
Volume 46B, issue 1, January 2012
ISSN 0191-2615
ELSEVIER
TRANSPORTATION
RESEARCH
AN INTERNATIONAL JOURNAL
Part B: Methodological
Editor-in-Chief: Fred L. Mannering

Transportation Research Part B 46 (2012) 463–479

Contents lists available at SciVerse ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Inverse optimization with endogenous arrival time constraints to calibrate the household activity pattern problem

Joseph Y.J. Chow *, Will W. Recker [1]

Institute of Transportation Studies, University of California, 4000 Anteater Instruction and Research Bldg., Irvine, CA 92697-3600, United States

ARTICLE INFO

Article history:
Received 27 February 2011
Received in revised form 10 November 2011
Accepted 11 November 2011

Keywords:
Inverse optimization
Activity based
Mixed integer linear programming
Pickup and delivery problem
Goal programming
Common prior

ABSTRACT

A parameter estimation method is proposed for calibrating the household activity pattern problem so that it can be used as a disaggregate, activity-based analog of the traffic assignment problem for activity-based travel forecasting. Inverse optimization is proposed for estimating parameters of the household activity pattern problem such that the observed behavior is optimal, the patterns can be replicated, and the distribution of the parameters is consistent. In order to fit the model to both the sequencing of activities and the arrival times to those activities, an inverse problem is formulated as a mixed integer linear programming problem such that coefficients of the objectives are jointly estimated along with the goal arrival times to the activities. The formulation is designed to be structurally similar to the equivalent problems defined by Ahuja and Orlin and can be solved exactly with a cutting plane algorithm. The concept of a unique invariant common prior is used to regularize the estimation method, and proven to converge using the Method of Successive Averages. The inverse model is tested on sample households from the 2001 California Household Travel Survey and results indicate a significant improvement over the standard inverse problem in the literature as well as baseline prescriptive models that do not make use of sample data for calibration. Although, not unexpectedly, the estimated optimization model by itself is a relatively poor forecasting model, it may be used in determining responses of a population to spatio-temporal scenarios where revealed preference data is absent.

1. Introduction

The concept of a utility-maximizing household is not a new one (Becker, 1965; Evans, 1972). This concept of a rational human being (perhaps not strictly so, but presumed rational nonetheless) has served as a foundation for a number of models of households or consumers maximizing their utilities by making choices on allocation of their time. In particular, the field of activity-based demand models (Kim, 2008; Pinjari and Bhat, 2011) tries to capture this behavior using a number of different methodologies. Some (as also noted in Timmermans et al., 2002; Lee and McNally, 2003; Miller and Roorda, 2003) have argued that people do not behave as global optimizers but rather behave in a local, greedy fashion. This proposition has resulted in a number of activity-based modeling developments built from purely econometric or rule-based computational process models that either do not assume any normative aspects of traveler decision-making except that which can be observed, or rely on heuristic behavioral rules. While this has resulted in a number of operational models, these models tend to be data-hungry and do not adapt well to scenarios without revealed preference data.

* Corresponding author. Tel.: +1 949 824 5989.
E-mail addresses: ychow@uci.edu (J.Y.J. Chow), wwrecker@uci.edu (W.W. Recker).
[1] Tel.: +1 949 824 5989.

0191-2615/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.trb.2011.11.005

Others (Kitamura, 1988; Recker, 1995; Bhat, 2005) have argued that travel behavior can indeed be captured within a rational utility-maximizing framework. The challenge is in defining an appropriate framework such that this behavior neatly and theoretically ties in with the demand to participate in activities (and the subsequent derived demand for travel). Some of the economic-based utility maximization approaches, such as the Kuhn-Tucker models shown by Bhat (2008), have a strong theoretical foundation for time use allocation. However, as pointed out by Jara-Díaz (2003), economic utility maximization approaches alone lack a robust methodology for understanding the technical relations between time allocation and activity consumption that can often be highly constrained by space and time. In other words, there is no direct relationship between the choices of activities with the spatio-temporal constraints imposed by travel on a physical network. Some (Lam and Yin, 2001; Ramadurai and Ukkusuri, 2010) have proposed activity-based dynamic traffic assignment models to address the joint activity and route choices under a dynamic user equilibrium. However, imposing congestion as the rationale for scheduling a day's activities assumes that travelers weigh congestion effects as being paramount in activity choices when in fact there can be many different objectives and criteria.

To that end, Recker (1995) proposed a multiobjective model for explicitly capturing a household's decision-making in routing and scheduling of activities throughout the course of a day. This household activity pattern problem (HAPP) is based on a class of vehicle routing problems (specifically that of the pickup and delivery problem with time windows by Solomon and Desrosiers (1988)). He further demonstrated the connection to travel demand modeling practice by showing that the framework is a generalized treatment of traditional trip-based approaches (Recker, 2001), but one that is resistant to conventional estimation techniques.

Vehicle routing problems (VRPs) (reviewed by Toth and Vigo (2002)) represent a broad field of prescriptive models that provide optimal solutions to fulfill an objective subject to constraints through the use of mixed integer linear programming (MILP). While the HAPP model, which falls within this class, is a theoretically sound framework for understanding the technical relationship identified by Jara-Díaz, there are a number of issues preventing its effective use as an operational model for activity-based travel forecasting. The first and foremost issue is that the vehicle routing problems (and mathematical programming models, in general) on which HAPP is based are prescriptive in nature. While this can be useful for identifying bounds in travel behavior for various policy alternatives (Recker and Parimi, 1999), to this point there is no consistent mechanism for calibrating such a model based on observed travel patterns.

This calibration aspect of the modeling process has remained a challenge, primarily due both to its complexity as well as to the nature of solutions to the parameter estimation problem. In contrast to conventional trip-based disaggregate travel demand models in which the object of the estimation typically is the maximization of the likelihood of observing the outcome of a relatively simple choice (e.g., selection of a travel mode, travel route, or travel destination) or combinations of such choices (e.g., travel mode and route) from among a finite (countable) set of alternatives, activity-based models typically comprise a multitude of interrelated choices (e.g., travel mode-time of departure-participating actor pairings) for a sequence of activities that are drawn from an infinite set of alternatives. And, although a utility maximization representation that bears resemblance to those found in discrete choice models can also be specified for an activity-based formulation, such as that described here, it has some notable exceptions that greatly complicate its application in empirical demand analysis—(1) the set of feasible solutions (alternatives) in the choice set is infinite, while that for standard discrete choice models is countable (and, usually small); (2) the solution vector comprises of continuous, as well as discrete, variables; (3) while the overall solution represents a mutually exclusive choice, the solution vector itself is composed of components that are not generally mutually exclusive; (4) the components of the utility function are not directly interpretable as utility weights of attributes, but rather are related to these weights through a transformation matrix; and (5) the complexity of the constraint space generally precludes the type of closed-form probability result achievable with standard discrete choice models.

While it is possible to generate the probability distributions of optimal solutions to the standard discrete choice problem either directly or through simple Monte Carlo simulation of the error terms (coupled to any standard integer programming technique), in the case of activity-based models this direct approach clearly is not possible. A second, and perhaps thornier, problem is that the solution vector contains continuous (i.e., the starting times of activities), as well as discrete, variables (i.e., person and vehicle assignments); hence, an infinite number of alternatives exist. As such, the probability of any solution matching exactly the observed behavior is infinitesimally small. In dealing with continuous distributions, it is more proper to address the probability that an outcome lies within some "band of similarity" to the actual distribution. However, using such an approach to estimate the utility parameters of the HAPP model with meta-heuristics does not provide any fundamental connection between the estimated values and the households from which they were obtained (Recker et al., 2008).

To tackle the issue of parameter estimation in a more elegant manner, we propose using an inverse optimization approach to obtain a set of objective coefficients, subject to unobservable goal arrival parameters, such that the observed household pattern is optimal. Inverse optimization (Tarantola, 2005) is a theory that seeks out a set of model parameters such that observed decision variables are optimal. It has been effectively formulated as an inverse linear programming (LP) and network flow problem (Burton and Toint, 1992; Ahuja and Orlin, 2001), and effective solution algorithms have been proposed for mixed integer linear programming problems (Wang, 2009). The inverse problem is known to be an ill-defined problem (one-to-many) compared to its forward problem with unique solution (many-to-one). Least squares, maximum likelihood, and minimum perturbation are typical selection rules imposed on inverse problems to overcome this issue to obtain unique solutions.

We introduce the inverse household activity pattern problem (InvHAPP), and propose an elegant solution by reformulating the InvHAPP such that both the objective coefficients and goal arrival time constraints are jointly estimated with a MILP formulation. A bi-level approach is avoided by exploiting the structure of the routing problem with time windows. By making the arrival time constraints endogenous to the inverse model, the inverse household activity pattern problem with joint b-constraint estimation (InvHAPPb) is more directly related to the observed behavior. To our knowledge, there has not been a formulation of an inverse network flow or inverse LP that jointly estimates both the objective coefficients and the constraints without resorting to bi-level programming, much less for inverse MILP. For example, Dempe and Lohse (2006) propose a bi-level approach for inverse LPs to jointly estimate the objective coefficients and constraints. Their method of minimizing least squares from only observed decision variables does not guarantee uniqueness. To ensure that the solutions in our model are regularized, i.e. that overfitting does not occur due to the ill-posed problem, a common prior concept is assumed for the population so that an invariant and stable solution can be obtained.

2. Model formulation

2.1. Household activity pattern problem

Although more general forms were developed, Recker (1995) showed that, in its most basic form, in which each member of the household has exclusive unrestricted use of a personal vehicle and any activity can be completed by any member of the household, the household activity pattern problem (HAPP) can be formulated as a multiobjective variation of the well-known pickup and delivery problem with time windows (PDPTW) within the class of vehicle routing problems with time windows (VRPTW).

