts to develop activity rescheduling models (Joh, 2004; Joh et al, (2004), 2005b; Joh et al, 2006; Ruiz et al, 2006; Gan and Recker, 2008); however, clarity in this general area has been elusive.

The research presented here models the activity scheduling problem within an uncertain environment by taking into account the impacts brought by relevant uncertainties in the outcomes of stochastic factors affecting activity participation on the household decision process. Such factors, which generally have been left completely unspecified in prior research on the formation of household activity plans, are purported to lead to better prediction of the revealed travel behavior of a household. Obviously, the resulting system incorporating all of the relevant factors can be expected to be highly complex, and computationally challenging. In this research, optimization-based mathematical models are developed that focus on a particular subset of the uncertain aspects mentioned previously—the uncertainty of activity participation. The focus of the models presented here is on the development of an "optimal plan" for the scheduling of desired activities in the face of uncertainty, rather than on modeling how to best dynamically reschedule/adjust the pattern should such uncertainties arise in the course of executing a pattern—the "plan" retains its optimality as a planned course of action, regardless of the realization (or non-realization) of the uncertainties considered. In that sense, it is distinguished from dynamic rescheduling algorithms, such as that proposed by Gan and Recker (2008). The proposed models inherit the basic structure of the HAPP model (Recker, 1995) to predict travel behavior by household members constrained by spatial-temporal limitations and household interactions guided by presumed utility maximization principle, while embedding uncertain factors. However, it should be noted that there is no direct extension from any existing model; taking uncertain and dynamics characteristics into account dramatically impacts both the problem formulation and the resulting methodology to derive the solution.

The Stochastic Household Activity Pattern Problem (SHAPP) arises when the inherent stochastic nature of activity participation is accommodated in the modeling process; we pose the SHAPP problem within the context of random choice of activity participation, in which the prescribed, planned, agenda includes flexible activities. We derive estimates of the activity participation and linkages between activities using knowledge about distributions of the stochastic elements. Mathematically, the SHAPP problems fall into the category of static problems, in which an *a priori* plan is generated before the realizations of the stochastic variables, and subsequently, the actual activity/travel pattern is determined by accommodating the impacts of the future events. The SHAPP model attempts to analyze/predict the optimal path of household members through time and space as they complete a prescribed agenda of out-of-home activities, in which each activity of the prescribed household agenda has an *a priori* known probability of being completed or cancelled. The problem has been posed as a multi-stage stochastic mixed-integer linear program that determines an optimal *a priori* activity/travel schedule within the context of stochastic activities, and solved to optimality by means of the L-shaped method. Numerical experiments are conducted to demonstrate the model and comparative analyses between the SHAPP model and its deterministic counterpart are carried to evaluate the value of the stochastic model.

## 2. Model Formulation

The SHAPP problem for which there are $n$ potential activities in the prescribed agenda of a particular household, represented by the set $A\{i=1,...,n\}$, with corresponding locations $P^{+}=\{1,...,n\}$, is essentially a network optimization problem that can be defined on a complete graph $G=(N,E)$, with vertex set: $N=P^{+}\cup P^{-}\cup\{0\}\cup\{2n+1\}$, where $"0"="2n+1"="\text{home}"$, $P^{+}=\{1,...,n\}$, $P^{-}=\{n+1,...,2n\}$ and the edge set is denoted by $E=\{(i,j):i,j\in N,i\neq j\}$; Subset $P^{+}$ denotes the set of locations (activity nodes) at which an activity is to be performed, while $P^{-}$ represents the set of virtual return home destination nodes for each planned activity (i.e., the terminal node indicating the arrival at the home location following completion of the tour during which the respective activity was performed), and is geometrically identical with home node “$0$”. The elements $i\in P^{+}$

and $i+n \in P^{-}$ thus represent the "commencement" and eventual "end-of-tour" locations of the *i*-th activity, respectively. Node $i+n$ is referred to as the companion of $i$, or vice versa. Vertex "$2n+1$" designates the node representing the final completion of all of the day's activities; its physical location is identical to that of the home node "$0$" as well. Each activity $i$ in set $A$ is constrained by a specified time window restriction $\left[a_i, b_i\right]$ regarding the earliest/latest starting time for participation in activity. Furthermore, within set $A$ it is presumed that there is a subset of activities drawn from $A$ that comprises activities that are more or less flexible, and that might be cancelled during the day. Matrices $C=\left(c_{ij}\right)$ and $T=\left(t_{ij}\right)$ are defined on $E$ and represent the travel times and travel costs incurred when travel to/from activities, respectively. Let $p_i > 0$ denote the probability that a particular activity $i \in A$ is performed. Activities with $p_i = 1$ are referred to as deterministic activities, while activities with $0 < p_i < 1$ are stochastic activities with random choice of participation, i.e., with $\left(1-p_i\right)$ probability of cancellation. Assume that there is at least one stochastic activity in set $A$. The above assumption leads to the presence of one or more stochastic nodes both in set $P^{+}$ and also in $P^{-}$. (Throughout the paper, the term "stochastic activity" is equivalent to "stochastic node.") Furthermore, since trips made by any vehicles must depart from home and end by returning home, nodes "$0$" and "$2n+1$" are deterministic, i.e., $p_0 = p_{2n+1} \equiv 1$. Figure 1 displays an illustrative example of such a network. In the figure, circles drawn with a solid line comprise the "commencement" of activity node set, i.e., elements of $P^{+}$, whereas those with dashed lines represent the "end-of-tour" node set, i.e., elements of $P^{-}$. Each link connecting the activity locations is associated with travel time and travel cost. In addition, time window restrictions are enforced at each node. In the example shown in Figure 1, there are $n$ activities in the prescribed agenda, in which the $i$-th activity has probability $p_i$ of being performed while the remainder of the activities are deterministic. As shown in Figure 1, node $i$ and its accompanying node $n+i$ are stochastic with probability $p_i$; i.e, node $i$ may or may not be visited depending on the realization of the random variables.

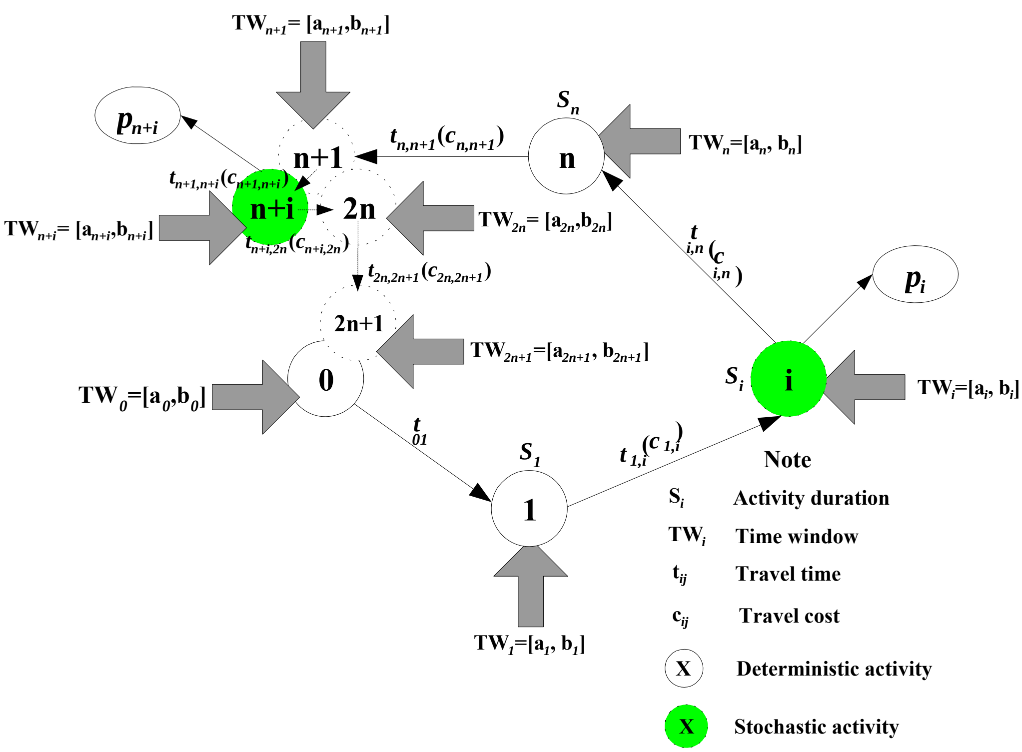


**Figure 1. An Illustrative SHAPP Network**

The basic question addressed by SHAPP is: given prescribed *uncertainties in exactly which activities will be performed*, what will be the optimal *planned pattern of travel* of the household members subject to such certain sets of constraints as time, money, and human resources? In addressing this question, stochastic information is presumed to be known exactly at the outset of developing the plan, together with the deterministic information applicable to the decision making. However, the actual decision to either complete or cancel a stochastic activity is revealed only at some time between the time that the activity schedule is drawn and, at the latest, upon leaving the activity immediately preceding it in the schedule. A two-stage stochastic integer linear program with recourse is proposed herein to capture the random choice of activity participation. In the two-stage optimization framework, only partial model parameters are available in the first stage (prior to the realization of any stochastic elements), with complete data being available in the second stage. Specifically, the determination of an *a priori* activity/travel schedule by the household—how the planned activities, with the presence of stochastic nodes, are to be implemented in terms of a start time, by whom, and by which vehicle—is made prior to the second stage, in which the stochastic information is realized. In the second stage, activities are performed in an order consistent with the optimal *a priori* activity/travel schedule, with the remaining schedule and its associated optimality subject to recourse actions that are employed to adjust the preplanned schedule in the face of the realization of participation/non-participation in those preceding activities for which participation was uncertain.

Although there are a myriad of potential recourse actions that could be considered upon realization of stochastic activities (e.g., selecting a different location for a conflicting activity, reducing the duration of a yet-to-be-completed activity, reordering remaining activities, etc.), we here restrict such recourse to simply eliminating any cancelled activity and completing the remaining activities according to the *a priori* schedule. We argue that inclusion of all of the possible recourse actions not only renders the *np*-hard problem unmanageable but is also not likely to be consistent with the detail that goes into a household's pre-planned schedule. There is consistent evidence from empirical studies that in the overwhelming number of cases, simple elimination of conflicting activities is the recourse action taken under such circumstances. For example, in a study involving 443 rescheduling decisions, Clark and Doherty (2009) found that the great majority of rescheduling involved "adding an activity (214, 48.3%), deleting an activity (65, 14.7%), and modifying the start time (74, 16.7%), end time (38, 8.6%), or both (36, 8.1%)." (In the formulation herein, both the possibilities of addition and deletion of activities are encompassed in the specification of the probability of occurrence.) Other changes to the planned schedule, including changing location, were observed to account for only 0.03% of cases. In another study of 5,968 activities completed in the Chicago, Illinois, area, Auld et al (2011) found that the locations of activities were almost entirely preplanned, with timing decisions, including start time and duration decisions, being more impulsive. In a study of schedule modifications involving 422 respondents reporting a total of 41,698 scheduling decisions, Joh et al (2006a) conclude that most modifications in schedules relate to timing adjustments, with start or end time, change or shifting of an activity to another time of the day accounting for 84% of all the modifications. They further conclude that other modifications, including location, are relatively rare. Other studies by these investigators and others (see e.g., Auld, et al, 2009a, b; Mohammadian and Doherty, 2005, 2006; Doherty et al, 2002; Ruiz and Roorda, 2008) offer consistent empirical evidence that the overwhelming majority of changes in scheduling, vis-à-vis a pre-planned schedule, involve such temporal adjustments as deletion of the conflicting activity, and/or altering the duration and/or start-end times of succeeding activities—insertion of new (unplanned) activities or changes in location of activities are rare. And, although here we explicitly consider only deletion of conflicting activities, with any necessary adjustments to subsequent start/end times handled by imposition of penalty functions, we remark that such penalties are easily translatable to equivalent decreases in utility of performance of the associated activities brought on by changes in duration.

The overall costs generated by the recourse actions comprise the recourse costs. SHAPP thereby consists of deriving an *a priori* optimal path of household members through time and space, as they complete a prescribed agenda of out-of-home activities that yields the minimum current cost and the expected recourse (future) costs. The problem falls into the category referred to as "*a priori* optimization problems," which is a natural solution approach to problems in which it is undesirable to re-compute an optimal solution whenever the random variables are realized. This approach makes sense in the context in which the performance of an activity is uncertain owing to future circumstances that may alter an individual's preferences as to whether to perform or cancel; e.g., the unavailability of a companion, or the rescheduling of an appointment. Under such circumstances, it would be logical to assume that the stochastic activity be included in the preplanned activity/travel schedule at the beginning of the day, but subject to a given likelihood that it may not occur. The model output is judged to be superior to the output derived from its deterministic counterparts in terms of flexibility when facing with unexpected events.

## 2. Model Formulation

The household activity/travel scheduling decision process herein, embedding inherent stochastic nature of activity participation, is cast within the format of a two-stage stochastic integer linear program with recourse. Distinguished from that generated by the deterministic version, such as by the HAPP model, the solution derived herein is reflective of compromise efforts that household members may devote to determining the activity/travel schedule in an environment that incorporates uncertainties regarding activity participation or travel time and durations. Following the notation of Recker (1995), the problem considered here is formulated as a stochastic mathematical program with the decision variables defined in Table 1:

| Decision Variables | Definitions |
|---|---|
| $X_{uw}^{v}, \forall u, w \in N, v \in V$ | Binary decision variable: $X_{uw}^{v} = 1$ if the linkage between node pair $u$ and $w$ is connected by vehicle $v$ and zero otherwise; |
| $H_{uw}^{\alpha}, \forall u, w \in N, \alpha \in \eta$ | Binary decision variable: $H_{uw}^{\alpha} = 1$ if household member $\alpha$ travels between node pair $u$ and $w$, and otherwise zero; |
| $T_u, \forall u \in P$ | The starting time of a particular activity $u$; |
| $T_0^v, T_{2n+1}^v, \forall v \in V$ | Respectively, the first departure time from home and the last return home time for a particular vehicle $v$; |
| $Y_u, \forall u \in P$ | The total accumulation of sojourns on a particular tour immediately following completion of activity $u$; |
| $\overline{T}_0^{\alpha}, \overline{T}_{2n+1}^{\alpha}, \forall \alpha \in \eta$ | Respectively, the first departure time from home and the last return home time for a particular household member $\alpha$ in the first stage problem; |

**Table 1 Decision Variables of SHAPP Problem**

The output $\mathbf{X}_i = \left[ X_{uw}^v, \forall u, w \in N, v \in V;\ H_{uw}^\alpha, \forall u, w \in N, \alpha \in \eta;\ T_u, \forall u \in P \right]_i$ of the optimization of a particular household, say $i$, during some time period, relative to minimizing the overall expected cost is specified by the decision variables defined above. It is noted that $\mathbf{X}_i$ consists of integer and continuous variables, resulting in a mixed integer stochastic linear program. In this formulation, decisions are assumed to be made simultaneously among the available choice set, constrained by the time-space prism. As with the HAPP model by Recker (1995), the formulation follows a utility maximization/disutility minimization principle. That is to say, the model is based upon the assumption that the activity/travel decisions made by the household associated with uncertain activity participation is governed by minimizing the expected disutility to the household of the total activity/travel package, which is represented by the respective objective function.

Let $\xi = (\xi_i)$ be a vector of Bernoulli random variables, where $\xi_i$ is equal to 1 if and only if node $i$ has a probability $p_i < 1$ of being present, independent of other activities. It follows that the nodes of the graph $G$ (See Figure 1) can be divided into two groups—group 1 is referred to as the deterministic node set with $p_i \equiv 1$, and group 2 is regarded as the stochastic node set with $0 < p_i < 1$. As specified, SHAPP is approached as a two-stage stochastic integer linear program. In the first stage, an *a priori* path through time and space has to be determined—before the realizations of the uncertain activities—that is governed by minimizing the incurred current cost. In the second stage, during which information on the realization of a stochastic activity is revealed, remaining activities are completed according to a prescribed schedule that incorporates any resulting incurred cost, referred to as the recourse cost for adjustment. As such, the overall objective of the SHAPP model is to derive a set of activity/travel patterns within minimum overall cost subject to the physical constraints, with a model output $\mathbf{X}_i$.

As outlined above, the utility considerations that drive household activity/travel schedule decisions in the deterministic HAPP model would also be in play under a stochastic environment. However, in this case, the optimal course of action is driven by the expected utility/disutility of activity/travel decisions; i.e., by the minimization of the expected value of total disutility with respect to the distribution of $\boldsymbol{\xi}$ in the second stage problem. Let $Z\left(\mathbf{X}_i, \boldsymbol{\xi}\right)$ be the disutility of the second stage solution if $\mathbf{X}_i$ is the first stage solution, and $\boldsymbol{\xi} = \left(\xi_i\right)$ is the random vector representing realizations of uncertain activity participation. The objective function of the second stage problem is then written as

$$\min \; E_{\xi} Z\left(\mathbf{X}_i, \boldsymbol{\xi}\right) \tag{1}$$

where $E_{\xi}$ denotes the expectation.

Recall that the model herein is represented as a two-stage stochastic programming problem, and hence Equation (1) can be rewritten in the two-stage framework. The first stage problem determines an optimal *a priori* activity/travel schedule $\mathbf{X}_i$ with cost of $C\left(\mathbf{X}_i\right)$. In the second stage, with the realization of a particular instance of $\boldsymbol{\xi}$, where $\boldsymbol{\xi} = \left(\xi_i\right)$ is a vector of Bernoulli random variables that represent the realizations of the random variables associated with uncertain activity participation, there result particular recourse costs, denoted as $Q(\mathbf{X}_i,\boldsymbol{\xi})$, that include such components as time window violations due to uncertain activity participations and potential disutility variations.

Let $Q(\mathbf{X}_i)$ denote the second stage expected recourse cost function. Thus, we have

$$Q(\mathbf{X}_i) = E_{\xi} Q(\mathbf{X}_i,\boldsymbol{\xi}) \tag{1a}$$

As such, the expected disutility of $\mathbf{X}_i$ can be broken into parts, each of which contributes to the cost incurred in the first stage problem, and the minimization of the expected value with respect to the distribution of $\boldsymbol{\xi}$ in the second stage problem, as:

$$\min E_{\xi} Z(\mathbf{X}_i,\boldsymbol{\xi}) = \min\left\{C(\mathbf{X}_i) + Q(\mathbf{X}_i)\right\} \tag{1b}$$

which implies that the recourse cost function can be rewritten as:

$$Q(\mathbf{X}_i) = E_{\xi} Z(\mathbf{X}_i,\boldsymbol{\xi}) - C(\mathbf{X}_i) \tag{1c}$$

Equation (1c) indicates that for a given feasible solution $\mathbf{X}_i$, the value of recourse cost can be derived once the relevant expected cost and deterministic cost are available. The subsequent section is devoted to the methodology to evaluate the recourse cost function, which will ultimately be used to generate optimality cuts in the so-called L-shaped method (See Section 4).