Following the notation of Solomon and Desrosiers (1988), Recker (1995) uses an analogous notation to describe the spatio-temporal movements of a household. In the analogy to the PDPTW, activities are viewed as being "picked up" by a particular household member (who, in this basic case, is uniquely associated with a particular vehicle) at the location where performed and, once completed (requiring a service time s_i) are "logged in" or "delivered" on the return trip home. Multiple "pickups" are synonymous with multiple sojourns on any given tour. The scheduling and routing protocol relative to some household objective produces the "time–space diagram" commonly referred to in travel/activity analysis.

The general HAPP structure can be summarized by Eqs. (1) and (2). The full set of objective components and constraints is contained in Eqs. (1)–(16), (18)–(24) from Recker (1995) and displayed in Appendix A for completeness.

$$\min \beta_i^T Z(X_i) \tag{1}$$

subject to

$$AX_i \leqslant 0 \tag{2}$$

where

$$X_i = \begin{bmatrix} X^v \\ T \\ Y \end{bmatrix}, \quad X^v = \left[X^v_{uw} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}\right], \quad T = [T_u \geqslant 0], \quad Y = [Y_u = Integer \geqslant 0]$$

Z is an objective vector with a corresponding coefficient vector β_i for a given household i. X^v is a vector of route choices for vehicle v belonging to household i; T is a vector of arrival times to each activity; Y is a tour accumulation vector analogous to the load variable in the PDP; and u and w are the respective origins and destinations associated with travel required to satisfy a particular activity. In behavioral terms, Y can be used to represent a number of capacity-related factors, such as a traveler's maximum number of tours, maximum distance traveled, or amount of fuel consumed. A is a matrix of the spatio-temporal constraint coefficients detailed in Appendix A.

A variety of different objectives can be defined for the basic HAPP formulation. Although clearly other objectives can also be considered, three objectives chosen from Recker (1995) are considered in the HAPP variations presented in this research, as shown in Eqs. (3)–(5). The significance of each objective is discussed by Recker (1995) and detailed analysis of these objectives assumed for the sample data is shown in Section 4. The original notation of Recker (1995) is maintained – t^v_{uw} is the travel time by vehicle v from the location of activity u to the location of activity w, T_{u+n} is the time of the return to home from activity u, and T^v_{2n+1} is the time of the final return home trip for vehicle v.

$$Z_1 = \sum_{v\in V}\sum_{u\in N}\sum_{w\in N} t^v_{uw}X^v_{uw} \tag{3}$$

$$Z_2 = \sum_{u\in P^+}(T_{u+n} - T_u) \tag{4}$$

$$Z_3 = \left(T^v_{2n+1} - T^v_0\right), \quad v \in V \tag{5}$$

Eq. (3) minimizes travel time t^v_{uw}. Eq. (4) is a measure of the delay in returning home due to multiple sojourn tours. Eq. (5) is a measure of the travel day length for each vehicle.

Careful attention needs to be paid to what is actually observable from travel diaries versus what is required as a data input for the HAPP model. In typical travel diaries, the observed data are comprised of only some of the X and the T variables (the "pickup" activities only, for that matter). For example, a traveler returning home after making a trip chain of two or more activities can unload their Y values in multiple ways, which (because in the formulation, the sequence is simply a mathematical construct) cannot be observed directly—this difference is important when determining the appropriate fitness measure for the inverse problem, which will be discussed in more detail. The more complex formulation is used because there are variations that make use of the variables, such as household member interactions as noted in Recker (1995) but ignored in this paper for simplicity.

Of particular importance, the hard time window constraints in the HAPP formulation are generally non-observable—the observable data are restricted to what the travelers actually did, and not within what temporal limitations the "optimal" pattern was executed. To account for this limitation in the observable data required to fully specify the constraints, the basic HAPP formulation can be extended to include soft time window constraints using a goal programming approach that avoids nonlinear constraints (Calvete et al., 2007). For simplicity, the departure times are assumed to be directly related to the arrival times (since the activity durations are typically known from diary information) so that the soft time windows are applied only to activity arrivals. The hard time window objective Z' is revised to include early ($Z_{e,u}$) and late ($Z_{l,u}$) arrival penalties, as shown in Eq. (6).

$$\min Z' + \sum_{u \in P^+} e_u Z_{e,u} + \sum_{u \in P^+} l_u Z_{l,u} \tag{6}$$

Constraint (11) in Recker's (1995) original formulation, i.e., $a_u \leqslant T_u \leqslant b_u, u \in P^+$ (11; Recker, 1995) in which a_u and b_u define the time window for activity u, is replaced with relations (7) and (8).

$$T_u + Z_{e,u} - Z_{l,u} = b_u, \quad u \in P^+ \tag{7}$$

$$Z_{e,u}, Z_{l,u} \geqslant 0 \quad u \in P^+ \tag{8}$$

The constraints shown here require a single arrival time goal, although they can be modified to handle soft time windows. Deviations in arrival time are quantified by the non-negative penalty values.

2.2. Inverse household activity pattern problem with soft time windows

If we consider a straightforward formulation of InvHAPP with soft time windows (InvHAPPSTW), only the objective coefficients are estimated in the objective function such that observed patterns result in an optimal objective value.

This InvHAPPSTW formulation solution methodology is directly based on the InvMILP described by Wang (2009). Each objective in the HAPPSTW is defined as an additional dummy variable so that the full set of variables is $[X, T, Y, Z]$. This allows the coefficients of Z to be estimated directly. The following equality constraints are added to the constraint set in Eq. (2).

$$\sum_{v \in V} \sum_{u \in V} \sum_{w \in N} t_{uw} X^v_{uw} - Z_1 = 0 \tag{9}$$

$$\sum_{u \in P^+} (T_{u+n} - T_u) - Z_2 = 0 \tag{10}$$

$$(T^v_{2n+1} - T^v_0) - Z_{2+v} = 0, \quad v \in V \tag{11}$$

The InvHAPPSTW can be defined by Eq. (12), assuming a minimum perturbation from initial objective coefficients under L_1 norm. Note that L_p norm is a distance measure defined as the p-dimensional distance between two objects. For example, L_1 norm is equivalent to the rectilinear (Manhattan) distance between two objects, whereas L_2 norm ("least squares") minimization refers to Euclidean distance and is a conventional "best selection rule" in estimation theory. Here, L_1 norm is used because it allows us to maintain a linear model for the inverse optimization problem.

$$\min_\beta w^T|\beta_0 - \beta| : [X, T, Y, Z]^\beta \in \arg\min_Z\{\beta^T Z : A[X, T, Y, Z] \leqslant b, \quad [Z, T, Y, Z] \geqslant 0, \quad X \in Z\} \tag{12}$$

where w is a vector of weights that can be used to customize the significance of perturbations for each coefficient; β_0 is an initial set of coefficients for all the variables, $[X,T,Y,Z]$, but constraining all the coefficients preceding $[X,T,Y]$ to be 0 and letting the objective dummy coefficients preceding Z to be equal to 1; β is the set of desired coefficients and the decision variables in this problem; $[X, T, Y, Z]^\beta$ is the set of observed decision variables and corresponding dummy variables; A is the set of constraints in Eqs. (2), (9)–(11); Z is a set of objectives, such as Eqs. (3)–(5), and Eq. (6) for InvHAPPSTW: $Z = [Z_1; Z_2; Z_3; Z_e; Z_l]$, where Z_e is an nx1 vector of early arrival deviations and Z_l is an nx1 vector of late arrival deviations.

The initial set of objective coefficients β_{0Z}'s (the Z subscript denotes the set of coefficients corresponding to the Z variables) and corresponding objective coefficients β_Z's relating to the soft time window penalties are equivalent to the e_u and l_u in Eq. (6). The rationale for setting the β_{0Z}'s equal to 1 is to assume initially that the households are indifferent between the objectives. This is the most suitable initial guess given no additional information.

The challenge in solving the inverse MILP is the duality gap in integer programming problems (Williams, 1996), which prevents using the formulation from Ahuja and Orlin (2001) directly. To solve the MILP version, Wang (2009) proposed a cutting plane algorithm along with an LP-relaxed form. An equivalent LP-relaxed form with cutting plane constraints for the InvHAPPSTW is shown in the following equations:

$$\min_{\psi,\zeta,\eta} w^T\zeta + w^T\eta \tag{13}$$

subject to

$$A^T\psi \geqslant \beta_0 - \zeta + \eta \tag{14}$$

$$(\beta_0 - \zeta + \eta)^T[X,T,Y,Z]^\beta \leqslant (\beta_0 - \zeta + \eta)^T[X,T,Y,Z]^j; \quad \forall [X,T,Y,Z]^j \in S^i, \quad j \in i \tag{15}$$

$$\zeta_{[X,T,Y]}, \quad \eta_{[X,T,Y]} = 0 \quad \psi, \zeta_Z, \eta_Z \geqslant 0 \tag{16}$$

where w is the vector of weights for the equivalent problem that needs to match the pre-defined β_0's; if $\beta_0 = [0,0,0,1]$ then $w = [0,0,0,1]$ so that the HAPP decision variables alone are not considered in the inverse objective; ψ is the vector of dual variables corresponding to the constraint set A comprising the original HAPP constraints of Recker (1995) together with those shown in Eqs. (7)–(11).