Given an *a priori* optimal activity/travel schedule $\mathbf{X}_i$, the disutility function of household $i$ , $C(\mathbf{X}_i)$, of the first stage problem can take the form of a myriad of different utility formulations beyond simple times and costs, as demonstrated in Recker (1995), Recker and Parimi (1999) and Kang and Recker (2009). Here, for simplicity, we take the example of one expressed as a weighted sum of the total travel times and costs:

$$C\left(\mathbf{X}_i\right) = \lambda_{tt} \cdot \sum_{v \in V} \sum_{u \in N} \sum_{w \in N} t_{uw}^{v} \cdot X_{uw}^{v} + \lambda_{tc} \cdot \sum_{v \in V} \sum_{u \in N} \sum_{w \in N} c_{uw}^{v} \cdot X_{uw}^{v} \tag{2}$$

where $\lambda_{tc}, \lambda_{tt}$ are relative weights assigned to the respective terms. With the incorporation of uncertain activity participation, the components specified by $C\left(\mathbf{X}_i\right)$ are thus random variables themselves. Then, under the usual assumption of additive part-wise utilities/disutilities, the expected disutility function $E_{\xi} Z\left(\mathbf{X}_i, \xi\right)$ can be written as a sum of the previously defined components multiplied by their respective probability of occurrence, e.g., in the case of the disutility defined by Equation (2):

$$E_{\xi} Z\left(\mathbf{X}_i, \xi\right) = \lambda_{tt} \cdot \sum_{v \in V} \sum_{u \in N} \sum_{w \in N} \overline{p}_{uw}^{v} \cdot t_{uw}^{v} \cdot X_{uw}^{v} + \lambda_{tc} \cdot \sum_{v \in V} \sum_{u \in N} \sum_{w \in N} \overline{p}_{uw}^{v} \cdot c_{uw}^{v} \cdot X_{uw}^{v} \tag{3}$$

The terms specified by $E_{\xi} Z\left(\mathbf{X}_i, \xi\right)$ jointly represent the expected cost associated with vehicle flows and activity assignment incurred by a household when stochastic elements related to uncertain activity participation are inherently considered in the decision process. In particular, for the example presented, the terms in Equation (3) represent the expected overall travel time and travel cost, respectively, which have been expressed as the sum over all links $(u, w)$ of the travel time $t_{uw}^{v}$ or travel cost $c_{uw}^{v}$ multiplied by the probability $\overline{p}_{uw}^{v}$ that a particular link from node $u$ to $w$ is traversed by vehicle $v$ given an *a priori* solution $\mathbf{X}_i$. Therefore, if the probability of presence of direct travel between $u$ and $w$ can be derived, the expected travel time and travel cost of a given activity/travel schedule $\mathbf{X}_i$ can be subsequently obtained. To compute the expected travel time or travel cost of a given *a priori* solution, all possible combinations of the realizations of random instances $\xi_i$ should be considered—requiring an exponential number of operations.

In the literature, significant effort has been devoted to efficient methodologies in order to evaluate the expected travel cost of a given *a priori* tour for SVRP with stochastic customers or PTSP with stochastic customers (Jaillet, 1988; Laporte et al., 1994; Gendreau et al., 1995, 1996). Although the previous efforts clearly provide insights to the current research, the existing methodologies are applicable only to the vehicle routing problem with customers exclusively requesting “pick up” or “delivery” and, based on the assumptions made in these works, cannot be applied to PDPTW and its variants—for example, to the SHAPP problem. Ho and Haugland (2004) propose an efficient method to compute the expected travel cost of an *a priori* solution of the PDARP with random service request, which consists of a more general case of SVRP problem in which customers request pickup and delivery at the same time. Because the SHAPP problem shares substantial resemblance to the PDARP problem, the methodology is adopted herein to compute the expected travel cost/time of an *a priori* activity/travel pattern. The method can be summarized as follows. Let $\tau^v$ denote the set of locations visited by vehicle $v \in V$ of an *a priori* activity/travel pattern $\mathbf{X}_i$—a path through time-space prism starting from home “0” and then eventually ended by returning to home “2n+1”. (Note that home and final destination are physically identical.) Let $l^v$ be the number of nodes included in set $\tau^v$ exclusive of nodes “0” and “2n+1”. Then, re-label the nodes of activity locations in set $\tau^v$ by the order of completion precedence, thus denoting $\tau^v$ by $\left\{i_0^v, i_1^v, i_2^v, \ldots, i_{l^v}^v, i_{l^v+1}^v\right\}$, where $i_0^v = "0"$ and $i_{l^v+1}^v = "2n+1"$. For example, with the above notations, Equation (3) can be rewritten as follows with respect to the new labels:

$$E_\xi Z\left(\mathbf{X}_i, \xi\right) = \lambda_{tt} \cdot \sum_{v \in V} \sum_{u=0}^{u=l^k} \sum_{w=u+1}^{w=l^k+1} \overline{p}_{i_u^v, i_w^v}^{v} \cdot t_{i_u^v, i_w^v}^{v} + \lambda_{tc} \cdot \sum_{v \in V} \sum_{u=0}^{u=l^k} \sum_{w=u+1}^{w=l^k+1} \overline{p}_{i_u^v, i_w^v}^{v} \cdot c_{i_u^v, i_w^v}^{v} \tag{4}$$

Given two nodes $i_u^v$ and $i_w^v \in \tau^v \setminus \{0\}$, $0 \le u < w \le l^v + 1$ indicates that node $i_u^v$ is traversed prior to node $i_w^v$ by vehicle $v$. Furthermore, the following subsets of $\tau^v$ can be drawn:

$$\tau_{uw}^{v,1} = \begin{cases} \{u\}, & \text{if } i_w^v = i_u^v + n \\ \{u, w\}, o.t.w & \end{cases} \tag{5a}$$

$$\tau_{uw}^{v,2} = \left\{ r \in \{u+1, \ldots, w-1\} : i_r^v \le n \right\} \tag{5b}$$

$$\tau_{uw}^{v,3} = \left\{ r \in \{u+1, \ldots, w-1\} : i_r^v \ge n+1, i_r^v = i_{r'}^v + n, r' \in \left\{ (1, \ldots, u-1) : u-1 \le n \right\} \right\} \tag{5c}$$

For any given instance of the problem, whether or not link $\left(i_u^v, i_w^v\right)$ is in the resulting trip chain of vehicle $v$ can be determined as follows:

$$\overline{p}_{i_u^v, i_w^v}^{v} = 0 \quad \text{if } \left\{i_u^v, i_w^v\right\} \notin \tau^v \tag{6a}$$

$$\overline{p}_{i_u^v, i_w^v}^{v} = 0 \quad \text{if } u \ge w \tag{6b}$$

$$\overline{p}_{i_u^v, i_w^v}^{v} = 0 \quad \text{if } u = 0, w = l^v + 1 \tag{6c}$$

$$\overline{p}_{i_u^v, i_w^v}^{v} = 0 \quad \text{if } \exists\, h \in \{u+1, \ldots, w-1\} \ni i_w^v = i_h^v + n, u \ge 1 \tag{6d}$$

$$\overline{p}^{v}_{i^v_u,i^v_w} = 0 \quad \text{if } \exists\, k \in \{u+1,\ldots,w-1\} \ni i^v_k = i^v_u + n,\, u \geq 1; \tag{6e}$$

$$\overline{p}^{v}_{i^v_u,i^v_w} = \prod_{h\in\tau^{v,1}_{uw}} p_{i^v_h} \prod_{r\in\tau^{v,2}_{uw}} \left(1-p_{i^v_r}\right) \prod_{k\in\tau^{v,3}_{uw}} \left(1-p_{i^v_k}\right), \text{ otherwise.} \tag{6f}$$

Equation (6a) simply states that the probability of any node pair that is not included in path $\tau^v$ is equal to zero. Equation (6b) imposes that the sequence order should not be violated in the second stage problem, which indicates that the probability of link $\left(i^v_u, i^v_w\right)$ being directly chained by vehicle $v$ is zero if node $i^v_u$ is visited later than node $i^v_w$ in the first stage, i.e. $u > w$. Equation (6c) enforces that the probability of the direct linkage of node pair "0" and "2n+1" is zero. Equations (6d) and (6e) rule out the probability of node pair $u$ and $w$ being directly linked if chain $\left\{i^v_{u+1},\ldots,i^v_{w-1}\right\}$ contains the "end-of-tour" node of $i^v_u$ or "activity commencement" node of $i^v_w$. Equation (6f) provides the general form to compute the probability of presence ( $p^v_{i^v_u,i^v_w}$ ) if chain $\left\{i^v_{u+1},\ldots,i^v_{w-1}\right\}$, drawn from $\tau^v$, contains both the nodes designating locations at which activity is performed as well as their corresponding ultimate destination of the "return to home" trip of a subset of the uncertain activities. Hence, the expected travel time and travel cost associated with an *a priori* activity/travel pattern $\mathbf{X}_i$ can be obtained through the calculation of $\overline{p}^{v}_{i^v_u,i^v_w}$. Although applied to the specific example of a disutility function comprising only travel times and cost, this process is easily verified to be applicable to the computation of any linear compensatory specification of $E_{\xi} Z\left(\mathbf{X}_i, \boldsymbol{\xi}\right)$ once the probability of presence of each node, i.e., the subset of uncertain activities drawn, is exactly known.

The above discussions address the computational methodology to evaluate the expected costs resulting from travel time/cost with respect to the random choice of activity participation. It remains to incorporate the expected costs associated with time window violations, if any. Recall that the recourse action employed herein states that the adjustment to the predetermined *a priori* activity/travel path followed in the second stage is accomplished by simply skipping the cancelled uncertain activities once the random variable $\xi$ defining stochastic activity participation is realized. More precisely, during the execution process, any cancelled activities are simply skipped and travel proceeds to the next location in the sequence, which ostensibly leads to early arrival at the related activity location that may violate the specified time windows. In the recourse model, it is presumed that the early arrival is allowed, but incurs a penalty. Alternatively, as a result of particular realizations of the random variable $\xi$, infeasibility due to late arrival also incurs a penalty cost when evaluating the expected cost of a given activity/travel path. As a complementary component of the expected cost of a given activity/travel path, the incorporation of the expected time window violation costs is expected to provide better understanding in activity/travel behavior under uncertain environment.

Incorporation of the expected cost associated with time window violations is accomplished by an extension of the approach taken by Campbell and Thomas (2008) to compute the expected cost associated with deadline violations of an *a priori* tour for single vehicle of a PTSPD problem. Their approach constructs an *a priori* PTSP route in light of time constraints by embedding recourse models with time constraints and stochastic customer presence to handle the time-definite delivery problem in a stochastic environment. In its original development, the time restriction on each customer is modeled simply as deadlines, rather than time windows associated with start/end time as in the SHAPP problem; their methodology is applicable only to the single vehicle case in which no constraints other than the precedence conditions are considered. Alternatively, the SHAPP problem is a more difficult and comprehensive problem that involves routing of multi-vehicle, multi-member households with time-windows, ride-loads and other operational constraints. In the extension of their methodology to the case of multiple vehicles, multiple tours $\tau^v$ may exist, in which the activities are ordered by their respective start times. Each tour denoted by $\tau^v$ is considered separately, and then the respective components are combined together to derive the overall penalties.

For a given $\tau^v$, let $w\left\{1 \le w \le l^v\right\}$ be the $w-th$ node of the ordered tour, and $i_w^v$ corresponds to the associated prescribed activity. Assume that a "per-unit-time" penalty is incurred if time window violation occurs. Let $\lambda_{tpl}^{i_w^v}\left(i_w^v \in \tau^v \text{ and } 1 \le w \le l^v\right)$ and $\lambda_{tpe}^{i_w^v}\left(i_w^v \in \tau^v \text{ and } 1 \le w \le l^v\right)$ denote the relative late/early arrival penalty weights assigned to the $i_w^v$-*th* activity, respectively. In essence, the kernel of the evaluation procedure is to determine the probability that a time window violation—either early arrival or late arrival—of a particular activity occurs. A random variable $R_{i_w^v}$ is defined denoting whether or not an activity with uncertain participation is realized:

$$R_{i_w^v} = \begin{cases} 0, \text{ if activity } i_w^v \text{ is not realized} \\ 1, \text{ if activity } i_w^v \text{ is realized} \end{cases} \tag{7}$$

Recall that decision variable $T_{i_w^v}\ \left(i_w^v \in \tau^v\right)$ denotes the random variable indicating the start time of a particular activity; let $t$ represent the outcome of random variable $T_{i_w^v}$. Noting that a cancelled activity cannot contribute to any time window violations, the computation of the penalty term is applicable only to the realized activities. Alternatively, attention should be concentrated on evaluating the probability that participation in activity $i_w^v$ is started at time instant $t$ by vehicle $v$ given that activity $i_w^v$ is realized, which can be written as:

$$g^v\left(i_w^v,t\right)=P\left(T_{i_w^v}=t \middle| R_{i_w^v}=1\right) \tag{8}$$

Given $i_w^v$ is realized, the start time of activity $i_w^v$ is at time instant $t$ if and only if the precedent activity, say $i_u^v$, starts at time $\left(t-t_{i_u^v,i_w^v}-S_{i_u^v}\right)$, and these two activities are directly connected. Then, the above probability is equal to:

$$g^v\left(i_w^v,t\right)=P\left(T_{i_w^v}=t \middle| R_{i_w^v}=1\right)=\sum_{u=0}^{u=w-1} g\left(i_u^v,t-t_{i_u^v,i_w^v}-S_{i_u^v}\right)\cdot \overline{p}^{\,v}_{i_u^v,i_w^v},\ w\geq 1 \tag{9}$$

where $\overline{p}_{i_u^v,i_w^v}$ is specified by Equations (6a) through (6f). Under the assumption that the departure initiating each tour (by definition, each tour $\tau^v$ starts from home) in any *a priori* activity/travel pattern strictly follows the pre-determined start time, the following initial conditions are obtained:

$$g(i_0^v,t)=\begin{cases}1, & t=T_0^v\\ 0, & t\neq T_0^v\end{cases} \tag{10}$$

As such, the value of probability $g^{v}\left(i_{w}^{v},t\right)$ can be recursively derived based on the given initial conditions. Then, the overall expected penalty cost associated with delay in activity participation can be written as:

$$\sum_{v\in V}\sum_{w=1}^{l^{v}}\sum_{t=b_{i_{w}^{v}}}^{LT_{i_{w}^{v}}}\lambda_{tpl}^{i_{w}^{v}}\cdot g\left(i_{w}^{v},t\right)\cdot\left(t-b_{i_{w}^{v}}\right) \tag{11a}$$

where $LT_{i_{w}^{v}}$ refers to the latest potential starting of activity $i_{w}^{v}$, which is virtually the time instant to perform activity $i_{w}^{v}$ in the case that all the preceding activities of the *w*-th activity are completed, i.e. $T_{i_{w}^{v}}$ according to the given activity/travel pattern. Note that if the value of $T_{i_{w}^{v}}$ is less than that of $b_{i_{w}^{v}}$, no late start penalty is generated. Similarly, the early arrival penalty term for a given $\tau^{v}$ can be specified by

$$\sum_{v\in V}\sum_{w=1}^{l^{v}}\sum_{t=ET_{i_{u}^{w}}}^{a_{i_{w}^{v}}}\lambda_{tpe}^{i_{w}^{v}}\cdot g\left(i_{w}^{v},t\right)\cdot\left(a_{i_{w}^{v}}-t\right) \tag{11b}$$

where $ET_{i_{w}^{v}}$ denotes the earliest potential starting time of activity $i_{w}^{v}$. It is equivalent to the earliest time that activity $i_{w}^{v}$ could be started, which can be computed as follows:

$$\widehat{ET_{i_{w}^{v}}}=T_{0}^{v}+t_{0,i_{w}^{v}}\text{, where }0\leq u<w \tag{12}$$

As implied above, $\widehat{ET_{i_{w}^{v}}}$ results from traveling directly from home to activity $i_{w}^{v}$ while skipping the absent activities: from activity 1 to activity (*w*-1)-*th* along route $\tau^{v}$. It should be noted that $\widehat{ET_{i_{w}^{v}}}$ represents the lower bound approximation of the earliest time that we could start the *w-th* activity considering that there may be deterministic activity between the first activity and the (*w*-1)-*th* activity, which cannot be skipped. Intuitively, no early start penalty is incurred when the earliest start time $\widehat{ET_{i_{w}^{v}}}$ is always later than $a_{i_{w}^{v}}$.

Based on the above arguments, the expected disutility $E(\mathbf{X}_i, \boldsymbol{\xi})$ associated with a given activity/travel pattern is defined by Equations (3) through (11), and thus the relevant expected recourse cost $Q(\mathbf{X}_i)$ can be derived. However, it should be noted that to compute the exact value of the objective function—Equation (1)—of a two-stage stochastic program as we have formulated is challenging. The evaluations of the expected recourse cost would require solving the integer recourse problem for all possible realizations of the variables in the case of discrete distributed random variables herein, which may take exponential time to obtain the optimal solution.

The optimization specified in Equations (1) is subject to the same spatial and temporal constraints contained in Recker (1995) and displayed for completeness in the APPENDIX. These constraints generally capture the physical conditions, ensuring that each member of the household, as well as each vehicle used by the household, has a consistent continuous, path thorough time-space prisms that result in the household's agenda, being successfully completed by taking into account the effects of a subset of uncertain activities. Generally, the constraint set can be divided into four classes as follows: (I) those constraints related to activity assignment and vehicle movement, together with temporal constraints that confine the starting time of an activity (Equations (A1)-(A25)); (II) those constraints state the relationship between household member decisions and activity implementations (Equations (A26)-(A40)); (III) those coupling constraints that define the interactions between household member decisions and the vehicle flow variables (Equations (A41)-(A43)); (IV) those constraints that specify the non-negativity restriction on decision variables (Equation (A44)).

## 3. Illustrative Example and Discussion

The SHAPP problem herein attempts to determine the optimal *a priori* activity/travel solution with minimum expected cost. A feasible *a priori* activity/travel path generally can be found by solving the deterministic HAPP model (Recker, 1995) based on the assumption that each prescribed activity in the agenda is performed and all operational constraints (e.g., the vehicle-person/activity exclusion constraint, or time window constraints, etc.) are satisfied. We argue that an optimal activity/travel pattern from the deterministic HAPP model, if available, is not necessarily the optimal solution to the corresponding SHAPP problem. Furthermore, in some extreme cases resulting from random variable realizations, no activity/travel pattern obtained through HAPP model is feasible to serve as an *a priori* solution for the SHAPP problem. In this section, a simple example is presented to demonstrate the impacts of uncertain activity participation on the determination of an *a priori* activity/travel pattern within the context of a particular household. The particular numerical example presented demonstrates the differences in cost between the deterministic and the stochastic setting and further shows that a modeling process inherently capturing the random choice of activity participation can substantially improve model performance. In this example, we consider a single household member with exclusive, unrestricted use of a personal vehicle who plans to engage in three out-of-home activities, one of which—activity 2— is tightly constrained in terms of its time window of availability. The appropriate characteristics defining the activity agenda are described by Table 2:

| Variable | Variable Set |
|---|---|
| household members | $\eta = \{1\}$ |
| available vehicles | $V = \{\mathrm{V}_1\}$ |
| out-of-home activities | $A = \{1,\ 2,\ 3\}$ |
| activity durations | $S = [s_1,\ s_2,\ s_3] = [1.5, 1.0, 2.0]$ |
| time availability windows | $\left[a_i, b_i\right] = \begin{bmatrix} 14.00 & 24.00 \\ 16.00 & 16.15 \\ 14.00 & 24.00 \end{bmatrix}$ |
| return-home windows | $\left[a_{n+i}, b_{n+i}\right] = \begin{bmatrix} 14.00 & 24.00 \\ 14.00 & 24.00 \\ 14.00 & 24.00 \end{bmatrix}$ |
| initial departure windows | $\left[\overline{a}_0^{\alpha}, \overline{b}_0^{\alpha}\right] = \left[14.00 \quad 16.00\right]$ |
| end-of-day windows | $\left[\overline{a}_{2n+1}^{\alpha}, \overline{b}_{2n+1}^{\alpha}\right] = \left[14.00 \quad 24.00\right]$ |
| travel cost budget | $\mathbf{B}_{\mathbf{t}}^{\mathbf{1}} = 8.00$ |
| maximum number of sojourns in any tour | $D_s = 4$ |