Eq. (15) is the cutting plane introduced by Wang (2009) to replace the strong duality condition that cannot be met in an integer programming (IP) problem. The cutting plane algorithm for InvHAPPSTW is shown:

0. Initiate $i = 0$, and $S^i = \varnothing$.
1. Let $i = i + 1$. For a given iteration i, solve the relaxed LP Eqs. (13)–(16) such that $\beta^* = \beta_0 - \zeta^* + \eta^*$.
2. Solve HAPPSTW using β^* as the coefficients.
3. If the $[X, T, Y, Z]^i$ obtained from Step 2 results in condition (17), stop. Else, let $S^{i+1} := S^i \cup [X, T, Y, Z]^i$ to generate a new cutting plane in Eq. (15) and repeat Step 1.

$$(\beta_0 - \zeta^* + \eta^*)^T[X,T,Y,Z]^\beta \leqslant (\beta_0 - \zeta^* + \eta^*)^T[X,T,Y,Z]^i \tag{17}$$

A serious limitation of the InvHAPPSTW is that only the objective coefficients are changed. The b_u's are constant values based on median arrival times, obtained from travel survey data from the population sample. In the formulation, the penalty weights e_u and l_u are allowed to adjust. Inverse solutions end up having either $e_u = 1$ and $l_u = 0$ or vice versa, depending on observed arrival time, due to being under-identified. For aggregate population penalty estimates, an average value can be obtained, but if a single household is being evaluated it is not as accurate to say that the household member does not have any value to being early or late just because they happened to be late or early. For example, if a household is observed to arrive at an activity at 9 AM when the median ("social norm") arrival time turns out to be 8 AM, an inverse model that solves only for objective coefficients tends to set a weight of 0 to the late arrival penalty so that the household is not actually penalized. This under-identification problem is due to a specification of the problem such that there is one observed data point (observed arrival time) for two parameters to be estimated (early and late penalties).

Several considerations need to be made to allow joint estimation of both the objective coefficients and the constraints. First, arrival times b_u need to be made endogenous to the inverse model in a similar fashion to the coefficients. For example, an inverse problem can be solved to perturb the coefficients and the arrival times minimally such that observed patterns are optimal. This means that the b_u, $Z_{e,u}$, and $Z_{l,u}$ become decision variables in the inverse problem. To maintain a linear objective function, the arrival time penalty rates are held constant. The travel time and other objectives can then be weighted relative to these values so that some relative heterogeneity of the rates can be achieved across households, if not across activity types.

A new problem arises from adding deviations to the goal arrival times, however. Consider $b_u = b_{0,u} - \zeta^*_{2,u} + \eta^*_{2,u}$ for each activity u. There are three optimality conditions in an LP that need to be met: primal feasibility ($Ax \leqslant b$), dual feasibility ($A^Ty \geqslant c$), and strong duality ($c^Tx = b^Ty$). The primal feasibility condition cannot be used to solve for the b_u's because it only relates the b_u's to the $Z_{e,u}$ and $Z_{l,u}$ values, not to the variables in the dual feasibility condition. The dual feasibility condition is used to determine shifts for the objective coefficients, but it does not depend on any changes to b_u. For the MILP, the replaced cutting plane constraint (15) for handling the duality gap for the third condition does not relate to b_u either. There is no direct connection between the coefficients of Z_1, Z_2, and Z_3 to the b_u's through the $Z_{e,u}$'s and $Z_{l,u}$'s.

3. Proposed inverse household activity pattern problem with arrival time constraint estimation

3.1. Inverse problem formulation

In the proposed inverse formulation, the issues described at the end of Section 2.2 are resolved by acknowledging that constraint (7) is an equality constraint that directly relates the b_u's to the $Z_{e,u}$'s and $Z_{l,u}$'s. Rather than finding an inverse objective to perturb the b_u's, an objective is defined by perturbing the $Z_{e,u}$'s and $Z_{l,u}$'s instead. Since the $Z_{e,u}$'s and $Z_{l,u}$'s both relate to the coefficients of Z_1, Z_2, and Z_3 through the cutting plane constraints in Eq. (15), a connection can then be established. The InvHAPPb can be represented by Eq. (18).

$$\min_{\beta,Z_e,Z_l} w_1^T|\beta_0-\beta| + w_2^T|Z_{0,e}-Z_e| + w_3^T|Z_{0,l}-Z_l| : [X,T,Y,Z]^{\beta,Z_e,Z_l} \in \arg\min_Z\{\beta^T Z : A[X,T,Y,Z] \leqslant b[X,T,Y,Z] \geqslant 0, \quad X \in \mathbb{Z}\} \tag{18}$$

where w_1, w_2, and w_3 are vectors of weights corresponding to the objective coefficients, early arrival penalties, and late arrival penalties; $Z_{0,e}$ is an initial seed value of deviation determined as for each activity u as $Z_{0,e,u}$ = max(0, $b_{0,u} - T_u$). $b_{0,u}$ is the median arrival time of activity type corresponding to activity u; $Z_{0,l}$ is an initial seed value of deviation determined as for each activity u as $Z_{0,l,u}$ = max(0, $T_u - b_{0,u}$); b is a vector of constraint values of which a subset of size nx1 corresponds to the equality constraints for goal arrival times.

Note that the core difference between this formulation and the InvMILP is that both coefficients and dummy variables are being estimated with respect to observed variables. The equivalent LP-relaxed formulation with cutting plane constraints is shown in the following equations:

$$\min_{Z_e,Z_l,\psi,\zeta,\eta,b_u} w_1^T\zeta_1 + w_1^T\eta_1 + w_2^T\zeta_{2,e} + w_3^T\zeta_{2,l} + w_2^T\eta_{2,e} + w_3^T\eta_{2,l} \tag{19}$$

subject to

$$A^T\psi \geqslant \beta_0 - \zeta_1 + \eta_1 \tag{20}$$

$$A_{Z_e,Z_l}[Z_{0,e} - \zeta_{2,e} + \eta_{2,e}; Z_{0,l} - \zeta_{2,l} + \eta_{2,l}] - b_n = 0 \tag{21}$$

$$Z_{0,e} - \zeta_{2,e} + \eta_{2,e} \geqslant 0 \tag{22}$$

$$Z_{0,l} - \zeta_{2,l} + \eta_{2,l} \geqslant 0 \tag{23}$$

$$(\beta_0 - \zeta_1 + \eta_1)^T Z_{123}^{\beta} + e_n^T[Z_{0,e} - \zeta_{2,e} + \eta_{2,e}] + l_n^T[Z_{0,l} - \zeta_{2,l} + \eta_{2,l}] \leqslant (\beta_0 - \zeta_1 + \eta_1)^T Z_{123}^j + e_n^T Z_e^j + l_n^T Z_i^j, \quad \forall [X,T,Y,Z]^j \in S^i,\ j \in i \tag{24}$$

$$\zeta_{1,[X,T,Y]} = 0, Z_e, Z_l, \psi, \eta_{1,Z}, \eta_{1,Z}, \zeta_{2,e}, \zeta_{2,l}, \eta_{2,e}, \eta_{2,l} \geqslant 0, \quad b_n \geqslant 0 \tag{25}$$

where ζ_1 and η_1 are equivalent to ζ and η in the InvHAPPSTW, where they are two non-negative vectors such that $\beta = \beta_0 - \zeta_1 + \eta_1$; $\zeta_{2,e}$, $\zeta_{2,l}$, $\eta_{2,e}$, and $\eta_{2,l}$ are corresponding deviation vectors for the early arrival penalties and late arrival penalties; A_{Z_e,Z_l} is the constraint set corresponding to equality constraint (7); b_n is a vector of n arrival time goals that will be indirectly solved by the problem, where $b_u \in b_n$; Z_{123}^{β} is the set of observed objective values composed of Z_1, Z_2, and Z_3; e_n is a vector of n early arrival penalty rates, where $e_u \in e_n$; l_n is a vector of n late arrival penalty rates, where $l_u \in l_n$.

Relationship (20) is the same dual feasibility condition used in the InvHAPPSTW formulation. Eq. (21) is the additional primal feasibility conditions pertaining to the variables as expressed by the equality constraint (7) in terms of the decision variables. Relationships (22) and (23) ensure that the final arrival penalty deviations, Z_e and Z_l, are non-negative. Relationship (24) is the cutting plane constraint to fulfill the strong duality condition, keeping account of the decision variables. Relationships (25) consist of the non-negativity constraints.

The solution to the InvHAPPb results in the following outputs: objective coefficients β, acceptable arrival deviations Z_e and Z_l, goal arrival times b_n, and dual variables y. As the formulation is structurally the same as the InvMILP, the same cutting plane algorithm described in Section 2.2 can be applied here to solve this problem. Proof of convergence is provided by Wang (2009), and consists of first proving that a finite number of iterations can be achieved for an LP with a finite number of extreme points, followed by showing that the convex hull of an inverse MILP is similar to LP.

3.2. Inference properties

While the inverse formulation provides a method of fitting the observed data to an optimization problem, it does not answer the question of how to infer traveler behavior from n observed data samples. By itself, the InvHAPPb assumes a selection rule of minimizing L_1 norm from a prior, but no discussion has been provided yet on the appropriateness of the prior in terms of consistency or stability. Further, the problem pertains to estimating the parameters for a single observed sample, but extensions need to be made to a population of samples. In particular, it is necessary to justify the selection rule (in this case, an L_1 norm perturbation from a prior) as logically and statistically consistent.

Csiszár (1991) evaluates different selection rules (or projection rules comprised of sequences of selection rules from prior information) for inferring parameters of inverse problems based on various axioms that do not depend on probabilistic measures. He shows that if a selection (projection) rule is regularized, it should satisfy various consistency, distinctness, and continuity axioms. Furthermore, the consistency axiom suggests that parameters should remain consistent if additional constraints are added to the parameter space. This conclusion is supportive of a regularized inference process for evaluating "extreme" scenarios in travel behavior that require additional constraints in a HAPP framework while maintaining consistent parameters, such as with the incorporation of new destinations or modes of travel.

Regularization of the InvHAPPb is the key to achieving such consistency. Tenorio (2001) discusses regularizing inverse problems to come upon physically meaningful, stable solutions. He points out that the use of prior information can provide regularity for inverse problems. Maréchal and Lannes (1996) present the regularization problem in the context of prior information as an optimization problem that minimizes distance from a prior while maintaining an acceptable fit to the data. This form of estimation is encountered in other areas of transportation research, namely OD estimation (Maher, 1983).

In the HAPP estimation problem, there exists another source of information for providing regularization in addition to observed patterns and external, dedicated choice experiments. This information is the distribution of the parameters across the sample population. The use of a prior in the HAPP estimation problem is justified by the "common prior" concept defined by Harsanyi (1967) for Bayesian games with incomplete information. The common prior in the HAPP estimation problem depends on the expected beliefs of players. Each player is a household who holds an initial belief of what everyone else in the population believes is the distribution of values represented by HAPP parameters. The estimation problem is then the result of each player revealing his or her values given a prior expectation of the population's values. It can be shown that if players do not agree to disagree then a common prior always exists and is unique (Feinberg, 2000).

In the InvHAPPb, there are two sets of parameters: the constant arrival time penalties required for estimating arrival time constraints and the objective coefficients. Given a sample population of households, it is then possible to assume the existence of a unique common prior of objective coefficients relative to the constant arrival time penalties. Since the common prior is defined as each player's common belief of what the expected weights should be, a mean set of posterior parameters that differs significantly from the original prior would suggest an inconsistency. The solution method is to obtain an invariant set of parameters where the posterior distribution should be equivalent to the prior distribution, called the parametrization invariant prior. An invariant prior estimator is unbiased and results in the minimum variance. Although methods for obtaining parametrization invariant priors exist, the problem is less straightforward when the functional operator is a constrained optimization problem such as the InvHAPPb. The problem of finding an invariant common prior can be viewed as finding P^* under Eq. (26), where $f(\beta|P)$ is the average of the posterior solutions obtained from inverse optimization.

$$P^* = f(\beta|P^*) \tag{26}$$

Eq. (26) can be viewed as a fixed point problem without guarantee for an asymptotically stable solution. This is due to the complexity of the InvHAPPb, where f is not necessarily differentiable. Since it has been shown that common priors always exist and are unique, a Method of Successive Averages (MSA) algorithm is adopted.

Proposition 1. The Method of Successive Averages converges to a parametrization invariant common prior for a population of inverse mixed integer programming solutions.

Proof. Since the prior represents a set of objective weights and the weights are finite, there always exists a bounded equivalent set of parameters corresponding to any posterior by scaling the weights. As such, even if there is no asymptotically stable fixed point, there exists a stable range. MSA always converges to an expected point given a bounded range, due to the Law of Large Numbers. Since the common prior always exists and is unique, and is defined as the set of beliefs that converge upon the expected values of the population, this is a sufficient condition for convergence to the invariant common prior. □

Method of Successive Averages for obtaining invariant common prior
1. Let P_0 = {1} as a set of uninformed prior parameters. Set i =0
2. Let β_{i+1} = InvHAPPb(P_i) and let $P_{i+1} = \frac{i}{i+1}P_i + \frac{1}{i+1}\sum_S \frac{\beta_{i+1,s}}{S}$, where S is the number of samples
3. If max $\lvert P_{i+1} - P_i\rvert < \varepsilon$,, stop, else let i = i + 1 and go to 2

Because there exists a unique set of objective parameters, it should not matter what initial set of priors is used to initiate the MSA algorithm. However, using external experimental data such as that used to assume the arrival time penalties can likely improve the speed of convergence. Even with experimental data to improve the convergence process, the MSA is a notoriously slow converging algorithm. Future research should consider faster variants such as that shown in Bar-Gera and Boyce (2006). Identifying relative weights from empirical studies for use as initial weights in the MSA would also help improve the methodology.

The calibrated HAPP model provides a normative mechanism for assigning route and scheduling choices to N sample households by determining the coefficients of a multiobjective utility function. Whereas traditional discrete choice models consider utility functions with error terms to account for unobserved preferences such as those of other households in the population, the calibrated HAPP model does not extend to those. However, a calibrated HAPP model can be used to infer

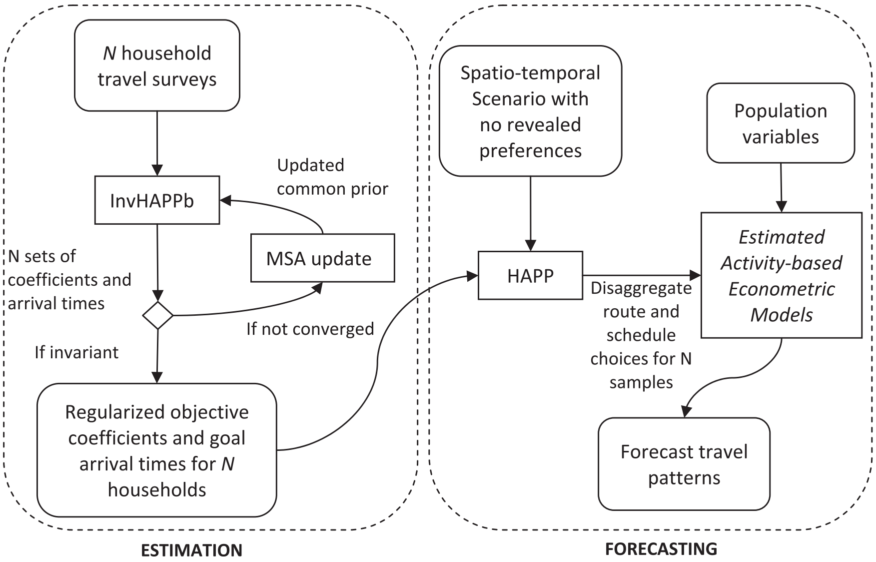

Fig. 1. HAPP estimation and integration within a forecasting process.

route and schedule choices of the *N* sample households for scenarios where spatio-temporal conditions are unobserved by assuming that the households behave rationally regardless of the spatio-temporal constraints. By integrating the HAPP as a disaggregate space–time assignment model, activity based models can benefit by having normatively inferred route and schedule choices for forecasting travel patterns in those scenarios. The integration of the InvHAPPb estimation process with the use of the calibrated HAPP model and further connections with other activity-based models is shown in Fig. 1. Section 5 provides a further discussion of illustrative uses of the model in the context of Fig. 1. Detailed application of the HAPP model with respect to these uses will be explored in future research.

4. Numerical tests

4.1. Illustrations of single household patterns

As an example, the InvHAPPb is applied to the 1-day observed household travel diaries of two households obtained from the 2001 California Household Travel Survey. HAPP runs are then made using the output coefficients and arrival times to compare the calibrated prescriptive model's values to the observed values. For the first household (Case I), further comparison is made with the InvHAPPSTW and with uniform indifferent coefficients with no prior information. The second household (Case II) shows an example of how the goal arrival time can end up being different from the observed arrival times. This model behavior is desired because it explicitly accounts for decision-maker trade-offs between arrival penalties and other objectives such as minimizing travel time.

Seed arrival times by activity type are based on median arrival times of the sample population, as shown in Table 1.

Table 1
Median arrival times by activity type from 2001 California Household Travel Survey.

Activity	Median arrival time	Activity	Median arrival time
Working at home	11:45	Childcare, daycare, after school care	9:36
Eating/preparing meals at home	15:00	Eat out (restaurant, drive through, etc.)	13:34
Watching TV/videos at home	16:30	Medical	11:55
Shopping by phone/TV/internet at home	14:25	Fitness activities	14:45
Exercising at home	8:30	Recreational (vacation, camping, etc.)	12:20
Other at home	3:00	Entertainment (movies, dance club, bar, etc.)	17:32
Wait for/get on a vehicle	13:40	Visit friends/relatives	15:00
Leave/park a vehicle	11:06	Community meetings, political or civic event	15:25
Boarding activities for airplane, rail, intercity bus	11:16	Occasional volunteer work	11:41
Getting off airplane, rail, intercity bus	15:30	Church, temple, religious meeting	17:30
Pick up someone or get picked up	15:00	Buy gas	13:35
Drop off someone or get dropped off	8:30	Incidental shopping (groceries, housewares, etc.)	14:09
Work	8:10	Major shopping (furniture, clothes, autos, etc.)	13:50
Work-related (sales calls, meetings, errands, etc.)	11:30	ATM, banking, post office, utilities	13:00
School (preschool to 12th)	8:00	Other personal or household business	13:05
School (post-secondary – college, vocational)	9:50	Be with another person at their activity	14:02