**Table 2  Parameters of Illustrative Example**

The travel time and travel cost matrices associated with the locations of the planned out-of-home activities are shown in Table 3:

| $t_{uw}^{v}$ | | | | | | $c_{uw}^{1}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **To** | 0 | 1 | 2 | 3 | | **To** | 0 | 1 | 2 | 3 |
| **From** | | | | | | **From** | | | | | |
| 0 | | 0.00 | 0.50 | 0.15 | 0.60 | 0 | | 0.00 | 1.00 | 0.30 | 1.20 |
| 1 | | 0.50 | 0.00 | 0.50 | 0.50 | 1 | | 1.00 | 0.00 | 1.00 | 1.00 |
| 2 | | 0.15 | 0.50 | 0.00 | 0.25 | 2 | | 0.30 | 1.00 | 0.00 | 0.50 |
| 3 | | 0.30 | 0.50 | 0.25 | 0.00 | 3 | | 0.60 | 1.00 | 0.50 | 0.00 |

**Table 3 Travel Time and Cost Matrices of Illustrative Example**

In addition, we assume that travel time is the only component determining the optimal *a priori* solution; i.e., the relative weights assigned to the respective terms of the objective function for the household defined by Equation (2) are: $\lambda_{tc} = 0.0$; $\lambda_{tt} = 1.0$.

The resulting optimal activity/travel tour obtained from the deterministic HAPP model-labeled Pattern I— is shown in Figure 2. In this *a priori* schedule, the optimal pattern has the person performing activity 2 first, followed by activity 3 and then activity 1, with total travel cost of 1.40. We note that, in the deterministic setting, the activity starting time window of activity 2 uniquely determines the activity chaining and the resulting initial departure and activity start time.

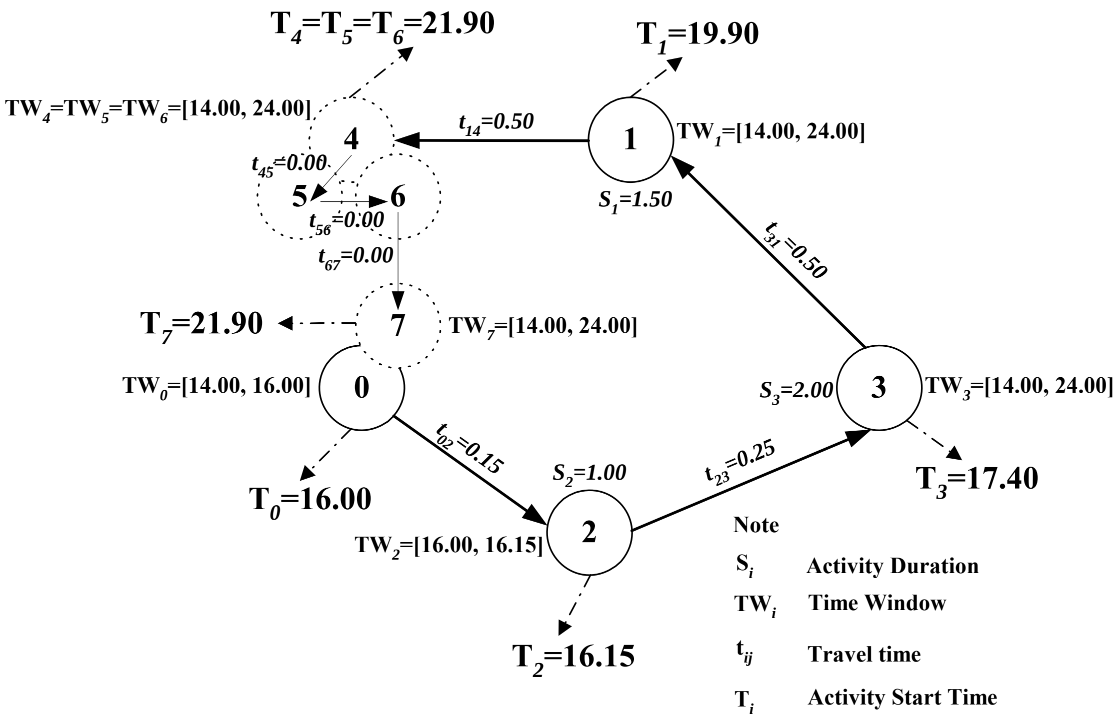


**Figure 2 Activity/Travel Pattern I**

The optimal solution depicted by Pattern I in Figure 2 is based on the assumption that each activity of the prescribed household agenda is deterministic. However, the problem in which each activity only has a known probability of being completed or cancelled is captured by the SHAPP formulation. For this simple example, we specify the respective probabilities of activity participation as follows:

$$\mathbf{p} = \left[ p_1, p_2, p_3, p_4, p_5, p_6 \right] = [1.00, 0.10, 0.50, 1.00, 0.10, 0.50]$$

As noted before, $p_0 = p_7 \equiv 1.00$, since they constitute the beginning and end nodes of the travel day. Moreover, we assume that each realized activity will be completed regardless of whether or not a time window occurs, while the cancelled activities are simply skipped. Therefore, the expected cost of Pattern I can be computed as shown in Section 2.

As displayed in Figure 2, Pattern I is specified by node set $\tau = \{0, 2, 3, 1, 4, 5, 6, 7\}$ and the number of nodes included in Pattern I (excluding the beginning home node) is $l = 6$. (The index of $v \in V$ is abbreviated throughout this section because only one vehicle is available in the illustrative example.) Then, re-label set $\tau$ by the order of completion precedence, and thus $\tau$ can be rewritten as follows:

$$\begin{array}{ll} \text{activity node} & \Rightarrow \\ & \\ \text{activity node} & \Rightarrow \\ & \\ \text{complete precedence} & \Rightarrow \end{array} \begin{bmatrix} 0 & 2 & 3 & 1 & 4 & 5 & 6 & 7 \\ \| & \| & \| & \| & \| & \| & \| & \| \\ i_0 & i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

According to Section 2, the expected cost of *a priori* activity/travel pattern $\mathbf{X}_i$, can be broken down into two components: expected travel time cost and expected time window violation cost. First, we compute the expected cost associated with link travel time; this involves computing probabilities of direct travel between any pair of nodes of $\tau$. Given Pattern I, we obtain the probability matrix $\overline{p}_{i_u, i_w}$ based on Equations (5) and (6) displayed in Table 4. By assumption, the household member executes the *a priori* activity/travel pattern and simply skips the cancelled activities until the outcomes of the random variables materialized. As a result, Pattern I has an expected cost of 1.30 and the time window restriction on Activity 2 is always satisfied regardless of the realizations of the uncertain activity participations during the execution of *a priori* activity/travel pattern.

$\overline{p}^{v}_{i_u, i_w}$

| From | To | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | 0.00 | 0.45 | 0.10 | 0.45 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 |
| 2 | | 0.00 | 0.05 | 0.00 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | | 0.00 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.45 | 0.45 |
| 5 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 |
| 6 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 |

**Table 4 Direct Travel Probability of Pattern I**

However, as outlined above, the optimal activity/travel pattern obtained through the deterministic HAPP model may not be optimal under uncertain environment from the perspective of minimizing the disutility to the household of the total activity/travel package. It is noticed that the tight time window associated with activity 2 drives the cost of Pattern I but the participation probability in Activity 2, $p_2 = 0.1$, is relatively low. Consider, for example, the alternative schedule, denoted by Pattern II, displayed in Figure 3. In this schedule, the household is scheduled to perform activity 2 beginning at 16.50, i.e., after the allowable latest start time specified by the time window. The resulting deterministic travel cost of Pattern II is 1.55, which is clearly inferior to Pattern I, the optimal obtained under the deterministic setting. However, Pattern II becomes preferable to Pattern I when the uncertainties of activity participation are taken into account. The probabilities of direct travel, computed based on Equations (5) and (6), for Pattern II are illustrated in Table 5. The expected cost of Pattern II is 1.17, which is 10.0% less than that of Pattern I. Moreover, the relevant time window associated with activity 2 is violated only 10% of the time, which is clearly a low probability event.