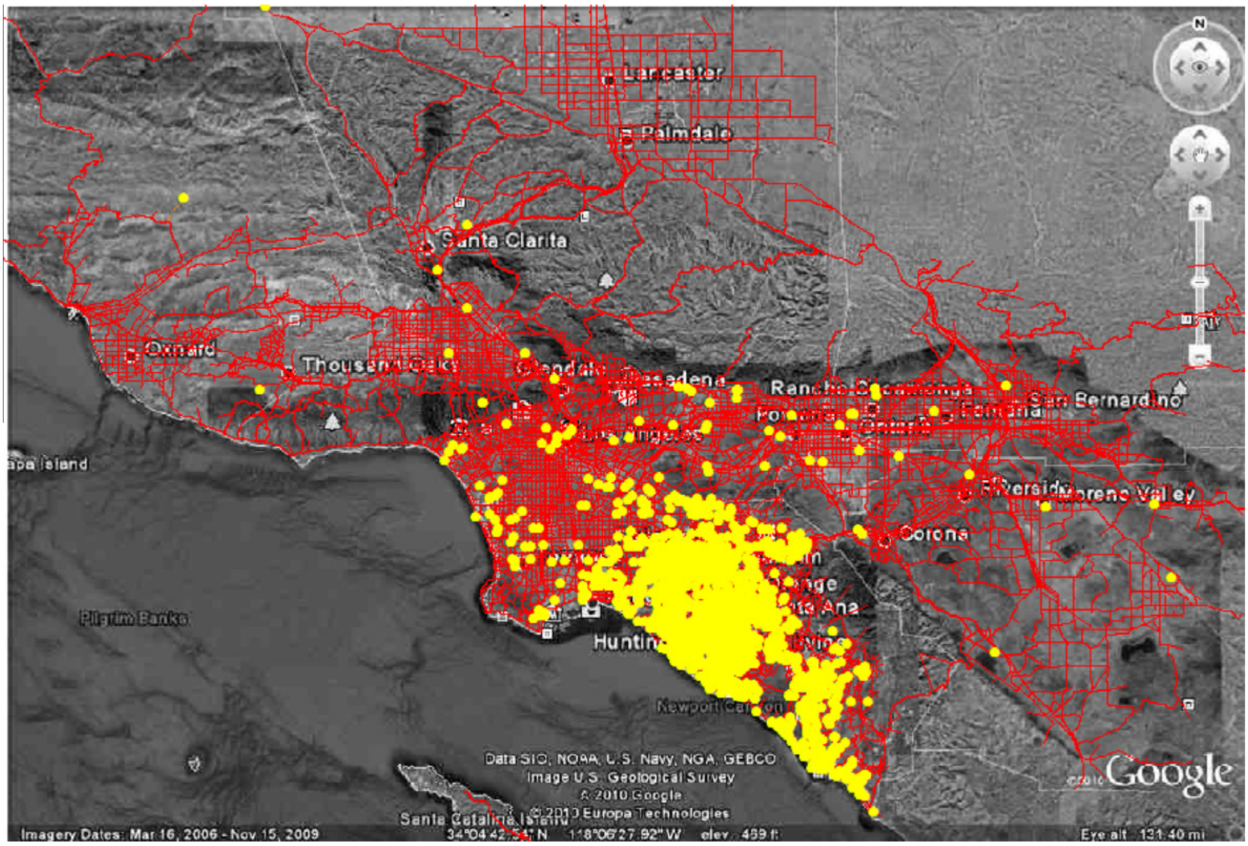

Fig. 2. The OCTAM Network and activity locations from households in Orange County, CA.

Constant values for the early and late arrival penalties are obtained from work commute values from Small (1982) and assumed the same for all activity types. The following relationship between early arrival, late arrival, and travel time were found for commuters: travel time – 0.106, early arrival – 0.065, and late arrival – 0.254. Assuming these relative weights are constant across population and activity types results in early arrival penalties of 0.613 per unit of early arrival and late arrival penalties of 2.396 per unit of late arrival.

A travel time matrix is obtained from the Orange County Transportation Analysis Model (OCTAM) TransCAD network, which is developed by the Orange County Transportation Authority (OCTA), and based on the network from the Southern California Association of Governments (SCAG). Of all the households located in Orange County that were sampled in the survey, a total of 2526 activity locations were identified, resulting in a 2526 $\times$ 2526 travel time matrix. The network overlaid upon a Google Earth image with all the Orange County household activity locations is shown in Fig. 2. Due to the coarseness of the network, some factored adjustments were made to the travel time matrix to more closely match the observed travel times in the travel diary.

- Case I: Single vehicle household (sample record no. 1048899)

From the travel diary, this single-vehicle household member is observed to leave home at 7:35, arrives at work at 8:10, goes to eat at 13:00, goes shopping at 15:35, and returns home for dinner at 16:50.

The InvHAPPb solution successfully converges in 10 iterations, taking 4.015 s on a 64-bit Intel Core i7 CPU with 2.67 GHz, 4 GB RAM, Windows 7 operating system running MATLAB on XP Mode calling CPLEX for the IP solver. The optimal objective coefficients estimated for the three objectives in Eqs. (3)–(5) are $\beta_1 = 1$, $\beta_2 = 0$, and $\beta_3 = 0.0746$. The goal arrival times are set to 8:10, 13:00, 15:35, and 16:50 – exactly corresponding to their observed arrival times. The HAPP solution using these calibrated parameters results in the exact same order of activities and arrival times as the observed activity arrivals. The only differences are the order with which activities are "dropped off" at home and the time which they depart from home or get unloaded, since the objective for minimizing length of time outside home, β_2, is set to 0 and the departure time from home is set to 0 by default. It duplicates the observed behavior exactly. The resulting objective coefficients inform us that this particular household sample did not place any value to reducing delay from trip chaining (strong preference to chain their trips) and for each hour of travel time saved they value 4.5 min of time reduced in their time spent outside the home.

The HAPP solution using purely indifferent coefficients with uniform weights of $\beta_1 = \beta_2 = \beta_3 = 1$, $e_u = 0.613$, and $l_u = 2.396$ result in different activity patterns with two trip chains instead of the single observed one. Instead of trip chaining from work to lunch to shopping before going home, this solution has the household member going to work, arriving at lunch at 12:37,

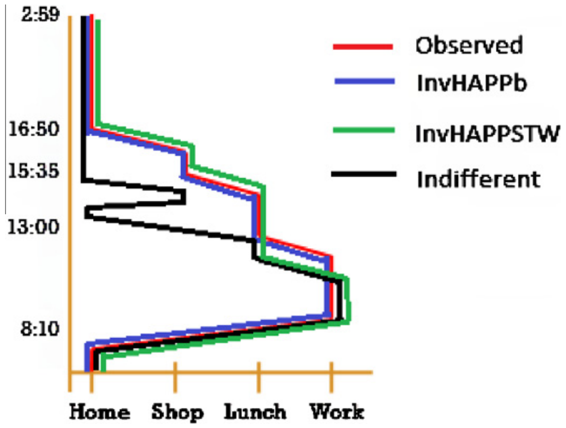

Fig. 3. Illustrative comparison of three household activity pattern problem outputs with observed patterns for California Household Travel Survey record no. 1048899.

returning home by 13:20 before heading back out shopping by 13:40, and finally returning home again to prepare for dinner at 14:45.

The HAPP solution to the InvHAPPSTW, which estimates only the objective coefficients, results in a single trip chain with the same order of activity arrivals, but different start times. Work starts at 8:10, but lunch begins at 12:37, shopping begins at 15:46, and the household member arrives back home at 16:51.

A comparison between the four patterns is shown in Fig. 3 representing an abstract spatio-temporal space. Clearly, the HAPP model run without any calibration of coefficients or arrival time goals can result in significantly different patterns from observed behavior. While InvHAPPSTW is much closer in terms of the ordering of activities, the lack of specification in the formulation for matching observed arrivals directly prevents an exact replication of the arrival times. InvHAPPb, on the other hand, is able to achieve this replication.

- Case II: Household with estimated arrival time different from observed arrival time (sample record no. 1049117)

In this example, the household member goes out for entertainment and is observed to arrive there at 12:45. The member returns home for "other at home" activities at 16:00. After two iterations and 0.116 s, the objective coefficients remain the same ($\beta_1 = \beta_2 = \beta_3 = 1$) while the goal arrival times are set to 16:04 for entertainment and 16:00 for "other at home". Note that the goal arrival times differ from the observed arrival times. Despite the difference in the calibrated arrival time parameters, the HAPP solution results in arrival at the entertainment activity at 12:45 and returning home at 16:00, matching the observed patterns exactly. This example demonstrates that the InvHAPPb can result in a perfect matching solution where the goal arrival times can be estimated to be different from observed arrival times.

4.2. Fitness comparisons among a sample of multiple households

The inverse formulation and estimation results provided in the previous section apply only to a method of fitting the observed data to an optimization problem involving a single observation; it does not address the inference problem associated with *n* observed data samples—the *de facto* estimation problem in travel behavior modeling. In this section we extend and apply the methods proposed to a set of 78 household samples drawn from the California Household Travel Survey data; for ease of presentation, households with 4 or fewer activities conducted during the day are selected. Among these samples, 65 are single-vehicle households while 13 are two-vehicle households. Carpooling is ignored in this numerical example for convenience. These samples are used to compare the goodness of fit of the InvHAPPb model with the InvHAPPSTW against a base model that uses uniform objective coefficients.

Differences can result either because of model error due to lack of data for each parameter being estimated, or because the inverse optimal solution lies on a plane that includes multiple extreme points. To measure how well each inverse model fits the actual observed data, two goodness-of-fit measures are defined. Both these ρ^2 measures represent the ratio of the squared error between inverse estimated values against observed patterns, to the squared error of the HAPP prescriptive model without any prior information against observed patterns. Since the value can potentially drop below 0 if the inverse optimal coefficients result in worse estimates than having uniform coefficient values, it is not quite equivalent to a true ρ^2 goodness of fit measure. However, it serves as a relative measure of fitness for different inverse models in being able to replicate observed patterns. For example, an $\rho_T^2 = 0.5$ suggests that the inverse model coefficients results in a prescriptive model that is twice as accurate as a prescriptive model with no prior information.

These measures are based only on how well the estimated parameters result in prescriptive HAPP models that can output the same or similar (exact duplicates, with the possible exception of the strictly-mathematical-construct variables in the formulation, noted previously) patterns to those observed. Therefore, it is not simply summing the error of all X's or T's between observed patterns and model outputs, since some of the X's and T's are not actually observed from a travel survey but assumed for use in estimating the parameters. As discussed, these include the drop-off activities and the unloading activities at home. One fitness measure is based on matching aggregated origin–destination trip patterns (OD) and one is based on activity arrival time patterns (T). Consider the following fitness measure for OD patterns in Eq. (27).

$$\rho^2_{\mathrm{OD}} = 1 - \frac{\sum_{\text{all ODs}}\left(\sum_{v\in V}\sum_{uw\in N^2} X^{obs}_{uw,v} - \sum_{v\in V}\sum_{UW\in n^2} x^{\mathrm{InvHAPP}}_{uw,v}\right)^2}{\sum_{\text{all ODs}}\left(\sum_{v\in V}\sum_{uw\in N^2} X^{obs}_{uw,v} - \sum_{v\in V}\sum_{UW\in n^2} x^{\mathrm{HAPP}}_{uw,v}\right)^2} \tag{27}$$

This measure aggregates all the activities into an OD matrix for a given day per household, and then the sum of squared errors is obtained between the model output's cells and the observed cells of the matrices.

For the arrival time measure, the drop-off times are ignored. Since there is no designation between multiple vehicles, the departure and arrival times from and to home are also ignored to avoid mistakenly treating correct patterns as errors (for example, it might be observed that vehicle 1 departs home at 8 AM and vehicle 2 at 9 AM, but the model might have vehicle 1 and 2 switched – which is still correct but not in a squared error comparison). In this case, only the arrival times to the activities themselves should be accounted for. The arrival time-based fitness measure is shown in Eq. (28).

$$\rho^2_T = 1 - \frac{\sum_k\sum_{v\in A}\left(T^{obs}_{u,k} - T^{\mathrm{InvHAPP}}_{u,k}\right)^2}{\sum_k\sum_{v\in A}\left(T^{obs}_{u,k} - T^{\mathrm{HAPP}}_{u,k}\right)^2} \tag{28}$$

First, the HAPP model is solved for all 78 samples using objective coefficients of unity, with arrival penalties of 0.613 and 2.396 for early and late arrivals. The resulting patterns are compared against observed patterns as shown in Eqs. (27) and (28). The sums of squared errors are reflected in Table 2 under the column "Base HAPP".

The InvHAPPSTW and InvHAPPb models are evaluated and reflected in the third and fourth columns, respectively, of Table 2; the computed relative ρ^2 values for OD and T measures are as defined in Eqs. (27) and (28). The InvHAPPSTW parameters result in 93.8% reduction in error relative to the base HAPP for the OD patterns, and a 55.9% reduction in arrival time error. On the other hand, the InvHAPPb results in similar OD error reduction (92.2%) while having a vastly improved arrival time reduction (99.3%) from the base HAPP. These improvements coincide with the Case I comparison in Section 4.1. As these results indicate, the InvHAPPb is a much better fitting model for estimating the parameters of the HAPP model than the InvHAPPSTW and consequently the standard InvMILP formulation.

4.3. Applying MSA to obtain an invariant prior

The MSA method is applied to the data to obtain an invariant common prior for both the 1-vehicle and 2-vehicle samples. The MSA algorithm converged with a tolerance of 0.001 after 58 iterations for the 65 1-vehicle sample and after 24 iterations for the 13 2-vehicle sample, as shown in Fig. 4.

Table 2
Comparison of InvHAPPSTW to InvHAPPb using alternative goodness of fit measures.

	Base HAPP	InvHAPPSTW HAPP	ρ^2	InvHAPPb HAPP	ρ^2
SSE of OD	128	8	0.938	10	0.922
SSE of T	6.5813×10^7	2.9052×10^7	0.559	4.3910×10^5	0.993

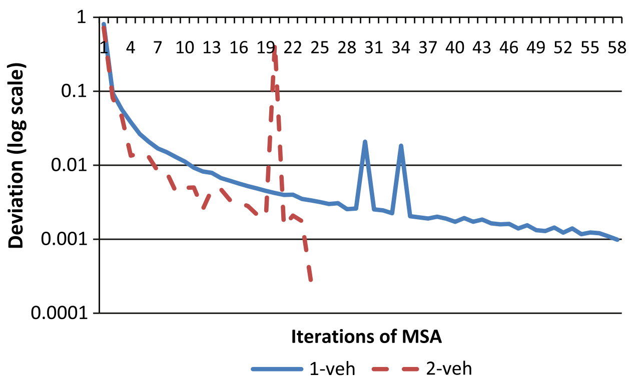

Fig. 4. Convergence of MSA for 1-vehicle sample common prior.

Table 3
InvHAPPb results for 1-vehicle sample using MSA-based common prior versus uninformed prior.

	Travel time	Return home delay	Length of day	Early arrival	Late arrival
Common prior from MSA (P^*)	1.2287	0.2715	0.0598	0.613	2.396
Mean InvHAPPb (P^*)	1.2831	0.2059	0.0606	0.613	2.396
Standard deviation (P^*)	0.4421	0.1151	0.1631	NA	NA
Uninformed common prior (P_0)	1	1	1	0.613	2.396
Mean InvHAPPb (P_0)	1.0752	0.6995	0.1972	0.613	2.396
Standard deviation (P_0)	0.6064	0.4562	0.3449	NA	NA

Table 4
Parameter correlation matrix from 1-vehicle sample InvHAPPb with MSA common prior results.

	Travel time	Return home delay	Length of day
Travel time	1	0.1077	0.4171
Return home delay		1	−0.0634
Length of day			1

Table 5
InvHAPPb results for 2-vehicle sample using MSA-based common prior versus uninformed prior.

	Travel time	Return home delay	Length of day (Vehicle 1)	Length of day (Vehicle 2)	Early arrival	Late arrival
Common prior from MSA (P^*)	1.4475	1.0000	0.1402	0.0742	0.613	2.396
Mean InvHAPPb (P^*)	1.4435	1.0000	0.1363	0.0285	0.613	2.396
Standard deviation (P^*)	0.0143	NA	0.1919	0.0376	NA	NA
Uninformed common prior (P_0)	1	1	1	1	0.613	2.396
Mean InvHAPPb (P_0)	1.0000	1.0000	0.3086	0.2799	0.613	2.396
Standard deviation (P_0)	NA	NA	0.4291	0.3469	NA	NA

Table 6
Performance of mean InvHAPPb parameters for a homogeneous HAPP.

	Base HAPP	HAPP (P^*)	ρ^2
SSE of OD	128	114	0.109
SSE of T	6.5813×10^7	6.6281×10^7	−0.007

The common prior along with the mean and standard deviation of the InvHAPPb runs are shown in Table 3, in comparison to the mean and standard deviations if InvHAPPb runs were made against uninformed prior (first iteration). The results clearly show that there is a significant reduction in variance of the distribution of the posterior due to the use of a common prior obtained from MSA. However, note also that the common prior does not result in an asymptotically stable fixed point, as the mean posterior parameters differ slightly. The correlation matrix determined from the InvHAPPb results for the three parameters is shown in Table 4. There does not appear to be much correlation among the three parameters, aside from some positive correlation between travel time and length of day—intuitively, this makes sense since increasing travel time would likely increase the length of day.

The results of the 2-vehicle household sample estimation are shown in Table 5, which also show a significant reduction in variance when using the MSA common prior. The results indicate that in both cases the average values of early and late arrivals relative to travel time are less than reported values for commuters. This is reasonable since the activities here include non-work activities that on average are less stringent about early or late arrivals. The 2-vehicle households tend to place a higher value on return home delay and length of day, which suggests a higher value of at-home time. This makes intuitive sense since larger households are likely to find more value in being at home with the added company and more family-oriented characteristics.

While the InvHAPPb can infer individual household objective parameters under the invariant prior, the HAPP model by itself is not designed to forecast travel behavior for other households as a statistical model, and no such claim is made here. In fact, running the HAPP model with the mean parameters obtained from the MSA-based InvHAPPb for the selected objectives actually results in goodness of fit measures that do not stray far from running a HAPP model with uninformed parameters, as shown in Table 6. This result is not unexpected in that the HAPP model, in its current form, is a deterministic constrained optimization formulation—the constraints themselves limited to only physical attributes of the feasible patterns that can arise—with no explicit endogenous consideration of variability in tastes or preferences among the households. Moreover, the discrete nature of the HAPP model imposes problems analogous to using linear regression models for discrete variables. Much of the variability is due to the optimization model itself, and not to the characterization of the common prior. Therefore, it is not expected that the HAPP model running on mean parameters would have a good fit. Instead, the value of an estimated HAPP model is in its integration as a normative instrumental method with descriptive, disaggregate behavioral models, as shown earlier in Fig. 1. Section 5 provides a detailed exposition on such potential uses.

5. Significance to activity-based travel analysis

A number of conceptual considerations are presented to evaluate the significance of this research with respect to activity-based travel analysis. Historically, modelers chose to use a normative traffic assignment problem under an assumption of user equilibrium to resolve the travelers' route choice behavior because of the difficulty of using such econometric methods as discrete choice models to handle the cumbersome number of alternative routes available to each traveler. Activity-based models have run into a similar issue. The HAPP model is essentially a disaggregate space–time assignment model that can be used to infer route and schedule choices for analyzing scenarios with unobserved spatio-temporal changes.

One example of the primary use of the HAPP model up until now is showcased in Recker and Parimi (1999). The HAPP model is used to enforce a normative behavior upon a population; in this case, people are enforced to behave in such a way that emissions (accounting for cold and hot starts) are minimized. The HAPP objective function is chosen to minimize CO emissions, and three scenarios are compared: optimal scheduling and travel linkages without ridesharing, optimal scheduling and travel linkages with ridesharing, and optimal scheduling and travel linkages with ridesharing using vehicles that incorporate (then) present-day emissions technology. These results are then compared to observed patterns to identify the potential aggregate impacts that can be made by such policies. Based on the estimation techniques presented here, such studies can now be revisited, with the HAPP model calibrated with multiple objectives that include minimizing CO emissions to find an invariant common prior so that the value of CO emissions savings relative to other objectives can be inferred from observed behavior. The HAPP model can then be used to evaluate a wider range of policies—for example, considering incentives that convert one objective to another through pricing or rewards programs.

A similar approach can be adopted for evaluating new constraints on activity patterns. As discussed earlier, Csiszár (1991) suggests that a consistent set of parameters should maintain consistency under more constrained circumstances. New constraints may be in the form of a new activity type, a new activity location, or a new vehicle technology or mode. Examples include alternative fueling infrastructure, transit options, and new land use developments. These constraints can all be evaluated by first estimating the parameters for a sample set using the proposed methodology. Additional constraint(s) are then introduced into the HAPP model for each member of the sample to measure the impacts, assuming the existing parameters remain unchanged relative to each other.