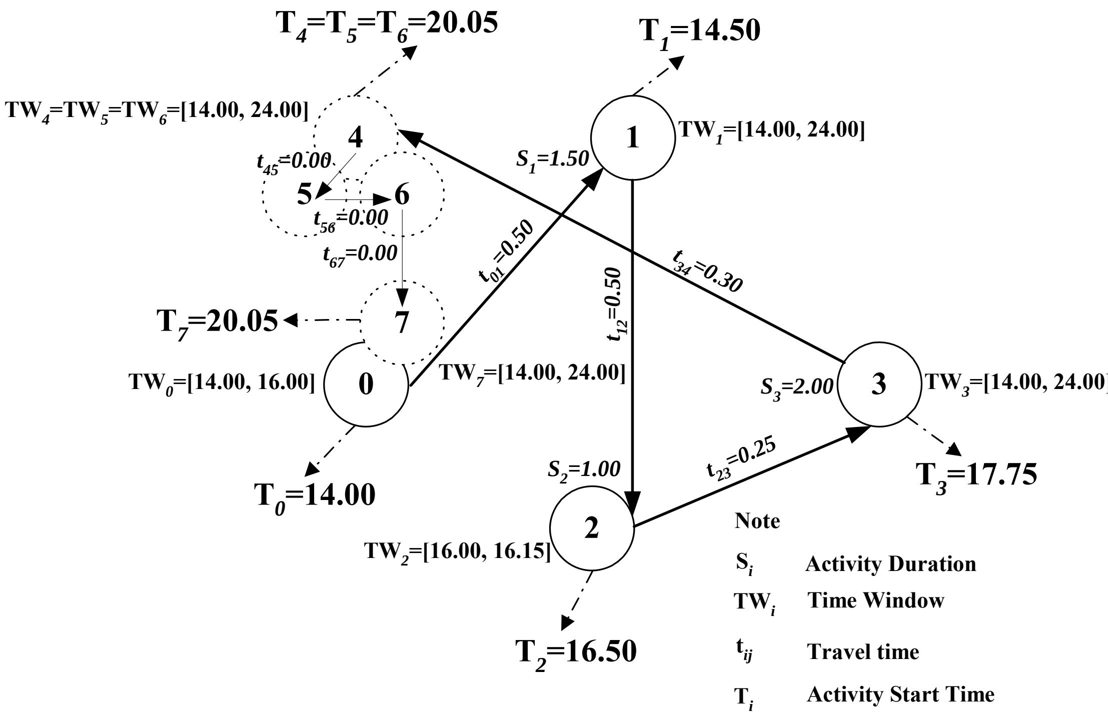


**Figure 3 Activity/travel Pattern II**

| $\overline{p}^{v}_{i_u,i_w}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **To** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **From** | | | | | | | | | |
| 0 | | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | | 0.00 | 0.00 | 0.10 | 0.45 | 0.45 | 0.00 | 0.00 | 0.00 |
| 2 | | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 |
| 3 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 | 0.00 | 0.00 |
| 4 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.45 | 0.45 |
| 5 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 |
| 6 | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 |

**Table 5 Direct Travel Probability of Pattern II**

A per-unit-recourse cost (late arrival) generated by time window violation of activity 2 greater than 3.71 is necessary before Pattern I is superior to Pattern II. This example demonstrates that an optimal solution of HAPP problem is not necessarily a good solution to the corresponding SHAPP problem, which reinforces the statement that travel decisions will be impacted when considering uncertainty concerning activity participation. Finally, it is clear that travel decisions under uncertain environment consist of compromises between deterministic activities and uncertain activities. The numerical results further indicate that, to the extent that uncertainties play a role in the decision process, model performance can be improved by incorporating explicit consideration of the uncertain activities with low probability of participation.

## 4. Solution Methodology

The problem defined by Equation (1) and Equations (A1) through (A44) is generally regarded as computationally challenging, which can be attributed to the nonlinear recourse function specified by Equation (1a) with presence of multi-dimensional integration to calculate expectation. Although the relevant variables are discrete in the SHAPP model, the total number of outcomes of a multi-dimensional random vector increases exponentially with problem size such that the calculations associated with the summations may be far too demanding to be practical. Hence, one may have to resort to approximations. In the literature, the so-called L-shaped method is widely applied to solve the two-stage stochastic programming with recourse to optimality (Gendreau et al., 1995 and Teng et al., 2004). The basic idea of the L-shaped method is to gradually approximate the recourse cost in the objective function from below by adding optimal cuts (which provides the lower bound of the approximated recourse cost) to the current constraints until optimality is reached. In this section, we employ the L-shaped method to solve the formulated SHAPP model to optimality.

Recall that for a given *priori* activity/travel pattern and a particular realized random vector $\boldsymbol{\xi}$, $C(\mathbf{X}_i)$ denotes the cost of the first stage solution, while $Q(\mathbf{X}_i, \boldsymbol{\xi})$ is the cost incurred by recourse actions, i.e. the sum of the time window violation if any, minus the decreases in disutility to the household. It implies that the term $Q(\mathbf{X}_i, \boldsymbol{\xi})$ is not always positive, and as a sequence, the expected recourse cost $Q(\mathbf{X}_i)$ defined by Equation (1c) thus can be of any sign. The above statement can be verified by the illustrated example shown in the previous section. Take Pattern I as an example. The total expected cost

$E_{\xi} Z(\mathbf{X}_i, \boldsymbol{\xi})$ is 1.30, while the cost of the first stage solution denoted by $C(\mathbf{X}_i)$ is 1.40; in this case, the resulting expected recourse cost $Q(\mathbf{X}_i)$ is -0.10, which is negative. Similar conclusions can be drawn for Pattern II. The negative recourse cost does not imply the failure of the model formulation; however, from the view of the solution methodology, a nonnegative recourse cost function is more preferable, due to the basic principle of the so-called L-shaped methodology that the penalty term is initially relaxed from the objective function, and is gradually approximated by a nonnegative term constrained by the optimality cut. As a result, Equation (1c) should be redefined to guarantee the non-negativity of the expected recourse cost. Gendreau et al. (1995) have discussed the similar problem and proved that the non-negativity can be ensured by defining a lower bound on the expected travel cost. The methodology has been adapted to the formulation herein, whiling taking into account the particular characteristics of the household activity problem, which essentially is a Pickup and Delivery Problem. First, rewrite Equation (1b) as below:

$$\min E_{\xi} Z(\mathbf{X}_i, \boldsymbol{\xi}) = \min\left\{\widetilde{C}(\mathbf{X}_i) + \widetilde{Q}(\mathbf{X}_i)\right\} \tag{1b'}$$

where $\widetilde{C}(\mathbf{X}_i)$ denotes the lower bound on the expected cost associated with the first stage solution, given that some activities in the prescribed agenda will be cancelled, and thus skipped in the second stage when the uncertain factors are realized. It is clear that $\widetilde{C}$ consists of the similar components defined by Equation (3); however, its components need to be redefined in order to satisfy the non-negativity restriction on the expected recourse cost. Based on the definitions given by Gendreau et al. (1995), we have the following substitution of terms in Equation (3):

$$\sum_{v\in V}\sum_{u\in N}\sum_{w\in N}\overline{p}_{uw}^{v}\cdot\tau_{uw}^{v}\cdot X_{uw}^{v}\rightarrow\sum_{v\in V}\sum_{u\in N}\sum_{w\in N}\tilde{\tau}_{uw}^{v}\cdot X_{uw}^{v}$$

where $\tilde{\tau}_{uw}^{v}=\begin{cases}\tau_{uw}^{v}, & \text{if } u=w+n \text{ or } w=u+n;\\ p_u p_w\tau_{uw}^{v}+\dfrac{1}{2}\left\{p_u\left(1-p_w\right)\min\limits_{k\neq u,w}\tau_{uk}^{v}+p_w\left(1-p_u\right)\min\limits_{k\neq u,w}\tau_{kw}^{v}\right\} & \text{otherwise}\end{cases}$

for $\tau=t,c$.

Thus, we have $\widetilde{Q}\left(\mathbf{X}_i\right)=Q\left(\mathbf{X}_i\right)+C\left(\mathbf{X}_i\right)-\widetilde{C}\left(\mathbf{X}_i\right)$, which is nonnegative.

Again, we take Pattern I shown in Section 3 as an example to verify the above specifications. The expected cost $E\left(\mathbf{X}_i,\xi\right)=Q\left(\mathbf{X}_i\right)+C\left(\mathbf{X}_i\right)=1.30$, while the lower bound of the expected cost of $\widetilde{C}\left(\mathbf{X}_i\right)=1.28$. As a result, the value of function $\widetilde{Q}\left(\mathbf{X}_i\right)$ is 0.02, which is positive.

After specifying the formulation of $\widetilde{Q}\left(\mathbf{X}_i\right)$, the L-shaped algorithm adapted to the SHAPP model can be summarized as below:

First, let $\eta$ denote the estimated lower bound on the expected recourse cost function $\widetilde{Q}\left(\mathbf{X}_i\right)$. Hence, at a given phase of the algorithm, a master problem can be formulated as follows:

Master Problem (MP):

$$\min \quad \widetilde{C}\left(\mathbf{X}_i\right)+\eta \tag{13a}$$

subject to:

Constraint Set I: (A1) through (A44)

Constraint Set II: set of optimality cuts (13b)

Note that in the initial Master Problem (MP), the Constraint Set II may only contain the constraint $\eta \geq 0$. In the subsequent iterations, newly identified optimality cuts—the approximation of the expected recourse cost—will be added to Constraint Set II.

Assume $\mathbf{X}_i^u$ and $\eta^u$ to be the current solution of the above MP; then, we are able to evaluate the second stage problem and derive the optimality cuts. Gendreau et al. (1995) derived valid optimality cuts for the vehicle routing problem (VRP) with stochastic demands and customers based on the structure of the model output, which essentially consists of the discrete vehicle routing decisions. We follow this idea to generate the optimality cuts by taking advantage of the particular structure of the model output of the SHAPP problem. Let $Z\left(\mathbf{X}_i^u\right) = E_{\xi}\left(\mathbf{X}_i^u, \xi\right)$, which denotes the expected cost of the second stage problem. We have the following proposition.