Pinjari and Bhat (2011) identify a number of important research directions for which current state-of-the-art activity behavior research needs improvement: intra-household interactions, time use allocation, activity scheduling, spatial dependencies, among others. While many of the choices in activity behavior can be modeled elegantly with econometric models, a number of the dimensions identified in the study show evidence that there is a general weakness in current models that the HAPP model is well defined to address.

The original model (Recker, 1995) introduced household member variables and allowed for such interactions between them as carsharing. Although these variables are not included in the current study for ease of presentation of the basic concepts introduced, they can be incorporated into an operational model so that, for example, pickups and drop off of children to school can be accounted for. Whereas the impact of a new transit mode to the joint choices of dropping off children to school, driver choice, and departure time is particularly challenging for a purely econometric model, it is elegantly handled by the HAPP as an estimated objective weight from which the variation would be fully accounted for under the new constraint.

Time use allocation, activity scheduling, and spatial dependencies all fall neatly under the HAPP framework as well. Pinjari and Bhat (2011) point to recent interest in applying random utility maximization and microeconomic theory to time allocation. Recker (2001) demonstrated some equivalencies in the HAPP formulation to random utility models with spatio-temporal constraints if unobserved random errors are introduced into the HAPP objective function. Scheduling and sequencing activities are directly accounted for in the formulation.

As discussed, while they can be used to consider additional constraints imposed on observed patterns, the HAPP model alone is not appropriate for forecasting household behavior. However, it can reduce much of the data burden if integrated properly with current state-of-the-art activity models such as CEMDAP, FAMOS, ALBATROSS, TASHA, TRANSIM, and MATSIM. For example, the CEMDAP microsimulation framework (Bhat et al., 2004) shown in Pinjari and Bhat (2011) can be modified into a hybrid framework, which we here call Comprehensive Econometric Microsimulator with Household Activity Pattern Problem (CEMHAPP), and shown in Fig. 5.

In this system, Box 1 and Box 2 represent the conventional CEMDAP model systems. A HAPP model is added on as a normative instrumental method. This model is estimated using the same sample data set or subset to derive the HAPP coefficients for each household. When the system is used to forecast scenarios that feature new constraints with no available data, e.g. evaluating alternative fuel vehicle usage with newly located refueling stations, the scenario can be added as constraints to HAPP with the estimated parameters. Running HAPP would infer the routing and scheduling choices from the sample in response to the scenario with spatio-temporal changes (e.g. requiring vehicles to visit a fueling station), for which Box 2 now has instrumented route and schedule choices to forecast scheduling features. If scenarios do not require analyzing new

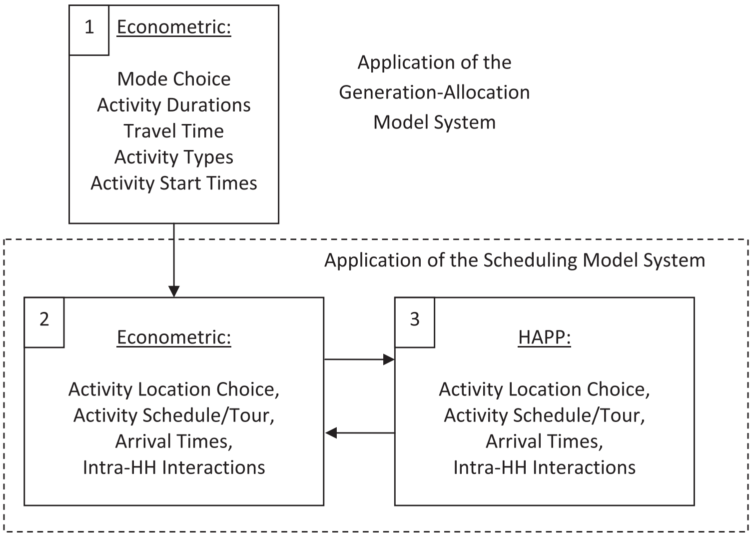

Fig. 5. Conceptual framework for CEMHAPP.

constraints, then CEMHAPP reverts to CEMDAP. Clearly this approach of integrating HAPP with econometric models can also be extended to computational process models and agent-based models.

6. Conclusion

A new inverse household activity pattern problem is proposed that jointly estimates objective coefficients and goal arrival times for a given Household Travel Survey sample. The formulation exploits the structure of the vehicle routing problem to avoid a bi-level approach, allowing the use of existing solution algorithms in inverse optimization to be used. Consistency is achieved by using the Method of Successive Averages to derive an invariant common prior for the objective parameters so that posterior inverse solutions would result in the same distribution as the prior. Numerical tests of the inverse model show a nearly 100% fit in matching the scheduling of activities to observed patterns for a number of samples obtained from the 2001 California Household Travel Survey. The numerical test also shows that the mean parameters estimated from the inverse model for the identified objectives do not perform well as a forecasting tool. Instead, other uses of the model are conceptually explored, primarily as a disaggregate, activity-based analog of the traffic assignment problem for activity-based travel forecasting. Revisiting earlier studies conducted with the model indicate that the ability to consistently estimate parameters can improve the analysis along a number of dimensions. Its real strength, though, potentially comes from the opportunity to integrate with existing state-of-the-art models. As an add-on, the HAPP model can be used to infer a sample population's rational responses to new spatio-temporal constraints for which data might not be readily available. This provides rational route and schedule choices from which the state-of-the-art models can extrapolate to forecast travel behavior in those scenarios.

Like with econometric models, proper identification of objectives for use in the HAPP model may improve the accuracy. Basic objectives of minimizing travel time, delay from return home, length of time outside home per vehicle, and arrival time penalties alone may not be the most significant factors in the household activity scheduling process. Other objectives can be added and calibrated to determine whether they are significant. Insignificant results would tend toward zero relative to the other objectives. Empirical studies should be conducted to compare the effectiveness of objective identification for the HAPP model. In addition, the HAPP model with calibration does not include unobserved error in the objective function. Future research should consider uncertainty in routing and scheduling for each household. One such ongoing research examines fuzzy arrival time constraints. Currently the calibrated HAPP model only addresses route and schedule choice, but it is also possible to incorporate destination choice by modifying the problem to become a selective routing problem or multicommodity profitable tour problem (Feillet et al., 2005; Chow and Liu, 2012). Incorporating needs fulfillment over a long term period (Arentze and Timmermans, 2008) can further integrate the proposed model with existing activity-based models. Due to the NP-hard nature of the HAPP model, VRP heuristics need to be identified for solving the HAPP and inverse formulation without sacrificing the optimality. Having done so, more comprehensive empirical studies should be conducted to validate the model's consistency for the same household samples over multiple scenarios when it relies on heuristics to solve.

Potential applications of the methods proposed here are not limited to activity-based modeling. This work is also purported to be significant in providing a basis for evaluating incomplete data for freight transport. For example, in urban truck activities it is typically necessary to obtain GPS data along with a full truck diary (Hunt and Stefan, 2007). If a HAPP-type model were used to infer some of the necessary behavior it may be possible to build an urban truck distribution model using only GPS data. That work is already underway.

Acknowledgments

The partial support of the University of California Multi-Campus Research Program and Initiative (MRPI) on Sustainable Transportation is acknowledged. Helpful comments from two anonymous reviewers are much appreciated. Any errors and opinions expressed are solely those of the authors.

Appendix A

Basic HAPP formulation of Recker (1995)

Notation: Following Solomon and Desrosiers (1988) the following notation is used:

$A = \{1,2,\ldots,i,\ldots,n\}$	set of out-of-home activities scheduled to be completed by travelers in the household
$V = \{1,2,\ldots,\upsilon,\ldots,\lvert V\rvert\}$	set of vehicles used by travelers in the household to complete their scheduled activities
$P^{+} = \{1,2,\ldots,i\ldots,n)$	set designating location at which each activity is performed
$P^{-} = \{n+1, n+2,\ldots,n+i,\ldots,2n\}$	set designating the ultimate destination of the "return to home" trip for each activity. (It is noted that the physical location of each element of P^{-} is "home".)
$[a_i, b_i]$	time window of available start times for activity i. (Note: b_i must precede the closing of the availability of activity i by an amount equal to or greater than the duration of the activity.)
$[a_{n+i}, b_{n+i}]$	time windows for the "return home" arrival from activity i
$[a_0, b_0]$	departure window for the beginning of the travel day
$[a_{2n+i}, b_{2n+i}]$	arrival window by which time all members of the household must complete their travel
s_i	duration of activity i
t_{uw}	travel time from the location of activity u to the location of activity w
c_{uw}^{υ}	travel cost from location of activity u to the location of activity w by vehicle υ
B_c	household travel cost budget
B_t^{υ}	travel time budget for the household member using vehicle υ
$P = P^{+} \cup P^{-}$	set of nodes comprising completion of the household's scheduled activities
$N = \{0, P, 2n+1\}$	set of all nodes, including those associated with the initial departure and final return to home
$X_{uw}^{\upsilon}, u, w \in N, \upsilon \in V, u \neq w$	binary decision variable equal to unity if vehicle υ travels from activity u to activity w, and zero otherwise
$T_u,\ u \in P$	time at which participation in activity u begins
$T_o^{\upsilon}, T_{2n+1}^{\upsilon}, \upsilon \in V$	times at which vehicle first departs from home and last returns to home, respectively
$Y_u,\ u \in P$	total accumulation of either sojourns or time (depending on the selection of D and d_u) on a particular tour immediately following completion of activity u

HAPP model

$$\textbf{Minimize Z} = \text{Household Travel Disutility} \tag{A.1}$$

subject to:

$$\sum_{\upsilon\in V}\sum_{w\in N} X_{uw}^{\upsilon} = 1, \quad u \in P^{+} \tag{A.2}$$

$$\sum_{w\in N} X_{uw}^{\upsilon} - \sum_{w\in N} X_{wu}^{\upsilon} = 0 \quad u \in P, \quad \upsilon \in V \tag{A.3}$$

$$\sum_{w\in P^{+}} X_{0w}^{\upsilon} = 1, \quad \upsilon \in V \tag{A.4}$$

$$\sum_{u\in P^{-}} X_{u,2n+1}^{\upsilon} = 1, \quad \upsilon \in V \tag{A.5}$$

$$\sum_{w\in N} X_{wu}^{\upsilon} - \sum_{w\in N} X_{w,n+u}^{\upsilon} = 0 \quad u \in P^{+}, \quad \upsilon \in V \tag{A.6}$$

$$T_u + s_u + t_{u,n+u} \leqslant T_{n+u} \quad u \in P^{+} \tag{A.7}$$

$$X_{uw}^{\upsilon} = 1 \Rightarrow T_u + s_u + t_{uw} \leqslant T_w, \quad u, w \in P, \quad \upsilon \in V \tag{A.8}$$

$$X_{0w}^{\upsilon} = 1 \Rightarrow T_0^{\upsilon} + t_{0w} \leqslant T_w, w \in P^+, \quad \upsilon \in V \tag{A.9}$$

$$X_{u,2n+1}^{\upsilon} = 1 \Rightarrow T_u + s_u + t_{u,2n+1} \leqslant T_{2n+1}^{\upsilon}, \quad u \in P^-, \quad \upsilon \in V \tag{A.10}$$

$$a_u \leqslant T_u \leqslant b_u, \quad u \in P \tag{A.11}$$

$$a_0 \leqslant T_0^{\upsilon} \leqslant b_0, \quad \upsilon \in V \tag{A.12}$$

$$a_{2n+1} \leqslant T_{2n+1}^{\upsilon} \leqslant b_{2n+1}, \quad \upsilon \in V \tag{A.13}$$

$$X_{uw}^{\upsilon} = 1 \Rightarrow Y_u + d_w = Y_w u \in P, \quad w \in P^+ \quad \upsilon \in V \tag{A.14}$$

$$X_{uw}^{\upsilon} = 1 \Rightarrow Y_u - d_{w-n} = Y_w u \in P, \quad w \in P^- \quad \upsilon \in V \tag{A.15}$$

$$X_{0w}^{\upsilon} = 1 \Rightarrow Y_0 + d_w = Y_w, \quad w \in P^+, \quad \upsilon \in V \tag{A.16}$$

$$Y_0 = 0, \quad 0 \leqslant Y_u \leqslant D, \quad u \in P^+ \tag{A.17}$$

$$X_{uw}^{\upsilon} = \begin{cases} 0 \\ 1 \end{cases} ; u, \quad w \in N, \quad \upsilon \in V \tag{A.18}$$

$$\sum_{\upsilon \in V} \sum_{u \in N} \sum_{w \in N} c_{uw}^{\upsilon} X_{uw}^{\upsilon} \leqslant B_c \tag{A.19}$$

$$\sum_{u \in N} \sum_{w \in N} t_{uw} X_{uw}^{\upsilon} \leqslant B_t^{\upsilon}, \quad \upsilon \in V \tag{A.20}$$

$$\sum_{w \in P^-} X_{0,w}^{\upsilon} = 0, \quad \upsilon \in V \tag{A.21}$$

$$\sum_{u \in N} X_{u,0}^{\upsilon} = 0, \quad \upsilon \in V \tag{A.22}$$

$$\sum_{u \in P^+} X_{u,2n+1}^{\upsilon} = 0, \quad \upsilon \in V \tag{A.23}$$

$$\sum_{w \in P^-} X_{2n+1,w}^{\upsilon} = 0, \quad \upsilon \in V \tag{A.24}$$

References

Ahuja, R.K., Orlin, J.B., 2001. Inverse optimization. Operations Research 49 (5), 771–783.
Arentze, T.A., Timmermans, H.J.P., 2008. A need-based model of multi-day, multi-person activity generation. Transportation Research Part B 43 (2), 251–265.
Bar-Gera, H., Boyce, D., 2006. Solving a non-convex combined travel forecasting model by the method of successive averages with constant step sizes. Transportation Research Part B 40 (5), 351–367.
Becker, G.S., 1965. A theory of the allocation of time. The Economical Journal 75 (299), 493–517.
Bhat, C.R., 2005. A multiple discrete-continuous extreme value model: formulation and application to discretionary time-use decisions. Transportation Research Part B 39 (8), 679–707.
Bhat, C.R., 2008. The multiple discrete-continuous extreme value (MDCEV) model: role of utility function parameters, identification considerations, and model extensions. Transportation Research Part B 42 (3), 274–303.
Bhat, C.R., Guo, J.Y., Srinivasan, S., Sivakumar, A., 2004. A comprehensive econometric microsimulator for daily activity-travel patterns. Transportation Research Record 1894, 57–66.
Burton, D., Toint, P.L., 1992. On an instance of the inverse shortest paths problem. Mathematical Programming 53 (1-3), 45–61.
Calvete, H.I., Galé, C., Oliveros, M.J., Sánchez-Valverde, B., 2007. A goal programming approach to vehicle routing problems with soft time windows. European Journal of Operational Research 177 (3), 1720–1733.
Chow, J.Y.J., Liu, H., 2012. Multicommodity profitable tour problems for an online activity routing system. In: Proceedings of the 91st Annual Meeting of the Transportation Research Board.
Csiszár, I., 1991. Why least squares and maximum entropy? An axiomatic approach to inference for linear inverse problems. The Annals of Statistics 19 (4), 2032–2066.
Dempe, S., Lohse, S., 2006. Inverse linear programming. Lecture Notes in Economics and Mathematical Systems 563 (Part I), 19–28.
Evans, A.W., 1972. On the theory of the valuation and allocation of time. Scottish Journal of Political Economy 19 (1), 1–17.
Feillet, D., Dejax, P., Gendreau, M., 2005. Traveling salesman problems with profits. Transportation Science 39 (2), 188–205.
Feinberg, Y., 2000. Characterizing common priors in the form of posteriors. Journal of Economic Theory 91 (2), 127–179.
Harsanyi, J.C., 1967. Games with incomplete information played by "Bayesian" players, I–III Part I. The basic model. Management Science 14 (3), 159–182.
Hunt, J.D., Stefan, K.J., 2007. Tour-based microsimulation of urban commercial movements. Transportation Research Part B 41 (9), 981–1013.

Jara-Díaz, S.R., 2003. On the goods-activities technical relations in the time allocation theory. Transportation 30 (3), 245–260.
Kim, H.K., 2008. Activity-based travel demand model with time-use and microsimulation incorporating intra-household interactions. PhD dissertation, University of California, Irvine, 235pp.
Kitamura, R., 1988. An evaluation of activity-based travel analysis. Transportation 15 (1–2), 9–34.
Lam, W.H.K., Yin, Y., 2001. An activity-based time-dependent traffic assignment model. Transportation Research Part B 35 (6), 549–574.
Lee, M.S., McNally, M.G., 2003. On the structure of weekly activity/travel patterns. Transportation Research Part A 37 (10), 823–839.
Maher, M.J., 1983. Inferences on trip matrices from observations on link volumes: a Bayesian statistical approach. Transportation Research Part B 17 (6), 435–447.
Maréchal, P., Lannes, A., 1996. Unification of some deterministic and probabilistic methods for the solution of linear inverse problems via the principle of maximum entropy on the mean. Inverse Problems 13 (1), 135–151.
Miller, E.J., Roorda, M.J., 2003. A prototype model of household activity/travel scheduling. Transportation Research Record: Journal of the Transportation Research Board 1831, 114–121.
Pinjari, A.R., Bhat, C.R., 2011. In: de Palma, A., Lindsey, R., Quinet, E., Vickerman, R. (Eds.), Activity-Based Travel Demand Analysis. Handbook of Transport Economics. Edward Elgar Publishing.
Ramadurai, G., Ukkusuri, S., 2010. Dynamic user equilibrium model for combined activity-travel choices using activity-travel supernetwork representation. Networks and Spatial Economics 10 (2), 273–292.
Recker, W.W., 1995. The household activity pattern problem: general formulation and solution. Transportation Research Part B 29 (1), 61–77.
Recker, W.W., 2001. A bridge between travel demand modeling and activity-based travel analysis. Transportation Research Part B 35 (5), 481–506.
Recker, W.W., Parimi, A., 1999. Development of a microscopic activity-based framework for analyzing the potential impacts of transportation control measures on vehicle emissions. Transportation Research Part D 4 (6), 357–378.
Recker, W.W., Duan, J., Wang, H., 2008. Development of an estimation procedure for an activity-based travel demand model. Computer-Aided Civil and Infrastructure Engineering 23 (7), 483–501.
Small, K.A., 1982. The scheduling of consumer activities: work trips. American Economic Association 72 (3), 467–479.
Solomon, M.M., Desrosiers, J., 1988. Time window constrained routing and scheduling problems. Transportation Science 22 (1), 1–13.
Tarantola, A., 2005. Inverse Problem Theory and Methods for Model Parameter Estimation. Society for Industrial and Applied Mathematics, Philadelphia, PA, 342pp.
Tenorio, L., 2001. Statistical regularization of inverse problems. SIAM Review 43 (2), 347–366.
Timmermans, H., Arentze, T., Joh, C.H., 2002. Analysing space–time behavior: new approaches to old problems. Progress in Human Geography 26 (2), 175–190.
Toth, P., Vigo, D., 2002. Preface. The Vehicle Routing Problem. Society for Industrial and Applied Mathematics, Philadelphia, PA.
Wang, L., 2009. Cutting plane algorithms for the inverse mixed integer linear programming problem. Operations Research Letters 37 (2), 114–116.
Williams, H.P., 1996. Duality in mathematics and linear and integer programming. Journal of Optimization Theory and Applications 90 (2), 257–278.