PROPOSITION 1:

Let $\left(\mathbf{X}_i^u, \eta^u\right)$ be the optimal solution to the master problem (MP). Define

$$\widetilde{Q}\left(\mathbf{X}_i^u\right) = Z\left(\mathbf{X}_i^u\right) - \widetilde{C}\left(\mathbf{X}_i^u\right) \tag{14}$$

which is the value of the expected recourse cost at the current solution, and let $\left\{E^u = (i,j) \in E : i, j \neq 0, 2n+1 \text{ and } X_{ij}^{v,u} = 1\right\}$. Then the valid optimality cut can be written as follows:

$$\eta \geq \widetilde{Q}\left(\mathbf{X}_i^u\right) \cdot \left( \sum_{(i,j)\in E^u} X_{ij}^v - 2n + |V| + 1 \right) \tag{15}$$

*Proof:*

Consider the discrete variable-vehicle flows. Observe that the optimal vehicle flow vector from $\mathbf{X}_i^u$ to the MP is characterized by $E^u$ and that

$$\sum_{(i,j)\in E^u} X_{ij}^{v,u} = 2n - |V| \tag{16a}$$

Any other solution $\mathbf{X_i^\lambda}$ with a different edge set $E^\lambda \neq E^u$, and we further have

$$\sum_{(i,j)\in E^\lambda} X_{ij}^{v,\lambda} \leq 2n - |V| - 1 \tag{16b}$$

Based on the above argument, for solution $\mathbf{X}_i^u$, the right hand side of Equation (15) is equivalent to:

$$RHS = \widetilde{Q}\left(\mathbf{X}_i^u\right) \cdot \left(2n - |V| - 2n + |V| + 1\right) = \widetilde{Q}\left(\mathbf{X}_i^u\right) \tag{16c}$$

Therefore, we have $\eta \geq Q\left(\mathbf{X}_i^u\right)$, and Equation (15) is satisfied.

For any solution $\mathbf{X}_i^\lambda \neq \mathbf{X}_i^\mathbf{u}$, the right hand side of Equation (15) can be written as below:

$$RHS \leq \left|\widetilde{Q}\left(\mathbf{X}_i^\lambda\right)\right| \cdot \left(0 + 1 - 1\right) = 0 \tag{16d}$$

and also variable $\eta$ is nonnegative. Therefore, variable $\eta$ must satisfy $\eta \geq 0$ for any feasible solution $\mathbf{X}_i^\lambda$.

We now outline the L-shaped method for the SHAPP problem:

Step 0: Set iteration index $u := 0$. Initialize the master problem (the MP) with constraint set (B1) through (B44) and constraint (4.4) only containing $\eta \geq 0$.

Step 1: Set $u = u + 1$; Solve the MP. If the MP is infeasible, go to Step 4. Otherwise, denote $\left(\mathbf{X}_i^u, \eta^u\right)$ as the optimal solution.

Step 2: Compute the value of the expected recourse cost of the current solution given by Equation (14).

Step 3: If $\eta \geq \widetilde{Q}\left(\mathbf{X}_i^u\right)$, the MP satisfies the optimality criterion, go to Step 4. Otherwise, add the optimality cut (Equation (15)) to constraint set (13b), and go to Step 1.

Step 4: Output the best-known solution and stop.

For the purpose of illustration, the proposed L-shaped algorithm has been applied to derive the optimal *a priori* activity/travel pattern of the illustrative example defined in Section 3. Parameter settings of objective function $C\left(\mathbf{X}_i\right)$ remain as those specified in Section 3. In addition, the per-unit recourse cost is set to be: $\lambda_{tpl} = 2.5$. The CPLEX solver is used to solve the resulting problem iteratively, and the “optimal” solution is displayed in Figure 4, with deterministic travel cost of 1.45, and expected travel cost of 1.17, but no time window violations in either scenario.

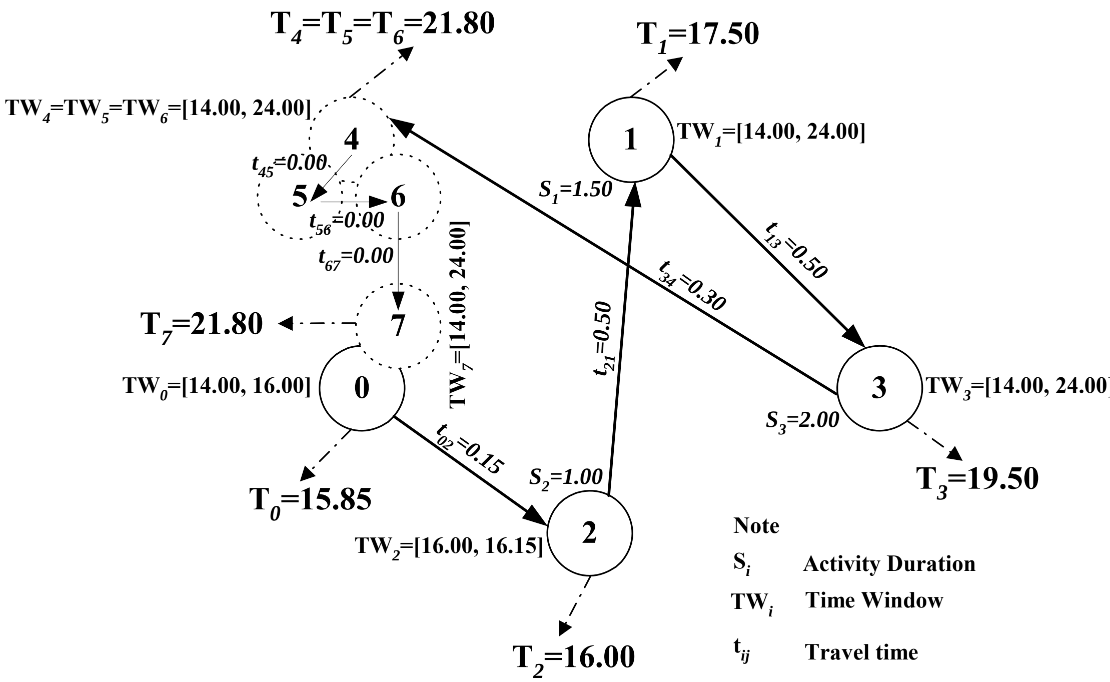


**Figure 4. Optimal Activity-Travel Pattern of Illustrative Example**

## 5. Summary and Conclusions

This paper has presented the SHAPP model, an extension to the well-known deterministic HAPP formulation that introduces random choice of activity participation as a consideration in the formation of planned activity patterns. SHAPP is formulated to replicate the activity/travel pattern within a household context while embedding the particular uncertain factors within the well-defined deterministic setting. We formulated the SHAPP model as a two-stage stochastic integer linear programming problem with recourse to determine an optimal *a priori* activity/travel pattern within the context of stochastic activities, while following a disutility minimization framework. Here we also propose a methodology to evaluate the expected cost of any given activity/pattern schedule to facilitate the algorithm to solve the stochastic programming problem, which is applicable to determine the time window violation penalty, if any. An illustrative example is provided to demonstrate the evaluation methodology. The experiment clearly indicates that the optimal activity/travel pattern obtained through deterministic activity-based models, such as by the HAPP model, may not be optimal in an uncertain environment. The result reinforces the claim that travel behaviors may be affected significantly when uncertainty concerning activity participation is brought into the prediction framework, further justifying the motivation to develop a household activity scheduling problem under uncertain environment. For the purpose of implementing the proposed model in large scale, the L-Shaped algorithm is tailored to solve the problem to optimality.

As a framework, the proposed SHAPP model is viewed as an initial step toward explicitly incorporating uncertain factors into activity-based behavioral travel analysis founded on mathematical programming principles. Despite the mathematical consistency and clarity afforded by the basic HAPP structure on which the current model is based, it must be remarked that the HAPP model (and most of its derivatives) merely constrains the utility maximizing principles assumed to be at play in the formation of household activity patterns to be executed in a manner consistent with the physical/mechanical constraints on activity participation imposed by Hägerstrand's time-space prism—there remains precious little behavior in this behavioral model. The results presented here represent an attempt to incorporate explicit recognition that, as a simultaneous decision model, the processes imbedded in HAPP are likely influenced by uncertainty over the planning horizon—uncertainty not exposed by the revealed patterns of the household typically obtained from household travel surveys. Because it is *np-hard*, a deliberate attempt toward simplification in the SHAPP model has been made in terms of the assumptions that underlay the uncertainty addressed in order to maintain tractability. We view this and other efforts to extend the model to a descriptive, rather than proscriptive, setting (Chow and Recker, 2012) as steps toward its use in the context of actually forecasting activity/travel behavior. Furthermore, although it is not intended to be used as a dynamic rescheduling algorithm, the proposed model can be applied to generate planned activity/travel schedules that form the initial pre-planned skeleton schedule on which dynamic rescheduling models, such as the HARP activity rescheduling model (Gan and Recker, *op cit*), rely.

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# APPENDIX

# HAPP CONTINUITY CONSTRAINTS

*(I) Spatial Connectivity and Temporal Constraints on Vehicles*

$$\sum_{w \in \Omega_V^v} \sum_{u \in P} X_{uw}^v = 0, \forall v \in V \tag{A1}$$

$$\sum_{v \in V} \sum_{w \in N} X_{uw}^v = 1, \quad \forall u \in P^+ \tag{A2}$$

$$\sum_{w \in N} X_{uw}^v - \sum_{w \in N} X_{wu}^v = 0, \quad \forall u \in P, v \in V \tag{A3}$$

$$\sum_{w \in P^+} X_{0w}^v \leq 1, \quad \forall v \in V \tag{A4}$$

$$\sum_{u \in P^-} X_{u,2n+1}^v \leq 1, \quad \forall v \in V \tag{A5}$$

$$\sum_{w \in N} X_{wu}^v - \sum_{w \in N} X_{w,n+u}^v = 0, \quad \forall u \in P^+, v \in V \tag{A6}$$

$$X_{uw}^v = 0 \text{ or } 1, \forall u, w \in N, v \in V \tag{A7}$$

$$X_{uw}^v + X_{wu}^v \leq 1, \quad u, w \in N, v \in V \tag{A8}$$

$$\begin{aligned} &X_{uu}^v = 0, \quad \forall u \in N,\ v \in V \\ &X_{0u}^v = 0, \quad \forall u \in P^-,\ v \in V \\ &X_{u0}^v = 0, \quad \forall u \in N, v \in V \\ &X_{u,2n+1}^v = 0, \forall u \in P^+, v \in V \\ &X_{2n+1,u}^v = 0, \forall u \in P^+, v \in V \\ &X_{n+u,u}^v = 0, \forall u \in P^+, v \in V \end{aligned} \tag{A9}$$

$$Y_u + d_w - Y_w \leq \mathrm{M}\left(1 - X_{uw}^v\right), \forall u \in P, w \in P^+, v \in V \tag{A10}$$

$$Y_u - d_{w-n} - Y_w \leq \mathrm{M}\left(1 - X_{uw}^v\right), \forall u \in P, w \in P^-, v \in V \tag{A11}$$

$$Y_0 + d_w - Y_w \le \mathrm{M}\left(1 - X_{0w}^{v}\right), \forall w \in P^{+}, v \in V \tag{A12}$$

$$Y_0 = 0 \tag{A13}$$

$$0 \le Y_u \le D^{s}, \forall u \in P \tag{A14}$$

$$T_u + S_u + t_{uw}^{v} - T_w \le \mathrm{M}\left(1 - X_{uw}^{v}\right), \forall u, w \in P, v \in V \tag{A15}$$

$$T_u + S_u + t_{uw}^{v} - T_w \ge -\mathrm{M}\left(1 - X_{uw}^{v}\right), \quad \forall\, u, w \in P, v \in V \tag{A16}$$

$$T_0^{v} + t_{0u}^{v} - T_u \le \mathrm{M}\left(1 - X_{0u}^{v}\right), \quad \forall u \in P^{+}, v \in V \tag{A17}$$

$$T_0^{v} + t_{0u}^{v} - T_u \ge -\mathrm{M}\left(1 - X_{0u}^{v}\right), \quad \forall u \in P^{+}, v \in V \tag{A18}$$

$$T_u + S_u + t_{u,2n+1} - T_{2n+1}^{v} \le \mathrm{M}\left(1 - X_{u,2n+1}^{v}\right), \quad \forall u \in P^{-}, v \in V \tag{A19}$$

$$T_u + S_u + t_{u,n+u}^{v} \le T_{n+u}, \quad \forall u \in P^{+}, v \in V \tag{A20}$$

$$a_u \le T_u \le b_u, \quad \forall u \in P \tag{A21}$$

$$a_0 \le T_0^{v} \le b_0, \quad \forall v \in V \tag{A22}$$

$$a_{2n+1} \le T_{2n+1}^{v} \le b_{2n+1}, \quad \forall v \in V \tag{A23}$$

$$\sum_{v \in V} \sum_{u \in N} \sum_{w \in N} c_{uw}^{v} X_{uw}^{v} \le B_c \tag{A24}$$

$$\sum_{u \in N} \sum_{w \in N} t_{uw} X_{uw}^{v} \le B_t^{v}, \quad \forall v \in V \tag{A25}$$

where $\mathrm{M}$ is a large positive number.

*(II) Constraints on Household Members*

$$\sum_{w \in \Omega_{\mathrm{H}}^{\alpha}} \sum_{u \in P} H_{uw}^{\alpha} = 0, \forall \alpha \in \eta \tag{A26}$$

$$\sum_{u \in N} \sum_{w \in N} t_{uw}^{v} H_{uw}^{\alpha} \le B_t^{\alpha}, \quad \forall \alpha \in \eta \tag{A27}$$

$$\sum_{\alpha\in\eta}\sum_{w\in N} H_{uw}^{\alpha} = 1, \quad \forall u \in A \tag{A28}$$

$$\sum_{w\in N} H_{uw}^{\alpha} - \sum_{w\in N} H_{wu}^{\alpha} = 0, \quad \forall u \in P, \alpha \in \eta \tag{A29}$$

$$\sum_{w\in P^{+}} H_{0w}^{\alpha} \le 1, \quad \forall \alpha \in \eta \tag{A30}$$

$$\sum_{u\in P^{-}} H_{u,2n+1}^{\alpha} \le 1, \quad \forall \alpha \in \eta \tag{A31}$$

$$\sum_{w\in N} H_{wu}^{\alpha} - \sum_{w\in N} H_{w,n+u}^{\alpha} = 0, \quad \forall u \in P^{+}, \alpha \in \eta \tag{A32}$$

$$H_{uw}^{\alpha} = 0 \text{ or } 1, \ \forall u, w \in N, \ \alpha \in \eta \tag{A33}$$

$$H_{uw}^{\alpha} + H_{wu}^{\alpha} \le 1, \quad u, w \in N, \alpha \in \eta \tag{A34}$$

$$\begin{aligned}
&H_{uu}^{\alpha} = 0, \quad \forall u \in N, \alpha \in \eta \\
&H_{0u}^{\alpha} = 0, \quad \forall u \in P^{-}, \alpha \in \eta \\
&H_{u0}^{\alpha} = 0, \quad \forall u \in N, \alpha \in \eta \\
&H_{u,2n+1}^{\alpha} = 0, \ \forall u \in P^{+}, \alpha \in \eta \\
&H_{2n+1,u}^{\alpha} = 0, \ \forall u \in P^{+}, \alpha \in \eta \\
&H_{n+u,u}^{\alpha} = 0, \ \forall u \in P^{+}, \alpha \in \eta
\end{aligned} \tag{A35}$$

$$T_u + S_u + t_{uw}^{v} - T_w \le \mathrm{M}\left(1 - H_{uw}^{\alpha}\right) \quad \forall u, w \in P, \ \alpha \in \eta, \ v \in V \tag{A36a}$$

$$T_u + S_u + t_{uw}^{v} - T_w \ge -\mathrm{M}\left(1 - H_{uw}^{\alpha}\right) \quad \forall u, w \in P, \ \alpha \in \eta, \ v \in V \tag{A36b}$$

$$\overline{T}_0^{\alpha} + t_{0u}^{v} - T_u \le \mathrm{M}\left\{\left(1 - H_{0u}^{\alpha}\right) + \left(1 - \sum_{w\in P^{-}} X_{wu}^{v}\right) - X_{0u}^{v}\right\} \quad \forall u \in P^{+}, \alpha \in \eta, v \in V \tag{A37a}$$

$$\overline{T}_0^{\alpha} + t_{0u}^{v} - T_u \ge -\mathrm{M}\left\{\left(1 - H_{0u}^{\alpha}\right) + \left(1 - \sum_{w\in P^{-}} X_{wu}^{v}\right) - X_{0u}^{v}\right\} \quad \forall u \in P^{+}, \alpha \in \eta, v \in V \tag{A37b}$$

$$T_u + t^v_{u,2n+1} - \overline{T}^{\alpha}_{2n+1} \le \mathrm{M}\left\{\left(1 - H^{\alpha}_{u,2n+1}\right) + \left(1 - \sum_{w \in P} X^v_{wu}\right) - X^v_{u,2n+1}\right\} \quad \forall u \in P^-, \alpha \in \eta, v \in V$$

(A38)

$$\overline{a}^{\alpha}_0 \le \overline{T}^{\alpha}_0 \le \overline{b}^{\alpha}_0, \quad \alpha \in \eta \tag{A39}$$

$$\overline{a}^{\alpha}_{2n+1} \le \overline{T}^{\alpha}_{2n+1} \le \overline{b}^{\alpha}_{2n+1}, \quad \alpha \in \eta \tag{A40}$$

*<u>(III) Vehicle and Household Member Coupling Constraints</u>*

$$\sum_{\alpha \in \eta} H^{\alpha}_{uw} = \sum_{v \in V} X^v_{uw}, \ \forall u \in N, w \in N \tag{A41}$$

$$T^v_0 - \overline{T}^{\alpha}_0 \le \mathrm{M}\left[\left(1 - H^{\alpha}_{0u}\right) + \left(1 - X^v_{0u}\right)\right] \quad \forall u \in P^+, \alpha \in \eta, v \in V \tag{A42a}$$

$$T^v_0 - \overline{T}^{\alpha}_0 \ge -\mathrm{M}\left[\left(1 - H^{\alpha}_{0u}\right) + \left(1 - X^v_{0u}\right)\right] \quad \forall u \in P^+, \alpha \in \eta, v \in V \tag{A42b}$$

$$T^v_{2n+1} - \overline{T}^{\alpha}_{2n+1} \le \mathrm{M}\left[\left(1 - H^{\alpha}_{u,2n+1}\right) + \left(1 - X^v_{u,2n+1}\right)\right] \quad \forall u \in P^-, \alpha \in \eta, v \in V \tag{A43a}$$

$$T^v_{2n+1} - \overline{T}^{\alpha}_{2n+1} \ge -\mathrm{M}\left[\left(1 - H^{\alpha}_{u,2n+1}\right) + \left(1 - X^v_{u,2n+1}\right)\right] \quad \forall u \in P^-, \alpha \in \eta, v \in V \tag{A43b}$$

*<u>(IV) Non-Negativity Constraints on Decision Variables</u>*

$$\begin{aligned} &X^v_{uw} \ge 0, u, w \in N, u \ne w;\ H^{\alpha}_{uw} \ge 0, \forall u, w \in N, \alpha \in \eta; T_u \ge 0,\ u \in P;\ T^v_0 \ge 0,\ \forall v \in V; \\ &T^v_{2n+1} \ge 0,\ \forall v \in V; \overline{T}^{\alpha}_0 \ge 0,\ \overline{T}^{\alpha}_{2n+1} \ge 0,\ \forall \alpha \in \eta; \end{aligned} \tag{A44}$$